Functions of one variable. Integration

1. Primitive function, indefinite integral. Main properties.

$(\int f(x)dx)' = f(x)$, $d\int f(x)dx = f(x)dx$, $\int dF(x) = F(x) + C$;

$\int \alpha f(x)dx = \alpha\int f(x)dx$, $\int[f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$.

Table of integrals:

1. $\int 0\cdot dx = C$; $\int dx = x + C$;
2. $\int x^{\alpha} dx = x^{\alpha+1}/(\alpha + 1) + C$ $(\alpha \neq -1)$;
3. $\int x^{-1} dx = \ln|x| + C$ $(x \neq 0)$;
4. $\int a^{x}dx = a^{x}/\ln a + C$ $(0 < a \neq 1)$, $\int e^{x}dx = e^{x} + C$;
5. $\int \sin x\, dx = -\cos x + C$; $\int \cos x\, dx = \sin x + C$;
6. $\int\frac{dx}{\cos^2 x} = \mathrm{tg}\, x + C$; $\int\frac{dx}{\sin^2 x} = -\mathrm{ctg}\, x + C$;
7. $\int\frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$, $\int\frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$;
8. $\int\frac{dx}{1+x^2} = \mathrm{arctg}\, x + C$, $\int\frac{dx}{a^2+x^2} = \frac{1}{a}\mathrm{arctg}\frac{x}{a} + C$;
9. $\int\frac{dx}{\sqrt{x^2+a}} = \ln|x + \sqrt{x^2+a}| + C$;
10. $\int\frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$.

2. 3 main integration methods:

the direct table implementation;

the change of the integration variable:

$$\int f(x)dx\,|_{x = \varphi(t)} = \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt,$$

or in the opposite direction

$$\int f(\varphi(t))\varphi'(t)dt\,|_{t = \psi(x)} = \int f(x)dx$$

where ψ is the function inverse to φ : $t = \psi(x) \Leftrightarrow x = \varphi(t)$;

integration in parts:

$$\int udv = uv - \int vdu\ , \qquad \int u(x)\, v'(x)\, dx = u(x)v(x) - \int v(x)\, u'(x)dx\ .$$

3. Integration of rational functions and trigonometric polinomials.

4. Definite integrals (Rieman). Definition, geometrical and physical sense.

Main properties:

$$\int_a^a f(x)dx = 0; \quad \int_a^b f(x)dx = -\int_b^a f(x)dx, \quad \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;$$

$$\int_a^b \alpha f(x)dx = \alpha\int_a^b f(x)dx; \quad \int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx.$$

Mean value theorem: $\int_a^b f(x)dx = f(c)(b - a), \ c \in (a, b)$.

Comparison: $a \le b, \ f(x) \le g(x) \ \Rightarrow \ \int_a^b f(x)dx \le \int_a^b g(x)dx$

in particular: $\int_a^b f(x)dx \le \int_a^b |f(x)|dx$

Newton-Leibnitz formula: $F'(x) = f(x) \Rightarrow \int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_{x=a}^{x=b}$

Change of the integration variable $x = \varphi(t)$:

$$a = \varphi(\alpha), b = \varphi(\beta) \ \Rightarrow \ \int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt$$

Integration in parts. Calculation of definite integrals.