

## Problem 5

The system to be stabilized is a third-order disturbed integrator,

$$\begin{aligned} \ddot{\sigma} &= h_0(t, \bar{\sigma}_2) \|\bar{\sigma}_2\|_h^{1+3q} + g(t, \bar{\sigma}_2)u, \quad \sigma, u \in \mathbb{R}, \\ \bar{\sigma}_2 &= (\sigma, \dot{\sigma}, \ddot{\sigma}), \quad |h_0(t, \bar{\sigma}_2)| \leq C, \quad 0 < K_m \leq g(t, \bar{\sigma}_2) \leq K_M, \end{aligned} \quad (1)$$

where  $\|\bar{\sigma}_2\|_h$  is a homogeneous norm corresponding to the weights

$$\deg \sigma = 1, \quad \deg \dot{\sigma} = 1 + q, \quad \deg \ddot{\sigma} = 1 + 2q, \quad q \geq -1/3,$$

and  $h_0, g: \mathbb{R}^4 \rightarrow \mathbb{R}$  are some Lebesgue-measurable functions,  $t \geq 0$ . The functions  $h_0, g, \|\cdot\|_h$  are concretized further.

The task is as follows:

1. To develop a universal homogeneous control template  $u = \alpha U_3(\bar{\sigma}_2, \beta, q)$ ,  $\beta \in \mathbb{R}^k, \alpha \in \mathbb{R}$ , globally asymptotically stabilizing system for each  $q$  for some properly chosen parameters  $\alpha, \beta$  and the whole state  $\bar{\sigma}_2(t)$  available. The  $\beta$  dimension  $k$  is chosen by the student.
2. To find appropriate values of  $\alpha, \beta$  for  $q = -1/3$  by simulation. Two-three initial values of the form  $\bar{\sigma}_2(0) = (\pm 2, \pm 2, \pm 2)$  are to be taken.
3. To apply a homogeneous output-feedback control including a filtering differentiator of the differentiation order  $n_d = 2$  and the filtering order  $n_f \geq 5$ . To show the exact finite-time stabilization for only  $\sigma(t)$  available by simulation, zero differentiator initial values and **some**  $\bar{\sigma}_2(0) = (\pm 2, \pm 2, \pm 2)$ .
4. To show the robustness of the system with respect to some large noise in sampled  $\sigma(t)$  (random noise of the magnitude  $\sim 0.1$ , high-frequency noise  $A \cos(\omega t)$  for  $\omega \sim 10^5$  and  $A \sim 10^3$  or even larger).
5. The integration is performed by the Euler method with the sampling/integration step  $\tau \leq 10^{-4}$ . In the presence of sampling noises, if the noise contains a random component,  $\tau \leq 10^{-6}$  is to be taken for better results. Harmonic components  $A \cos(\omega t)$ ,  $A, \omega \gg 1$ , usually require  $\tau \leq 10^{-5}$ .

The report should contain the template development, parametric values, essential graphs and the maximal values of  $|\sigma|, |\dot{\sigma}|, |\ddot{\sigma}|$  over the last 1/3 of the simulation interval for  $\tau = 10^{-4}, 10^{-5}$ .

The functions from (1) are as follows:

$$h_0 = \frac{\sigma^3 - 2\dot{\sigma} \operatorname{sign} \ddot{\sigma} + \ddot{\sigma} \operatorname{sign} \sigma}{|\sigma^3| + 2|\dot{\sigma}| + |\ddot{\sigma}|} \sin(100t), \quad g = 2 - \frac{(\sigma \operatorname{sign} \dot{\sigma} - 2\dot{\sigma} + \ddot{\sigma}^3) \cos(60t)}{|\sigma| + 2|\dot{\sigma}| + |\ddot{\sigma}^3|}, \quad \|\bar{\sigma}_2\|_h = [|\sigma|^{\frac{1}{2}} + |\dot{\sigma}|^{\frac{1}{2(1+q)}} + |\ddot{\sigma}|^{\frac{1}{2(1+2q)}}]^2$$

**Essential information appears in the document**

[https://www.tau.ac.il/~levant/Levant-SMC2019\\_for\\_students.pdf](https://www.tau.ac.il/~levant/Levant-SMC2019_for_students.pdf)

Any questions are welcome.

בהצלחה!