## Methods of Applied Mathematics - 1 0366-4553-01 Tentative Syllabus

- Differential and derivative. Ordinary differential equations. Phase space. Extended phase space. Theorems on the iniquity and existence of solutions. Solution extension up to the region boundary. Autonomous and non-autonomous cases.
- Dependence on the parameters and initial conditions. Local expansion of the solution with respect to parameter. Linearization.
- First integrals. Newton equation. Periodic solutions (cycles) in the plane.
- Matrix exponent and solution of linear differential equations. Abel-Liouville theorem and its application.
- Critical points of differential equations. Classification of critical points in  $\mathbf{R}^2$  and  $\mathbf{R}^3$ .
- \* Poincare-Bendixson theorems. Index of a critical point in the plane.
- Comparison theorems for the first-order differential equations.
- Comparison theorems for the second-order linear differential equations. Theorems by Sturm on zeros of the solutions.
- Lyapunov stability of a solution and of a critical point. Asymptotic stability. Lyapunov function. Theorems on stability and instability. Stability criteria by Rauss Hurwitz.
- Construction of a Lyapunov function for linear autonomous systems. Stability of a critical point according to the linear approximation of the equation. Solution evaluation of disturbed stable systems.
- Solution of differential equations by means of power series. Local evaluation of the solutions.
- \* Euler differential equations. Singular points of linear differential equation. Regular singular points. Frobenius method of solving differential equations by means of power series in vicinity of a regular singular point.
- Boundary value problem. Its standard form. Green function for non-homogeneous regular boundary value problems
- Self-adjoint boundary-value problems. Sturm-Liouville problem. Systems of eugenfunctions, their eugenvalues. Ortogonality and completeness.
- Fourier series of eigenfunctions, solution of non-homogeneous boundary-value problems.

*The syllabus is subject to change without preliminary announcement.* The topics marked by \* are likely to be omitted.