Functions of one variable. Differentiation

1. Function notion, function graph, boundedness. Monotone, inverse functions, even, non-even functions. Sequence limit, function limit, infinitesimals.

2. Continuity. Theorems on continuity: $f+g$, fg, f/g, cf, $f(g)$. Continuity of the elementary functions. Intermediate value theorem. The function inverse to the monotone one. The existence of the maximum and minimum of any continuous function in any closed segment.

3. Derivative - definition, the geometrical and phys. senses. Tangent line. Differential. Differentiation of the composite functions $f(g)$, the chain rule.. Inverse function and its derivative. The differentiation table. Invariance of the form of the first-order differential. High order derivatives and differentials.

4. Main theorems: Fermat, Roll, Lagrange. Increasing, decreasing regions. Local extremum. The rule of L'Hospital. Convexity. Asymptotes. Function study. The Taylor and Maclourin formulas.

$$(const)' = 0;$$

$$(x^{\alpha})' = \alpha\, x^{\alpha-1}; \quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}; \quad \left(\frac{1}{x}\right)' = -\frac{1}{x^2};$$

$$(\sin x)' = \cos x; \qquad (\cos x)' = -\sin x;$$

$$(\mathrm{tg}\, x)' = \frac{1}{\cos^2 x}; \qquad (\mathrm{ctg}\, x)' = -\frac{1}{\sin^2 x};$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}; \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}};$$

$$(\mathrm{arctg}\, x)' = \frac{1}{1+x^2}; \qquad (\mathrm{arcctg}\, x)' = -\frac{1}{1+x^2};$$

$$(a^x)' = \ln a \cdot a^x, \qquad (e^x)' = e^x;$$

$$(\log_a x)' = \frac{1}{\ln a \cdot x}, \quad (\ln x)' = \frac{1}{x}.$$

$$(Cu)' = Cu', \; C = const$$

$$(u + v)' = u' + v' \qquad (u \cdot v)' = u' \cdot v + u \cdot v' \quad (u/v)' = (u' \cdot v - u \cdot v')/v^2$$

$$(g(f(x))' = g'(f(x))\, f'(x) \qquad (f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$$

The Taylor formula:

$$f(x_0+\Delta x) = f(x_0) + f'(x_0)\frac{\Delta x}{1!} + f''(x_0)\frac{\Delta x^2}{2!} + ... + f^{(n)}(x_0)\frac{\Delta x^n}{n!} + R_n(x),$$

$$R_n(x) = f(x_0+\theta\Delta x)\frac{\Delta x^{n+1}}{(n+1)!},\ \theta\in(0,1).$$

Some useful Taylor (Maclourin) formulas:

$$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots+\frac{x^n}{n!}+R_n(x),\ R_n(x) = e^{\xi}\frac{x^{n+1}}{(n+1)!},\ \ \xi = \theta x,\ \theta\in(0,1)$$

$$\sin x = \frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+R_{2n+2}(x),\ \ R_{2n+2}(x) = c\frac{x^{2n+3}}{(2n+3)!},$$

$$|c| \le 1$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}-\ldots+(-1)^n\frac{x^{2n}}{(2n)!}+R_{2n+1}(x),\ \ R_{2n+1}(x) = c\frac{x^{2n+2}}{(2n+2)!},$$

$$|c| \le 1$$

$$\ln(1+x) = \frac{x}{1}-\frac{x^2}{2}+\frac{x^3}{3}-\ldots+(-1)^{n+1}\frac{x^n}{n}+R_n(x),\quad R_n(x) = \frac{x^{n+1}}{(1+\xi)^{n+1}(n+1)},$$

$$x\in(-1,\infty),\ \xi = \theta x,\ \theta\in(0,1)$$

$$(1+x)^{\alpha} = 1+\alpha x+\frac{1}{2}\alpha(\alpha-1)x^2+\ldots+\frac{1}{n!}\alpha(\alpha-1)\ldots(\alpha-n+1)x^n+R_n(x),$$

$$R_n(x) = \frac{(1+\xi)^{\alpha-n-1}}{(n+1)!}\alpha(\alpha-1)\ldots(\alpha-n)x^{n+1},\ x\in(-1,\infty),\ \xi = \theta x,\ \theta\in(0,1)$$