AIAA Journal of Guidance, Control and Dynamics, 23(4), 2000, 586-594

Aircraft Pitch Control Via Second Order Sliding Technique

A. Levant and A. Pridor
Institute for Industrial Mathematics, Beer-Sheva, Israel

R. Gitizadeh and I. Yaesh
Israel Military Industries, Ramat-Hasharon, Israel

J.Z. Ben-Asher
Technion IIT , Haifa 32000, Israel

Abstract. Control of high-performance low-cost unmanned air vehicles involves the problems of incomplete measurements, external disturbances and modeling uncertainties. Sliding mode control combines high precision with robustness to the aforementioned factors. The idea behind this approach is the choice of a particular constraint which, when maintained, will provide the process with the required features and remove, therefore, the plant's uncertainty. However, standard sliding modes are characterized by a high-frequency switching of control, which causes problems in practical applications (so-called chattering effect). A second order sliding controller implemented in the present paper features bounded continuously time-dependent control and provides higher accuracy than the standard sliding mode, while preserving precise constraint fulfillment within a finite time. It possesses, also, significant adaptive properties. The general approach is demonstrated by solving a real-life pitch control problem. Results of a computer simulation and flight tests are presented.

I. Introduction

Aircraft and missile systems are equipped with control systems whose tasks are to provide stability, disturbance attenuation and reference signal tracking, while their aerodynamic coefficients vary over a wide dynamic range due to large Mach-altitude fluctuations and due to aerodynamic coefficient uncertainties resulting from inaccurate wind tunnel measurements.

It is common practice, when designing a control system for an unmanned air vehicle (UAV), to represent the flight envelope by a grid of Mach-altitude operating points, and then to perform a linearization of the nonlinear state equations at the equilibrium points (so-called "trim" points) of the gridded flight envelope. The plant in fact becomes a differential inclusion (see Boyd *et al.* 1994) under continuously varying flight conditions. There are many possible ways of dealing with the control of such linear time-varying plants, the classical approach being to design a controller for a mid-envelope point and then to schedule the controller's gain according to a measured (or derived) parameter which represents flight conditions, such as dynamic pressure (McRuer, Ashkenas and Graham 1973). Gain scheduling is aimed at keeping one feature of the closed loop (e.g. natural frequency of the dominant poles) approximately constant throughout the flight envelope. In another method, H_∞ methods are invoked to design a collection of full (Doyle *et al.* 1989) or reduced order controllers (e.g. Peres *et al.* 1994, Yaesh & Shaked 1995), where, for each operating point in the flight envelope grid, a controller with a fixed structure results. The ensuing set of controllers is then transformed into a single gain-scheduled controller by obtaining a least squares fit of its parameters with respect to dynamic pressure, or Mach number, etc. A more systematic treatment results when it is assumed that the underlying differential inclusion resides within a convex polytopic, where a few extreme operating points in the flight envelope are selected as the vertices of a convex hull, and all other possible operating points are represented as convex combinations of these vertex points. A gain-scheduled controller is then designed using LMI (linear matrix inequalities) methods (Gahinet & Apkarian 1995).

All of the above-mentioned methods are linear design techniques and require either exact knowledge of system parameters, or, alternatively, assumption of some uncertainty model such as norm boundedness or polytopic uncertainties, thus allowing for robust controller design. Most of the currently available methods (e.g. Boyd et al. 1994) do not combine robustness (with respect to structural uncertainties) and gain scheduling. One possible approach is adaptive control design which includes some form of observation and parameter identification, and assumes that the uncertainty is inherent in a few parameters that are constant or slowly varying. Other approaches use dynamic inversion and feedback linearization (Brockett 1978). All of the above approaches require at least approximate knowledge of some parameters. On the other hand, the variable structure system (VSS) method may, in principle, be implemented for dynamic systems having only a qualitative description and a number of inequality restrictions. As will be shown in the sequel, this approach will provide a high-performance controller which is both gain-scheduled and robust, and uses a rather straightforward design procedure.

Sliding modes are the primary form of operation of VSSs. A sliding mode is a motion on a discontinuity set of a dynamic system and is characterized by a theoretically-infinite switching frequency. Such modes are used to maintain the given constraints with utmost accuracy, and are known for their robustness with respect to both external and internal disturbances. Unfortunately, standard sliding modes also feature high-frequency switching of input signals (controls). As the switching frequency tends to infinity, sliding mode motion trajectories approach trajectories of a smooth system described by a formal substitution of a smooth equivalent control (Utkin 1977, 1992) for the discontinuous control signal. However, such a mode might be unacceptable if the control signal has some physical significance, such as angular position or force. Indeed, high frequency switching may be destructive for end effectors or may cause system resonances. The accompanying (sometimes dangerous) vibrations are termed “chattering”.

The higher the order of an output variable derivative where the high frequency discontinuity first appears, the less visible the vibrations of the variable itself will be. Thus, the way to avoid chattering is to move the switching to the higher order derivatives of the control signal. The problem is how to preserve the main feature of sliding modes: exact maintenance of constraints under conditions of uncertainty. Such sliding modes were discovered and termed “higher order sliding modes” (HOSM) (Levantovsky 1985, Emelyanov et al. 1986 a,b,c, Chang 1990, Levant (Levantovsky) 1993, Fridman and Levant 1996, Bartolini *et al.* 1997a,b). The constraint to be kept being given by an equality of some output variable to zero, the order of the sliding mode is the order of the first discontinuous total time derivative of that variable. These sliding modes may attract trajectories in finite time like the standard ones or may be asymptotically stable. Being removed to the higher derivatives of the control, the switching is no longer dangerous, since it takes place within the inner circuits of the control system (mostly in a computer) and not within the actuator. HOSM may provide for up-to-its-order precision with respect to the measurement time step, as compared to the standard (first order) sliding mode whose precision is proportional to the measurement step.

A practical application of a second order sliding mode with finite time convergence is demonstrated in the present paper for the first time. The problem considered is to ensure the tracking of the pitch angle of a flight platform to some external signal given in real time while the platform is subject to unmeasured external disturbances. Some delay and noise are also present in the measurements, and the system contains an actuator whose behavior exhibits both delay and discretization effects. A new controller is implemented, which is a special hybrid modification of the earlier controllers by Levant (1993).

II. Variable Structure Systems: A Brief Description

The VSS Concept. The idea of a VSS is illustrated here by a simple example. Consider a dynamic system of the form:

$$\ddot{y} = g(t, y, \dot{y}) + 3u, \tag{1}$$

where $y \in \mathbf{R}$, and $u \in \mathbf{R}$ is the control. States y and $\dot{y}$ are available as measurements. The problem is to stabilize the system at the origin 0. If g were known, the problem would be easy, otherwise it is rather difficult. Let g be some unknown but bounded measurable function, $|g|<1$.

The problem could be solved if we could fulfill and precisely maintain the constraint condition $\sigma = y + \dot{y} = 0$. This may be done by VSS control of the type:

$$u = - \text{ sign } \sigma \tag{2}$$

or by

$$u = -\sqrt{1+\dot{y}^2}\ \mathrm{sign}\ \sigma. \quad (3)$$

Indeed, $u = -\sqrt{1+\dot{y}^2}$ sign σ allows for the control to be predominant in the expression for $\dot{\sigma}$,

$$\dot{\sigma} = \dot{y} + \ddot{y} = \dot{y} + g(t, y, \dot{y}) + 3u, \quad (4)$$

and this in turn leads to the inequalities $\sigma\dot{\sigma} \leq 0$, $|\dot{\sigma}| > 1$, thus implying finite-time convergence of σ to 0. The control law $u = -$ sign σ , on the other hand, provides only for local convergence within some vicinity of 0 where $|\dot{y}| < 1$. The resulting closed loop dynamic system is discontinuous and cannot be analyzed conventionally, since the motion on the line σ = 0 is indeterminate.

A special theory of such differential equations was developed by Filippov (1988) in the early 60s. The phase portrait of the closed loop system is shown in Fig. 1. After the system's trajectory eventually impinges upon the constraint surface σ = 0, the constraint σ = 0 is kept here in so-called sliding mode which is further referred to as "standard". The motion in the sliding mode is provided by "infinite-frequency switching" control and has to be understood as a limiting motion which is attained with gradual disappearance of various switching imperfections such as hysteresis or switching delays (Filippov 1988).

If the function g were known *a-priori*, the control law $u = -(\dot{y} + g(t, y, \dot{y}))/3$ found from (4) would ensure that the system remains on the sliding surface σ = 0 once it has reached it. Such a control is called "equivalent control" and is denoted by u_{eq}. The important fact is that, with no relation to a control providing for $\sigma \equiv 0$, the corresponding motion is described by (1) with u_{eq} substituted for u. That proposition was proved by Utkin (1977, 1992) for dynamic systems linearly dependent on control u with u_{eq} uniquely determined from the equation $\dot{\sigma} = 0$ and generalized by Bartolini and Zolezzi (1986) to include some cases of non-linear dependence on control. It was also shown (Utkin 1977, 1992) that in practical applications the average control value tends to the equivalent control.

Hence, the main advantages and disadvantages of the classical VSS methodology are the following:

- Highly precise maintenance of constraints is ensured in the face of heavy uncertainty conditions, which leads to system performance being insensitive to both external and internal disturbances.
- VSS control is characterized by high-frequency switching while in 1-sliding mode, and does not tend to any function of time when switching imperfections vanish and switching frequency tends to infinity. Only its average value tends to some specific smooth function. As a result the controlled system is subject to the "chattering" effect if the control is some physical or mechanical quantity.

The idea of 2-sliding mode. In order to avoid chattering, it was proposed to suitably modify the dynamics within a small vicinity of the discontinuity surface in order to avoid real discontinuities and at the same time preserve the important properties of the system as a whole. A transition to the modified system defined near the switching surface has to be sufficiently smooth. This idea is realized by insertion of a functional unit (Slotine, Sastry 1983) or of an auxiliary dynamic system (Emelyanov, Korovin 1981). In the present paper we are interested in the latter approach.

Let $\sigma = 0$ be the constraint condition to be fulfilled, while our dynamic system is of relative degree 1, thus implying that the control appears explicitly already in the first total time derivative of σ. Hence, control chattering corresponds to chattering in $\dot{\sigma}$ and *vice versa*. The idea is to keep exactly two constraint conditions $\sigma = \dot{\sigma} = 0$ instead of the one originally given, providing simultaneously for continuity of $\dot{\sigma}$. To this end the state space is inflated by the addition of the control variable u as a new coordinate. The total time derivative $\dot{\sigma}$ may now be regarded as a regular continuous function defined on the extended state space. The task is completed by prescribing the appropriate dynamics to the extended state space coordinates. For that purpose the time derivative $\dot{u}$ of the control may, for example, be treated as the new control variable. By maintaining $\sigma + \dot{\sigma} = 0$ the stated auxiliary problem may be solved via the standard VSS approach considered previously. This will lead to the joint fulfillment of the two constraints mentioned above *in infinite time* by means of a continuous control *u(t)*. Attaining the same goal *in finite time* is a more intricate problem. Provided σ and $\dot{\sigma}$ are continuous functions of the closed-loop system state, the motion in the mode $\sigma = \dot{\sigma} = 0$ is called "second order sliding mode" motion. It should be noted that the function $\dot{\sigma}(t,x,u)$ is considered as unknown, and only its current value $\dot{\sigma}(t,x(t),u(t))$ (or some approximation thereof) is available in real time by manipulation of the observed data or measurements.

In similar fashion, sliding modes of arbitrary order r are defined as modes keeping $\sigma = \dot{\sigma} = ... = \overset{(r-1)}{\sigma} = 0$ with continuous σ, $\dot{\sigma}$, ..., $\overset{(r-1)}{\sigma}$ and discontinuous (or undefined) $\overset{(r)}{\sigma}$ (Levantovsky 1985, Emelyanov et al. 1986a, Levant 1993, Fridman and Levant 1996). The terms "*r*th order sliding mode" are further abbreviated to "*r*-sliding mode". With discrete measurements, *r*-sliding modes may provide for sliding precision of up to the *r*th order with respect to the time interval between the measurements (Levant 1993). The most interesting *r*-sliding controllers are those endowed with finite time convergence properties, since only they can provide for the above-mentioned higher order precision and their information handling and storage requirements are usually more modest. Such finite-time convergent algorithms may be constructed for any order (Levant 1998b).

Standard 2-Sliding Controllers. Let the system to be controlled be described by some uncertain dynamic system

$$\dot{x} = f(t,x,u).$$

The aim is to fulfill and keep exactly the constraint $\sigma = 0$ by means of control $u \in \mathbf{R}$ continuously dependent on time. Here $\sigma = \sigma(t,x)$ is an output variable available in real time, and neither f nor dimension of x need to be known. The functions $f(t,x,u)$ and $\sigma(t,x)$ are supposed to be sufficiently smooth.

Assume that the standard (1-sliding) controller $u = -\mathrm{sign}\, \sigma$ provides for the existence of a 1-sliding mode $\sigma = 0$. Thus, it may also be assumed that there are some positive constants $K_m < K_M$, σ_0, C such that the inequalities

$$0 < K_m < \frac{\partial}{\partial u}\dot{\sigma}(t,x,u) < K_M, \quad \left|\frac{\partial \dot{\sigma}}{\partial t}(t,x,u) + \frac{\partial \dot{\sigma}}{\partial x}(t,x,u) f(t,x,u)\right| < C, \; u\dot{\sigma} > 0 \text{ with } |u| \geq 1$$

hold in some region $|\sigma| < \sigma_0$. Here $\dot\sigma(t,x,u) = \sigma'_t(t,x) + \sigma'_x(t,x)f(t,x,u)$. The algorithm to be constructed has to solve the problem for any dynamic system of the class defined by the constants $K_m < K_M$, σ_0, C. It will obviously be robust with respect to any disturbance or uncertainty of the mathematical model which do not stir the system from that class.

The above assumptions are needed in order to ensure the existence of an equivalent control $u_{eq}(t,x)$ defined in the region $|\sigma| < \sigma_0$, satisfying equation $\dot\sigma(t,x,u_{eq}) = 0$. $\dot u_{eq}$ is uniformly bounded by C/K_m, $|u_{eq}| < 1$. As a result this unknown function may be tracked by a bounded Lipschitz control. Any proposed algorithm has to hold the transient trajectories inside the region $|\sigma| < \sigma_0$ where the system possesses "good" properties. The following 2-sliding controllers (Levant 1993) solve the stated problem:

1. $$\dot u = \begin{cases} -u \text{ with } |u| > 1, \\ -\alpha_m \operatorname{sign}\sigma \text{ with } \sigma\dot\sigma \le 0,\ |u| \le 1, \\ -\alpha_M \operatorname{sign}\sigma \text{ with } \sigma\dot\sigma > 0,\ |u| \le 1; \end{cases} \tag{5}$$

2. $$u = u_1 + u_2, \qquad u_2 = \begin{cases} -\lambda\,|\sigma_0|^\rho \operatorname{sign}\sigma, & |\sigma| > \sigma_0, \\ -\lambda\,|\sigma|^\rho \operatorname{sign}\sigma, & |\sigma| \le \sigma_0, \end{cases} \qquad \dot u_1 = \begin{cases} -u, & |u| > 1, \\ -\alpha \operatorname{sign}\sigma, & |u| \le 1, \end{cases} \tag{6}$$

where $\alpha_M > \alpha_m > C/K_m$, $0 < \rho \le 0.5$, λ, α, $\sigma_0 > 0$. A few additional algebraic restrictions (Levant 1993) involving α_M, α_m, ρ, λ, α, C, K_m, K_M can be easily fulfilled with sufficiently large λ, α, α_m, α_M/α_m, and are omitted here. Controllers (5) and (6) are called twisting and super-twisting algorithms respectively. In practice the most convenient way to find the appropriate parameter values is to adjust them during computer simulation, otherwise redundantly large values may be achieved due to the rather rough evaluation technique. Note that control law (5) may be rewritten in a more compact form to be used in the sequel :

$$\dot u = \begin{cases} -u \text{ with } |u| > 1, \\ -r_1 \operatorname{sign}\sigma - r_2 \operatorname{sign}\dot\sigma \le 0,\ |u| \le 1, \end{cases}$$

where $r_1 > r_2 > 0$, $\alpha_M = r_1 + r_2$, $\alpha_m = r_1 - r_2$.

Let $t_0, t_1, t_2, ..., \; t_{i+1}-t_i = \tau > 0$, be the instants in time when σ is measured. Since the exact value of the derivative is not available in practice and only its sign is needed, $\dot\sigma$ is replaced by its first difference $\Delta\sigma_i = \sigma(t_i)-\sigma(t_{i-1})$ thus resulting in

$$\dot u = \begin{cases} -u(t_i), \; |u(t_i)| > 1, \\ -\alpha_M \text{sign}\, \sigma(t_i), \; \sigma(t_i)\Delta\sigma_i > 0, |u(t_i)| \le 1, \\ -\alpha_m \text{sign}\, \sigma(t_i), \; \sigma(t_i)\Delta\sigma_i \le 0, |u(t_i)| \le 1, \end{cases} \tag{7}$$

where $t \in [t_i, t_{i+1})$. Second order sliding precision with respect to the measurement time interval τ is ensured for the above controller (Levantovsky 1985, Emelyanov *et al.* 1993, Levant 1993). Also the second algorithm provides the same precision order with ρ=0.5. That is one order higher than with the 1-sliding control of the form $u = -\text{sign}\, \sigma_i$ (Levant 1993, 1998a).

Both 2-sliding controllers have their advantages and disadvantages. Algorithm (6) is very robust, since it does not require any information about $\dot\sigma$. It provides for proportionality of sup |σ| and sup $|\dot\sigma|$ to ε and $\varepsilon^{1/2}$ respectively in the presence of any measurement noise of magnitude ε (Levant 1998a). On the other hand, the control signal it produces is not Lipschitz when σ is small, and this may result in noise on the control signal. Algorithm (5) and its discrete measurement counterpart (7) produce Lipschitz control signals, but lose convergence properties when confronted with large measurement errors or very small measurement time steps.

Explanation of 2-sliding algorithms. A detailed explanation having been presented in a number of papers (Levant 1993, Emelyanov et al. 1993, Levant 1998a), the main points only are clarified here. Denote $\Lambda_u(\cdot) = \frac{\partial}{\partial t}(\cdot) + \frac{\partial}{\partial x}(\cdot) f(t,x,u)$. It is apparent that $\dot\sigma(t,x,u) = \Lambda_u(\sigma(t,x))$, $|\Lambda_u\Lambda_u\sigma| < C$. Calculate $\ddot\sigma$ for the control laws (5) and (6) respectively, assuming $|u| < 1$, $|\sigma| < \sigma_0$:

$$\ddot\sigma = \Lambda_u\Lambda_u\sigma + \frac{\partial\dot\sigma}{\partial u}\dot u = \Lambda_u\Lambda_u\sigma - \tilde\alpha \frac{\partial\dot\sigma}{\partial u}\text{sign}\, \sigma, \; \tilde\alpha = \begin{cases} -\alpha_M \text{sign}\sigma\,, \; \sigma\dot\sigma > 0 \\ -\alpha_m \text{sign}\sigma\,, \; \sigma\dot\sigma \le 0 \end{cases}; \tag{8}$$

$$\ddot\sigma = \Lambda_u\Lambda_u\sigma - \tfrac{1}{2}\lambda \frac{\partial\dot\sigma}{\partial u}\dot\sigma |\sigma|^{-\frac{1}{2}} + \frac{\partial\dot\sigma}{\partial u}\dot u_1 = \Lambda_u\Lambda_u\sigma - \tfrac{1}{2}\lambda\frac{\partial\dot\sigma}{\partial u}\dot\sigma|\sigma|^{-\frac{1}{2}} - \alpha\frac{\partial\dot\sigma}{\partial u}\text{sign}\, \sigma. \tag{9}$$

Equations (8) and (9) may be rewritten in the form $\ddot{\sigma} = \Lambda_u \Lambda_u \sigma - \overline{\alpha}\, \frac{\partial \dot{\sigma}}{\partial u}$ sign σ where $\overline{\alpha}$ is a time-varying parameter. Following is the set of *principles* which are used in the construction of the two aforementioned 2-sliding controllers :

1. $\frac{\partial \dot{\sigma}}{\partial u}$ is a definite (positive or negative) bounded quantity separated from 0 (it is taken positive here) .

2. The equivalent control $u_{eq}(t,x)$ never leaves some range available for u. It means here that u_{eq} remains within some range $[-u_0,\ u_0] \subset [-1,\ 1]$. Otherwise, the constraint $\sigma = 0$ could not be kept identically.

3. The time derivative $\dot{u}$ of the control must dominate in expressions (8), (9) for $\ddot{\sigma}$. It means formally that inequality inf $(\overline{\alpha}\, \frac{\partial \dot{\sigma}}{\partial u}) > \sup|\Lambda_u \Lambda_u \sigma|$ is satisfied providing for $\ddot{\sigma}\sigma < 0$ (it is sufficient here that inf $(\overline{\alpha}\, \frac{\partial \dot{\sigma}}{\partial u}) > C$). This causes $|\dot{u}| > \sup |\dot{u}_{eq}|$ to be held. It also leads to the projection of the trajectory onto the $\sigma\dot{\sigma}$ plane being rotated around the origin.

4. The idea is to leave the region $\sigma\dot{\sigma} > 0$ as soon as possible and remain within the region $\sigma\dot{\sigma} < 0$ for as long as possible. For that purpose $\overline{\alpha}|_{\sigma\dot{\sigma}>0}$ must be sufficiently large with respect to $\overline{\alpha}|_{\sigma\dot{\sigma}<0}$.

For purposes of comparison, consider a controller of the form:

$$\dot{u} = \begin{cases} -u \ \text{ with } |u| > 1, \\ -\alpha \ \text{sign}\sigma \ \text{ with } |u| \le 1, \end{cases}$$

where α is a positive constant (Emelyanov, Korovin 1981). Such a controller turns into the standard relay controller $u = -\text{sign}\ \sigma$ as $\alpha \to \infty$. Taking $\overline{\alpha} = \alpha$ results in

$$\ddot{\sigma} = \Lambda_u \Lambda_u \sigma + \frac{\partial \dot{\sigma}}{\partial u}\, \dot{u} = \Lambda_u \Lambda_u \sigma - \alpha \frac{\partial \dot{\sigma}}{\partial u} \text{sign}\ \sigma. \qquad (10)$$

Consider trajectories of (8), (9), (10) on the plane $\sigma,\ \dot{\sigma}$ in a small vicinity of the origin $\sigma = \dot{\sigma} = 0$ corresponding to the 2-sliding mode. Take for comparison trajectory segments lying in the half-plane $\sigma \ge 0$, and let $\alpha = \alpha_M$. Let $\sigma = 0,\ \dot{\sigma} = \dot{\sigma}_0$ at the initial time, and at the final time $\sigma = 0$ and $\dot\sigma = \dot\sigma_1, \dot\sigma'_1, \dot\sigma''_1$ for (10), (8), (9) respectively (Fig. 2 a). Then $\dot\sigma_1 \approx -\dot\sigma_0$, on the contrary $|\dot\sigma''_1/\dot\sigma_0|, |\dot\sigma'_1/\dot\sigma_0|$ can be shown to be less than some constant $\eta < 1$. Thus, successively continuing the trajectories from one half-plane to another, obtain trajectories of (8) and (9) converging to the origin (Fig 2 b, c). The convergence time may now be estimated by the sum of the achieved geometric series $|\dot\sigma_0|, |\dot\sigma_1|$, ... Indeed, consider the twisting algorithm (8). The absolute value of $\ddot\sigma$ varying between two positive constants, the convergence time is proportional to the sum of absolute values of intersections with axis $\dot\sigma$. The latter being estimated by a constant multiplied by $\sum_{i=0}^{\infty}\eta^i$, the convergence time is finite. The super-twisting algorithm (9) convergence time is similarly estimated.

Finite difference usage instead of $\dot\sigma$ *in algorithm* (5). Calculation of the exact derivative being extremely not robust, consider now the proposed replacement of $\dot\sigma$ by $\Delta\sigma_i = \sigma(t_{i+1}) - \sigma(t_i) = \dot\sigma(t_i)\tau + \eta(t_{i+1}) - \eta(t_i) + O(\tau^2)$, where η is the measurement noise. It is easy to see that sign $\Delta\sigma_i$ preserves its value if the noise is small with respect to $\dot\sigma\tau$. Thus, errors in the measurements of sign $\Delta\sigma_i$ are possible only in some vicinity of σ axis in the plane $\sigma, \dot\sigma$, while errors in the measurements of sign σ_i are possible for $|\sigma| \le \varepsilon$ only. Since $\alpha_M > \alpha_m > C/K_m$, the trajectory rotation is preserved with $|\sigma| > \varepsilon$, and the noise influence on the trajectories (Fig. 2b) is negligible for small noises. For larger noises a special measurement step feedback $\tau = \min[\max(\kappa|\sigma(t_i)|^{1/2},\tau_m), \tau_M]$ is to be applied, turning the twisting algorithm into a robust one. It preserves the accuracy $\sigma = O(\tau_m^2)$, $\dot\sigma = O(\tau_m)$ in the absence of noises and $\sigma = O(\varepsilon)$, $\dot\sigma = O(\varepsilon^{1/2})$ in the presence of measurement noises with magnitude ε, $\tau_m = O(\varepsilon^{1/2})$ (Levant 1993, 2000, to appear). In fact, simulation shows that with a relatively large measurement step characterizing the pitch control problem considered further the noise influence may be neglected. Nevertheless, another more robust controller was applied ensuring for algorithm convergence for larger noises as well.

Chattering avoidance. Any approach (adaptive, linear or other) attaining exact maintenance of the constraint $\sigma = 0$ will yield, as a result, the so-called zero-dynamics motion (Isidori 1989) described by the formal substitution of the smooth equivalent control for the real control. So any r-sliding controller does, $r = 1, 2, ...$ The difference is that with $r = 1$ the sliding control is a discontinuous infinite-frequency switching signal, its average value only being equal to the equivalent control, while the r-sliding control, $r > 1$, is an (r-1)-smooth function really coinciding with the unknown equivalent control, when the r-sliding mode is obtained. In the presence of a switching delay or discrete-time measurements the r-sliding control contains an infinitesimally-small high-frequency vibration component with the magnitude proportional to the switching delay (measurement step) powered to r-1 (that evaluation is true with respect to most of the known finite-time convergent controllers). Having infinitesimal energy, such a component cannot be distinguished from natural noises always present in the system and is harmless. Another usually more significant source of control vibration is the measurement error. The latter problem persists for any feedback control based on measurements of the real-time deviation from the constraint. For an instance, any linear control technique is, in general, more sensitive to such errors, for it usually requires relatively large gains in order to overcome the uncertainty. With discrete measurements the corresponding performance is often hardly distinguishable from a standard 1-sliding mode with a minimized control magnitude.

Let ε be the magnitude of the noise in the measurements of σ, noise being any function measurable in the Lebesgue sense. It may be shown (Levant 1993, 2000, to appear) that both 2-sliding controllers (the above-mentioned variable measurement step is to be used in the twisting controller) are almost insensitive to the noise frequency and provide for the magnitude of σ being of the order of ε, while control vibrations $u - u_{eq}$ and $\dot{\sigma}$ have maximal possible magnitudes proportional to $\varepsilon^{1/2}$.

Example. Consider a simple dynamic system

$$\dot{\sigma} = a(t) + 2u, \ \ |a(t)| \leq 1, \ |\dot{a}(t)| \leq 1.$$

where $a(t)$ is unknown. It satisfies all the assumptions with arbitrary σ_0, so that $\sigma_0 = \infty$ may be taken. In that case the twisting algorithm (5) with $\alpha_m = 2$ and $\alpha_M = 6$ is effective, and the super-twisting algorithm (6) may be taken in a simplified form

$$u = -2\,|\sigma|^{1/2}\,\mathrm{sign}\,\sigma + u_1, \qquad \dot{u}_1 = \begin{cases} -u, & |u| > 1, \\ -3\ \mathrm{sign}\,\sigma, & |u| \le 1. \end{cases}$$

Both controllers provide for $\sigma \equiv 0$ in the absence of measurement noises, providing for finite-time convergence of u to the unknown function $u_{\mathrm{eq}} = 0.5a(t)$.

Hybrid 2-sliding controller. The controller used in the sequel is derived from the following control law (that is already a discrete-measurement form):

$$u = u_1 + u_2, \qquad \begin{aligned} u_2 &= \begin{cases} -\lambda\,|\sigma_0|^{\rho}\ \mathrm{sign}\,\sigma(t_i), & |\sigma(t_i)| > \sigma_0, \\ -\lambda\,|\sigma(t_i)|^{\rho}\ \mathrm{sign}\,\sigma(t_i), & |\sigma(t_i)| \le \sigma_0, \end{cases} \\ \dot{u}_1 &= \begin{cases} -u(t_i) \ \text{ with } |u(t_i)| > 1, \\ -\alpha_{\mathrm{m}}\ \mathrm{sign}\sigma(t_i) \ \text{ with } \sigma(t_i)\Delta\sigma_i \le 0,\ |u(t_i)| \le 1, \\ -\alpha_{\mathrm{M}}\ \mathrm{sign}\sigma(t_i) \ \text{ with } \sigma(t_i)\Delta\sigma_i > 0,\ |u(t_i)| \le 1. \end{cases} \end{aligned} \tag{11}$$

Being a hybrid of the standard 2-sliding controllers (5), (6), this controller is very robust and has a faster convergence rate than (5), (6). Its convergence persists even with significant measurement errors. Indeed, the use of sign $\Delta\sigma_i$ improves the convergence properties of controller (6), having combined them with the convergence mechanism of the twisting controller. At the same time the super-twisting convergence mechanism is also preserved when sign $\Delta\sigma_i$ is invalid, if λ is taken large enough.

III. Flight Platform Control

Pitch control problem. The real-life control problem we are confronted with is the pitch angle control problem of the Delilah vehicle (Ben-Asher 1995). The Delilah vehicle is a small turbojet powered decoy equipped with active and passive Radio Frequency (RF) payloads which imitate a full-size aircraft. Its weight is 400 lbs, with a dash speed of about 770 fps, a stall speed of approximately 250 KEAS (equivalent air speed in Knots), and a flight ceiling of close to 30,000 ft. Starting 2 seconds after the Delilah - host vehicle separation, the pitch angle control loop must quickly track a pitch angle command profile which is defined in real time and in situations where the structural asymmetry in pitch due to manufacturing tolerances may possibly be rather large. Usually a high gain integrator in a PID (proportional + integral + derivative) loop would suffice for the task. However, a high integrator gain would need excessive damping and is rather difficult to achieve due to the intricate characteristics of the stepper motor servo with which the Delilah vehicle is equipped. Note that the Delilah vehicle is extremely rigid, and hence flexible modes are completely ignored in the discussion below.

Mathematical statement of the problem. The problem is to enforce the tracking of the flight platform's pitch angle θ to a signal $\theta_c(t)$ given in real time during the course of the flight, while being subjected to unmeasured external disturbances. The nonlinear rigid body dynamics of the system is of six degrees of freedom (DOF) where the aerodynamics forces and moments model is based on wind tunnel measurements. The Delilah vehicle is equipped with an air data system providing dynamic pressure, Mach number and altitude. Therefore, the controller adaptively depends on these flight conditions. For simplicity of the design process we have used a linearized model for each relevant envelope point on a given grid. The six DOF simulations were used to verify the design before flight tests. More specifically, the nonlinear dynamic system is replaced by its 5-dimensional numerical linearizations describing the vertical-plane motions and calculated at 42 equilibrium points within the "altitude - Mach number" flight envelope. Each system possesses the form

$$\dot{x} = Ax + C\begin{pmatrix} q \\ \theta \end{pmatrix} + Bu, \quad \dot{q} = a^t x + cq + bu, \quad \dot{\theta} = q,$$

where $a, B, x \in \mathbf{R}^3$, $q, \theta, u, b, c \in \mathbf{R}$, A, C - 3×3 and 3×2 matrices respectively, u is the control (horizontal stabilizer angle). The coordinates are two velocity components x_1, x_2 [ft/sec], height x_3 [ft], θ [rad] and $q = \dot{\theta}$ [rad/sec]. These coordinates, as well as u [rad], θ_C and $\dot{\theta}_c$ are bounded by some given constants. The corresponding parameters of the linearized system as well as most of the coordinates are not available in real time. The only measurements available are θ, θ_c, $\dot{\theta}$, $\dot{\theta}_c$, the dynamic pressure and the Mach number (which is equivalent to the altitude and the Mach number). The system has second relative degree (Isidori 1989), which implies that the control appears explicitly only in the second time derivative of θ. Some delay and noise are also present in the measurements, and the system contains an actuator whose behavior exhibits both delay and discretization effects. That actuator also imposes some bounds on the magnitude and velocity of the control variation.

Rewrite the dynamic equations as:

$$\frac{d}{dt}\begin{bmatrix} x \\ \theta \\ q \end{bmatrix} = G\begin{bmatrix} x \\ \theta \\ q \end{bmatrix} + Hu,$$

where the 42 values of G and H are given corresponding to the 42 points within the flight envelope, all of which represent open-loop stable systems. The control algorithm has to be effective for all of them. It has also to be sufficiently robust in order to preserve its properties when controlling a real system not very similar to the given set of linear models. We present two such matrix pairs G and H as follows:

a. altitude = 1334 m (4376 ft) , dynamic pressure p = 200 lb/ft^2, Mach number M = 0.4

$$G = \begin{bmatrix} -0.01213 & 0.05233 & -0.00007 & -31.91731 & -54.2126 \\ -0.07226 & -0.70408 & 0.001 & -4.02416 & 433.03 \\ 0.12422 & -0.99225 & 0 & 437.387 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.00615 & -0.03901 & 0 & -0.00001 & -0.596 \end{bmatrix}, \quad H = \begin{bmatrix} -2.062 \\ 46.2402 \\ 0 \\ 0 \\ 23.275 \end{bmatrix}$$

b. altitude = 1361 m (4464 ft), dynamic pressure p = 900 lb/ft^2, M = 0.85

$$G = \begin{bmatrix} -0.21389 & 0.0076 & -0.00024 & -32.17644 & -9.83138 \\ 0.45528 & -1.80203 & 0.00273 & 0.41842 & 903.7794 \\ 0.01088 & -1. & 0 & 929.14 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.03835 & -0.1119 & 0.0001 & 0.58069 & -18.18247 \end{bmatrix}, \quad H = \begin{bmatrix} -25.97 \\ 230.3706 \\ 0 \\ 0 \\ 136.15 \end{bmatrix}.$$

The previously mentioned bounds are: $|\theta_c| \le 0.21$ (12°), $|\dot{\theta}_c| \le 0.175$ (10°/s), $|\ddot{\theta}_c| \le 0.35$ (20°/s^2), $|\dddot{\theta}_c| \le 12$ (688°/s^3), $|x_1| \le 70$ ft/s, $x_2 \in [40, 70]$ ft/s, $x_3 \in [0, 30000]$ ft , $|u| \le 0.209$ (12°). Measurements are carried out at the rate of 64 times per second and the measurement error and measurement delays of θ, θ_c are 0.1° = 0.02 rad and 0.016 s respectively, while the error and delay for $q = \dot{\theta}$ and $\dot{\theta}_c$ are 0.02 rad/s and 0.005 s.

The *actuator* is an electro-mechanical unit which translates a controller output signal u to control surface motion v. Its output v must track its input u subject to the saturation constraint $|u| < 0.446$ rad (25°). The actuator receives values of u at 512 Hz and at each input sample time, the output changes with steps of 0, 0.2° (0.0035 rad) or - 0.2°. If the direction of the required shift (i.e. sign(u-v)) changes, v ceases to change for 1/32 s. During this delay the actuator continues to receive commands and any change of sign(u-v) causes a new delay to begin.

Primary statement of the problem. The first ("primary") statement of the problem did not allow for measurements of the derivatives $q = \dot{\theta}$ and $\dot{\theta}_c$. The difficulty with the "primary problem" statement can be seen from the fact that good tracking of θ_c by θ and boundedness of $\ddot{\theta}_c$ and $\dot{q} = \ddot{\theta}$ imply good tracking of $\dot{\theta}_c$ by q as well. Thus, any successful controller may also be considered as a quality differentiator for arbitrary signals $\theta_c(t)$ which are calculated in real time with some noise. Real time differentiation, as is well known, is an extremely complicated problem, and, thus, the primary problem is also much more sophisticated than the main one.

Nevertheless, the primary problem is successfully solved by the presented approach. Indeed, a robust differentiator (Levant 1998a) having been applied, the primary problem is actually reduced to the main one, certainly, with much larger errors in measurements of $\dot{\theta}$ and $\dot{\theta}_c$. While the general problem of differentiation in the presence of input noises is really ill-posed, it turns into a well-posed one if the basic input signal is assumed to have a derivative with a known Lipschitz constant. Such differentiator is based on the super-twisting controller (6) applied to the trivial dynamic system $\dot{y} = v$ with scalar control v and output y. Indeed, let $g(t)$ be an input signal, then, keeping $y = g(t)$ in 2-sliding mode, achieve $v = \dot{y} = \dot{g}(t)$. That differentiator is proved to provide for maximal differentiation error proportional to the square root of the maximal magnitude of Lebesgue-measurable input noises.

IV. Problem Solution

Solution concept. The relative degree of the controlled system being 2, a 1-sliding control cannot be directly implemented to control output θ. At the same time direct implementation of a 2-sliding control is possible, but requires discontinuous control influence which cannot be realized by the actuator. The way out is to maintain the constraint

$$\sigma = \Lambda(\theta-\theta_c) + (q - \dot{\theta}_c) = 0, \qquad (12)$$

where $\Lambda > 0$, $q = \dot{\theta}$. Keeping it in a standard 1-sliding mode is also impossible, for the discontinuous control cannot be followed by the actuator output, thus, 2-sliding mode is needed. At first glance, each of the standard 2-sliding controllers listed previously could have been implemented with sufficiently large constant gain. Nevertheless, this simple approach was shown not to work.

There are no sufficient control resources to keep $\sigma = 0$, even for one of the given 42 linear approximations. In other words, principle # 2 of the 2-sliding controller design is not fulfilled: the variation range of the equivalent control u_{eq} is larger than the admissible range of the control u itself. The situation changes only after the natural damping of the 2 fast stable (short period) modes, which takes about 2-3 seconds and places a restriction on the minimal transient-process duration. The control having been applied 2 seconds after the separation of the vehicle from the plane, that does not cause any trouble. However, the amplification gains still have to be chosen independently of any of the linearized approximations, otherwise the control will influence some of the linearized systems too strongly and some of the others too weakly.

The controller which would solve the problem with above-listed complications may be constructed based on the hybrid controller (11) as a sum of a functional and a dynamic unit of the form:

$$u = u_{\text{func}} + u_{\text{dyn}},$$

$$u_{func} = - K_1(p,M)\, \lambda\, |\sigma|^{1/2} \text{sign}\, \sigma,$$

$$\dot{u}_{\text{dyn}} = K_2(p,M) \begin{cases} -u, & |u| > U_M, \\ -r_1 \text{sign}\sigma - r_2 \text{sign}\dot{\sigma}, & |u| \le U_M. \end{cases}$$

Here U_M is the largest admissible value of the control. Dynamic pressure p and the Mach number M are measured during the course of the flight. Positive constant parameters $r_1 > r_2$ and λ are to be taken so as to provide for admissible tracking for one system singled out from the above-mentioned 42 linear approximation systems with $K_1 = K_2 = 1$. Factors $K_1(p,M)$, $K_2(p,M)$ are quasi-constant scaling factors corresponding to the ranges of u_{eq} and $\dot{u}_{\mathrm{eq}}$ respectively, and providing for fulfillment of the design principles 2, 3 for all the 42 systems with "equal" performance quality. Note, once more, that from the theoretical point of view these factors are not needed, for there are r_1, r_2, λ providing for finite-time convergence to 2-sliding mode for all the systems. But the performance will vary from system to system.

Other problems of the control design are the measurement step of 1/64 seconds and the actuator delay which transform infinitesimal overshoots into significant ones. All the above-listed good features of 2-sliding controllers presume the measurement step to be infinitesimally small. In fact it is so large in the considered case that the very use of 2-sliding control turns out to be questionable. In order to suppress overshoots and improve the tracking accuracy, the control gains must be as small as possible. Concurrently small gains lead to slow convergence. Thus, the solution is to make them dependent upon σ: the gains are to decrease with decrease of $|\sigma|$.

The actually implemented discretized 2-sliding controller is as follows:

$$u = u_{\mathrm{func}} + u_{\mathrm{dyn}}, \tag{13}$$

$$u_{\mathrm{func}} = - K_1(p(t_i),M(t_i))\{ \mu\, (q(t_i)\text{-}\dot{\theta}_c\,(t_i)) + \min[1, \lambda\, (|\sigma(t_i)|^{1/2}+|\sigma(t_i)|)]\mathrm{sign}\,\sigma(t_i)\}, \tag{14}$$

$$\dot{u}_{\mathrm{dyn}} = K_2(p(t_i),M(t_i))\begin{cases} -u(t_i), & |u(t_i)| > U_M, \\ -r_1(\sigma(t_i))\mathrm{sign}\sigma(t_i) - r_2(\sigma(t_i))\mathrm{sign}\Delta\sigma_i, & |u(t_i)| \le U_M. \end{cases} \tag{15}$$

$$r_1 = \beta_{0,1} + \min[\beta_{1,1}, \max(\beta_{2,1}, \gamma_1|\sigma(t_i)|)],$$

$$r_2 = \beta_{0,2} + \min[\beta_{1,2}, \max(\beta_{2,2}, \gamma_2|\sigma(t_i)|)].$$

The current time t satisfies here $t \in [t_i, t_{i+1})$. During the simulation a few terms were found useful to be added here to the functional component of the control. Mark that, while in sliding mode, they are negligible (any sliding control completely removes disturbances coming to the system through the control channel), and in the absence of the actuator, with really small measurement steps they are totally redundant. The term

$\mu(q - \dot{\theta}_c)$ is used to improve the stability properties of the open-loop system, and the term proportional to σ improves the convergence features of the algorithm. Parameters $\beta_{i,j}$ are to provide for faster convergence of the algorithm to the 2-sliding mode, preserving good sliding accuracy. They have to satisfy some conditions. In particular, $K_2(r_1 - r_2)$ must be larger than the derivative of the equivalent control $\dot{u}_{eq}$. It should be noted that, with σ small, r_i are constant, and the controller coincides with the simple hybrid controller (11). Variable parameters r_i speed up the transient process. It is worth mentioning that, with the parameters chosen in the sequel, this algorithm provides for ideal 2-sliding and results in asymptotically exact tracking, when the servo actuator is absent and the measurements are exact and continuous. After substitution of $\Delta\sigma_i$ for $\dot{\sigma}$, a robust algorithm is achieved. Its performance gradually deteriorates (without a drastic loss of accuracy) as the measurement step is enlarged and real system imperfections are introduced (Filippov 1988).

Realization of the proposed scheme. The controller was initially adjusted to control one particular system chosen from the aforementioned set of 42. The parameters of that particular controller were found from computer simulation. Direct analytical appointment of the parameters is possible here, but leads to redundantly large values due to extensive use of rough linear inequalities. It was chosen

$$\Lambda = 7, \qquad \lambda = 0.02, \qquad \mu = 0.05, \tag{16}$$

$$r_1 = 0.01 + \min[0.25, \max(0.03, 4|\sigma(t_i)|)], \tag{17}$$

$$r_2 = 0.01 + \min[0.18, \max(0.01, 3|\sigma(t_i)|)]. \tag{18}$$

Setting the amplification gains K_1, K_2. It was noted previously that in order to use 2-sliding controllers, control u has to be predominant in the expression for $\dot{\sigma}$, when u assumes its maximal value (principle 2), and its time derivative $\dot{u}$ has to be predominant in the expression for $\ddot{\sigma}$ (principle 3). In other words, the magnitudes of the control and its time derivative have to be larger than the maximum absolute values of the equivalent control and its time derivative respectively. The amplification factors for u and $\dot{u}$ are evaluated by the proportionality principle as follows:

$$K_1 = K_1^{\,0}\,\frac{\sup|u_{eq}^0|}{\sup|u_{eq}|},$$
$$K_2 = K_2^{\,0}\,\frac{\sup|\dot{u}_{eq}^0|}{\sup|\dot{u}_{eq}|}.$$

The upper index 0 indicates here that the corresponding quantity relates to the chosen linear system, for which the controller is already adjusted. $K_1^{\,0}$, $K_2^{\,0}$ are the simulation-based numbers. By taking into account the given bounds on the coordinates and evaluating the equivalent control and then its derivative from the equation $\dot{\sigma} = 7(q - \dot{\theta}_c) + (\dot{q} - \ddot{\theta}_c) = 0$ and its derivative respectively, the following formulae were achieved, after neglecting some small numerical quantities:

$$K_1 = 44/h_5.$$
$$K_2 = \frac{0.613}{h_5}\,[0.2|g_{2,5}g_{5,2}| + 30|g_{5,5}g_{5,2} + g_{2,2}g_{5,2}| + 15].$$

That allowed for the applicability of the controller to all of the 42 envelope points. The numerical coefficients 44 and 0.613 are the result of adjusting the controller to a particular system. K_1, K_2 are to be expressed now in terms of dynamic pressure p and the Mach number M. The gains in the following expressions were found by the least squares approximations for K_1 and K_2 and provide acceptable performance for all of the 42 given linearized systems:

$$K_1 \cong 221\,(p\text{-}80)^{-1} + 1.821\cdot 10^{-4}\,p + 0.1394\,(1.1\text{-}M)^{-1} - 0.9151\,M + 0.1763, \qquad (19)$$
$$K_2 \cong 57.05\,(p\text{-}80)^{-1} + 2.4181\cdot 10^{-4}\,p + 0.2333\,(1.1\text{-}M)^{-1} - 1.114\,M + 0.1695. \qquad (20)$$

For the 42 given systems, p and M vary within the limits 200 - 900 and 0.4 - 0.85 respectively.

V. Simulation and Flight Testing

6 Degree of Freedom (DOF) Simulations. The above VSS algorithm (12)-(20) has been realized in a 6 DOF nonlinear simulation of the Delilah Unmanned Air Vehicle (UAV) running at a frequency of 64 Hz. The simulation also includes elastic effects which have been neglected for simplicity in the analysis. For the first two seconds the vehicle flies in open loop. Two seconds following the ejection we begin applying VSS control to the longitudinal plane of the flight platform.

The pitch angle command is changed from the measured pitch at 2 sec. to the maximum pitch angle command $\theta_{c\,max} = 2\,°$ at the rate of $q_c = 3.52\,°/s$, that is :

$$\theta_c = \theta \quad \text{at } t = 2,$$

$$\theta_c = \min(\theta_c + q_c \cdot (t - 2), \theta_{c\,max}) \quad \text{at } t > 2 .$$

The measurement noise of the pitch rate in the simulation is a random zero-mean white sequence with a standard deviation of 0.1°/s. The pitch angle measurement is corrupted by a zero-mean random sequence with 0.01° standard deviation with a correlation time of about 2 seconds. Fig. 3 illustrates θ_c and θ versus time in the nominal simulation. The results of 100 Monte-Carlo runs are shown in Fig. 4 (θ versus time) and Fig. 5 (q versus time). These figures include mean and mean plus and minus two standard deviations of the 100 simulation runs.

The 2-sliding controller implementation assumes the controlled system to be smooth. The simulation grid-based system being non-smooth, the simulation results are to be worse than in reality. Nevertheless, it can be seen that the pitch angle and the pitch angle rate track their commanded signals remarkably well.

Flight Tests. The results of a flight test are shown in Fig. 6 (θ_c and θ) and 7 (q_c and q). The flight test validates the simulation and proves that the VSS pitch loop indeed provides exceptional tracking.

VI. Conclusions

The first practical application of the second order sliding technique was demonstrated in the present paper by a solution for the pitch control problem of the Delilah vehicle. The resulting controller's performance was analyzed via 6 DOF simulations and eventually flight-tested. The rather straightforward design procedure, together with the encouraging flight test results, invite further applications of this approach. One may think of applications to longer duration missions, where the control loop must also cope with time-varying disturbance signals which may stem from gusts, etc.

It has to be stressed that the tested controller demonstrates reasonable behavior even with large measurement time steps and time delays. The performance will be significantly enhanced according to any improvement in the above hardware-related factors.

Acknowledgements : The authors wish to thank Prof. Plotkin and Mr. Aron Pila for their useful suggestions and help in revising our manuscript.

References

Apkarian, P. and Gahinet, P. (1995) A convex characterization of gain-scheduled H_∞ controllers, *IEEE Transactions on Automat. Contr.,* Vol. **40(5)**, pp. 853-864

Bartolini, G. and Zolezzi, T., (1986). Control of Nonlinear Variable Structure Systems, *Journal of Math. Analysis and Applications*, **118(1)**, pp. 42-62.

Bartolini, G., Ferrara A., Punta E. and Usai E., (1997a). Application of a second order sliding mode control to constrained manipulators, EUCA, IFAC and IEEE *European Control Conference*, Bruxelles, Belgium

Bartolini, G., Ferrara A. and Usai E., (1997b). Output tracking control of uncertain nonlinear second order systems, *Automatica*, 33(12), 2203-2212

Ben-Asher, J. (1994). Optimal Trajectories for an unmanned air vehicle in the horizontal plane, AIAA J. of Aircraft, Vol. 32, pp. 677-679

Boyd, S. , El Ghaoui, L., Feron, E. ad Balakrishnanm V. (1994). Linear Matrix Inequalities in System and Control Theory, Society for Industrial and Applied Mathematics, Philadelphia, Chapters 4,7

Brockett, R.W. (1978). Feedback invariants for nonlinear systems, Proc. IFAC Congress, Helsinki

Doyle, J. K., Glover, K. , Khargonekar, P. and Francis, B. (1989). State space solutions to standard H_2 and H_∞ Control Problems. IEEE Trans. on Automat. Contr., Vol. AC-34, pp. 831-846.

Chang, L.W.(1990). A MIMO sliding control with a second order sliding condition. ASME WAM, paper no. 90-WA/DSC-5, Dallas, Texas.

Emelyanov, S.V. and Korovin, S.K., 1981, Applying the principle of control by deviation to extend the set of possible feedback types. Soviet Physics, Doklady, **26(6)**, 562-564.

Emelyanov, S.V., Korovin, S.K. and Levant, A. (1993). Higher-order sliding modes in control systems. *Differential Equations*, **29(11)**, 1627-1647.

Emelyanov, S.V., Korovin, S.K. and Levantovsky, L.V. (1986a). Higher order sliding modes in binary control systems. *Soviet Physics, Doklady*, **31(4)**, 291-293.

Emelyanov, S.V., Korovin, S.K. and Levantovsky, L.V. (1986b). Second order sliding modes in controlling uncertain systems. *Soviet Journal of Compute and System Science*, **24(4)**, 63-68.

Emelyanov, S.V., Korovin, S.K. and Levantovsky, L.V. (1986c). Drift algorithm in control of uncertain processes. *Problems of Control and Information Theory*, **15(6)**, 425-438.

Emelyanov, S.V., Korovin, S.K., Levantovsky, L.V. (1990). New class of second order sliding algorithms. *Mathematical Modeling*, **2(3)**, 89-100, [in Russian].

Isidori, A.(1989). *Nonlinear Control Systems*. Second edition, Springer Verlag, New York, Chapter 4.

Filippov, A.F. (1988), *Differential Equations with Discontinuous Right-Hand Side*, Kluwer, Dordrecht, the Netherlands, Chapters 1, 2.

Fridman, L., Levant, A. (1996). Sliding modes of higher order as a natural phenomenon in control theory. In *F. Garofalo, L. Glielmo (Eds.). Robust Control via Variable Structure & Lyapunov Techniques. - (Lecture Notes in Control and Information Science;* 217*)*, Springer-Verlag, London, 107-133.

Levant, A. (Levantovsky, L.V.) (1993). Sliding order and sliding accuracy in sliding mode control. *International Journal of Control*, **58(6)**, 1247-1263.

Levant, A. (1998a). Robust exact differentiation via sliding mode technique. *Automatica*, 34(3), 379-384.

Levant, A. (1998b). Arbitrary-order sliding modes with finite time convergence. *Proc. of the 6th IEEE Mediterranean Conference on Control and Systems*, June 9-11,1998, Alghero, Sardinia, Italy, World Sc. Publishing Co. Pte. Ltd., pp. 349-354.

Levant, A. (2000). Variable measurement step in 2-sliding control, *Kybernetika*, to appear.

Levantovsky, L.V. (1985). Second order sliding algorithms: their realization. In "*Dynamics of Heterogeneous Systems*", 32-43, Institute for System Studies, Moscow [in Russian].

Mc Ruer, D. , Askenas, I. and Graham, D. (1973) Aircraft dynamics and automatic control, *Princeton University Press, Princeton, New-Jersey.*

Peres, P. L. D., Geromel, Y. G. and Souza, R. (1994) H_∞ control design by static output feedback. In Proc. IFAC Symp. Robust Contr. Design, Rio de Janeiro, Brazil, Sept., 243-248.

Sira-Ramirez, H (1992). On the sliding mode control of nonlinear systems. *Systems & Control Letters*, **19**, 303-312.

Sira-Ramirez, H(1993). On the dynamical sliding mode control of nonlinear systems. *International Journal of Control*, **57(5)**, 1039-1061.

Slotine, J.-J. E., and Sastry, S.S. (1983). Tracking control of non-linear systems using sliding surfaces, with applications to robot manipulators. *International Journal of Control*, **38**, 465-492.

Utkin, V.I. 1977. Variable structure systems with sliding modes: a survey. *IEEE Transactions on Automatic Control*, **22**, 212-222.

Utkin, V.I. 1992. *Sliding Modes in Control and Optimization*, Springer Verlag, New York.

Yaesh, I. and Shaked, U. (1997). Minimum entropy static output-feedback control with an H_∞ - norm performance bound, *IEEE Trans. on Automat. Contr.*, **AC -42 (6)**, 853-858.

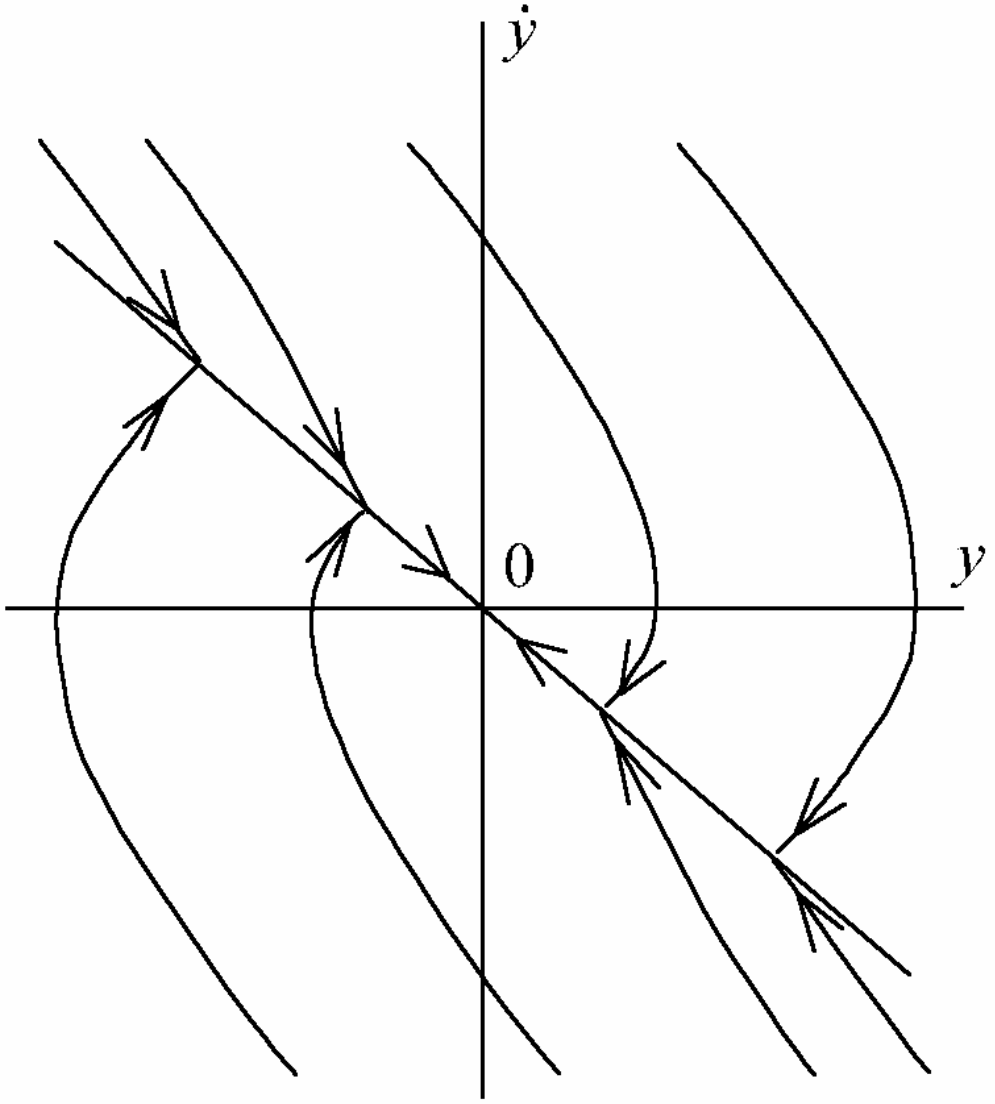

Fig. 1: Standard sliding mode

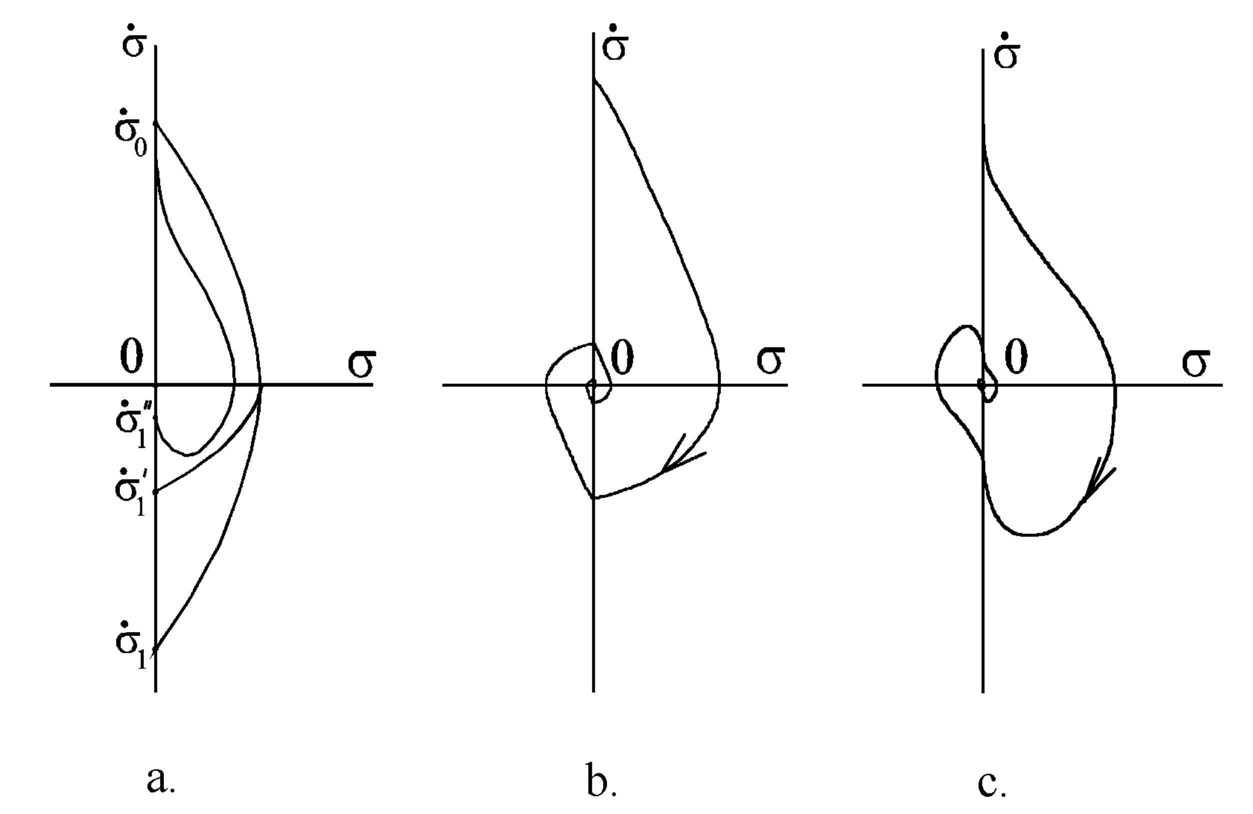

Fig. 2: Convergence of standard 2-sliding algorithms: a. Comparison of the algorithms; b. Twisting algorithm; c. Super-twisting algorithm

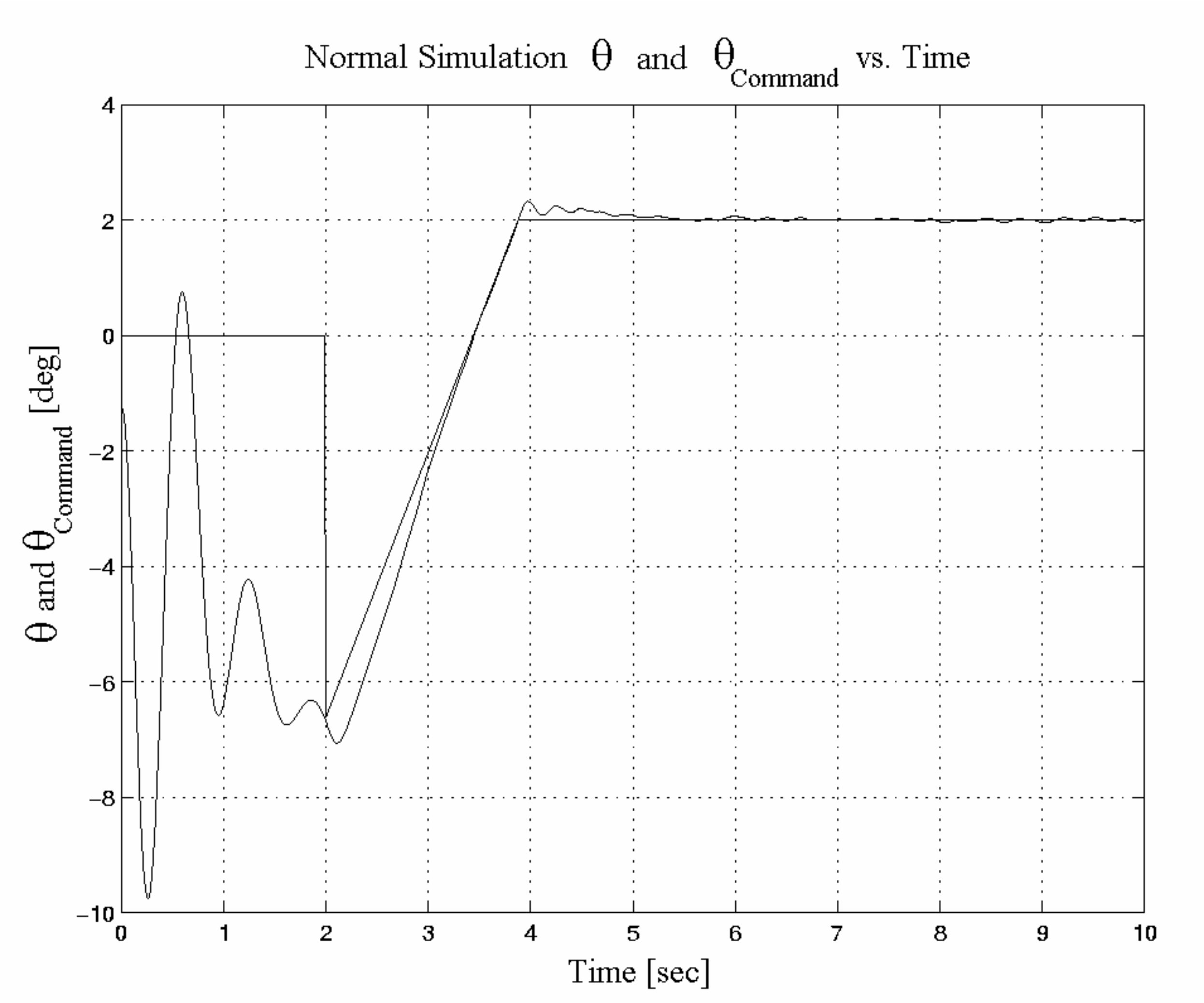

Fig. 3: θ_c and θ versus time in the nominal simulation

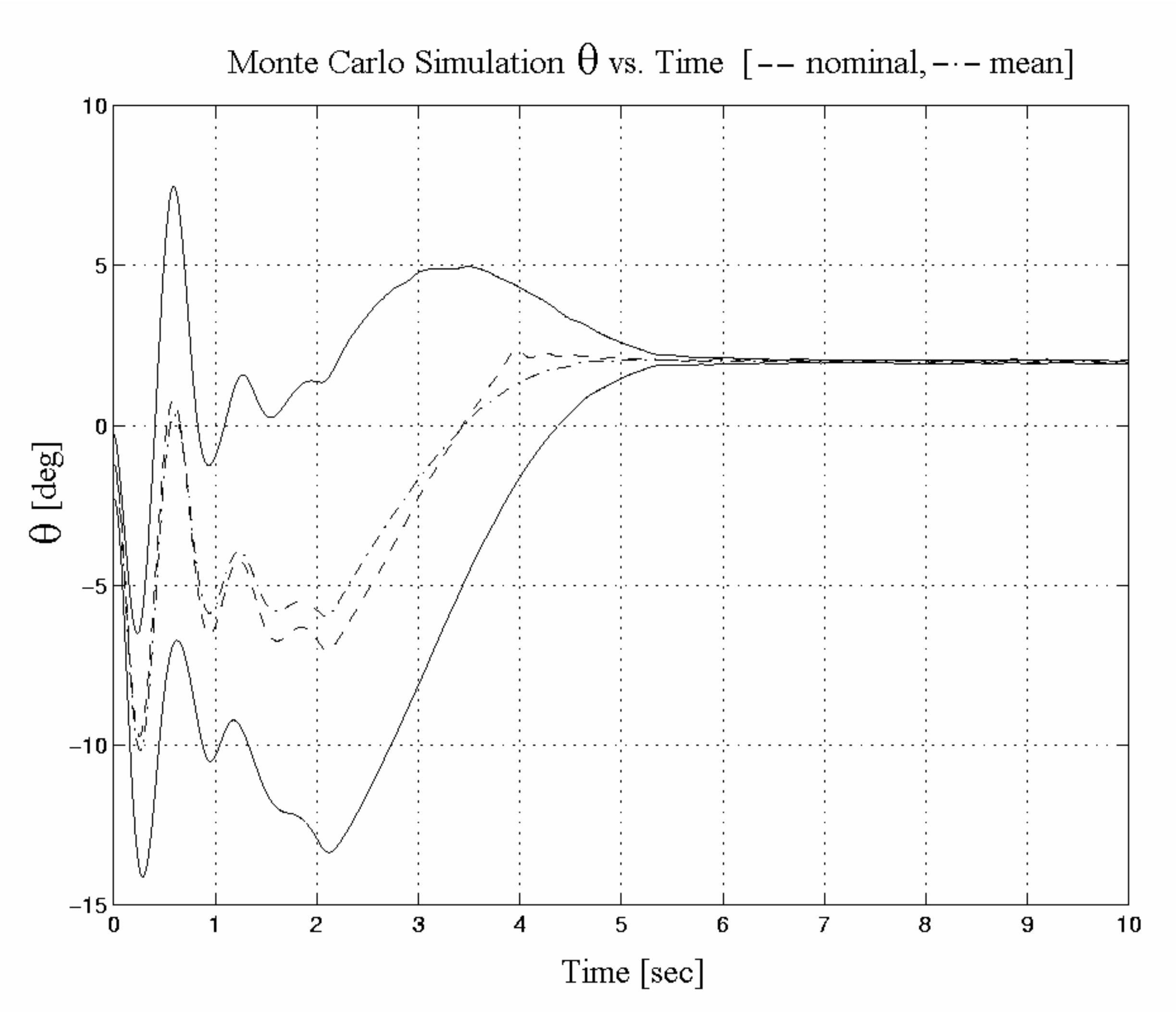

Fig. 4: The results of 100 Monte-Carlo runs: θ versus time

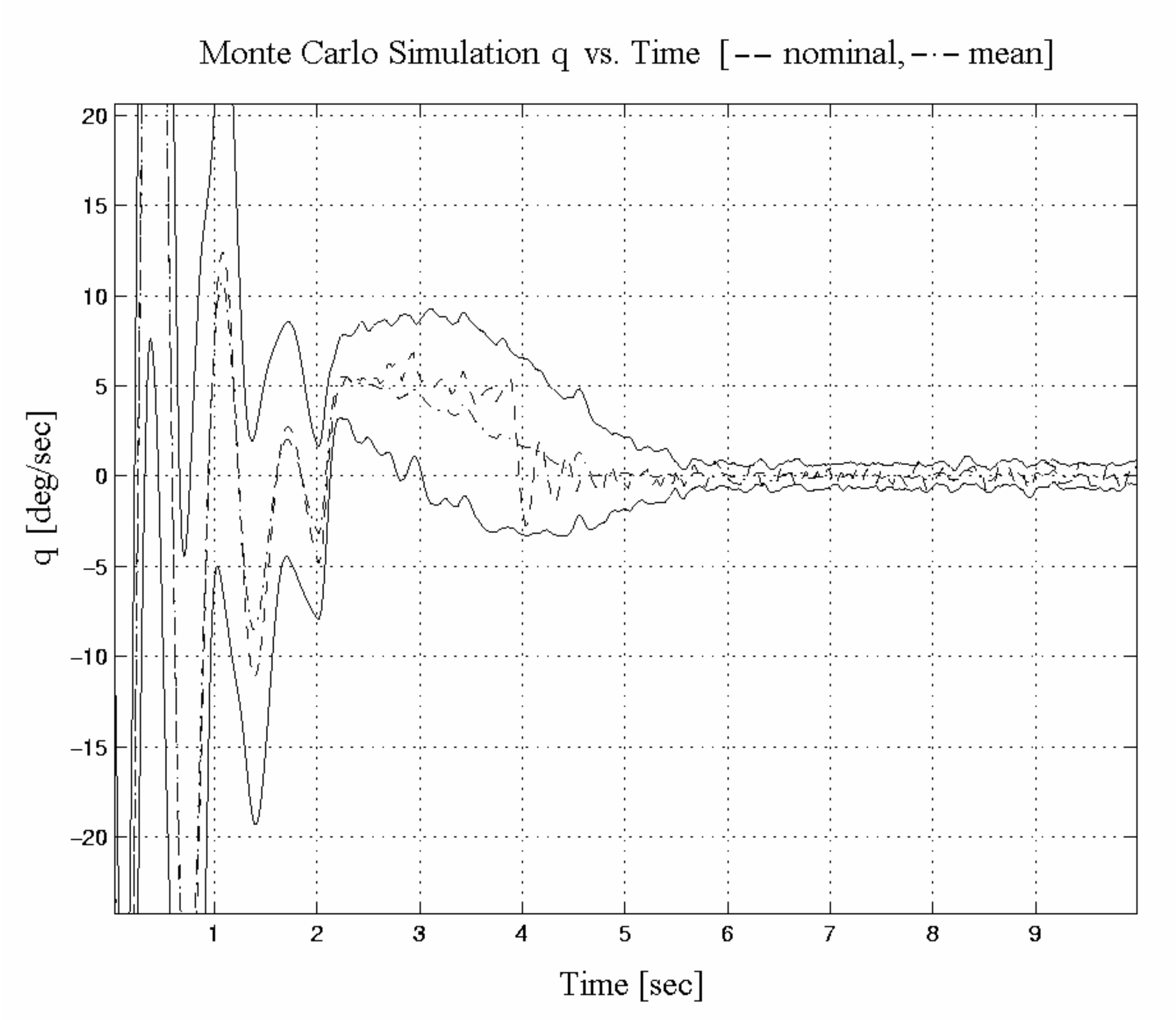

Fig. 5 The results of 100 Monte-Carlo runs: q versus time

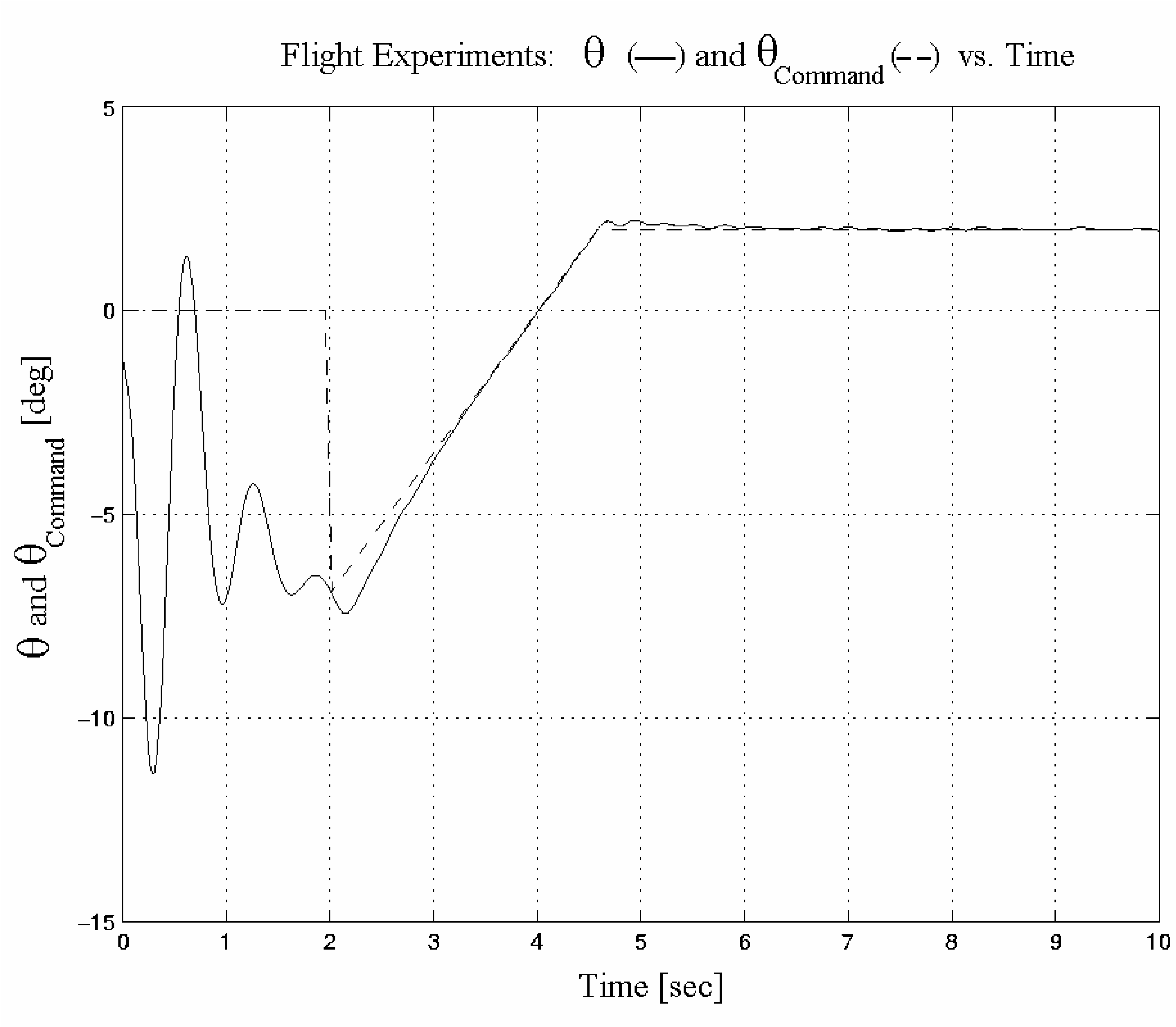

Fig. 6: Flight test: θ and θ_c versus time

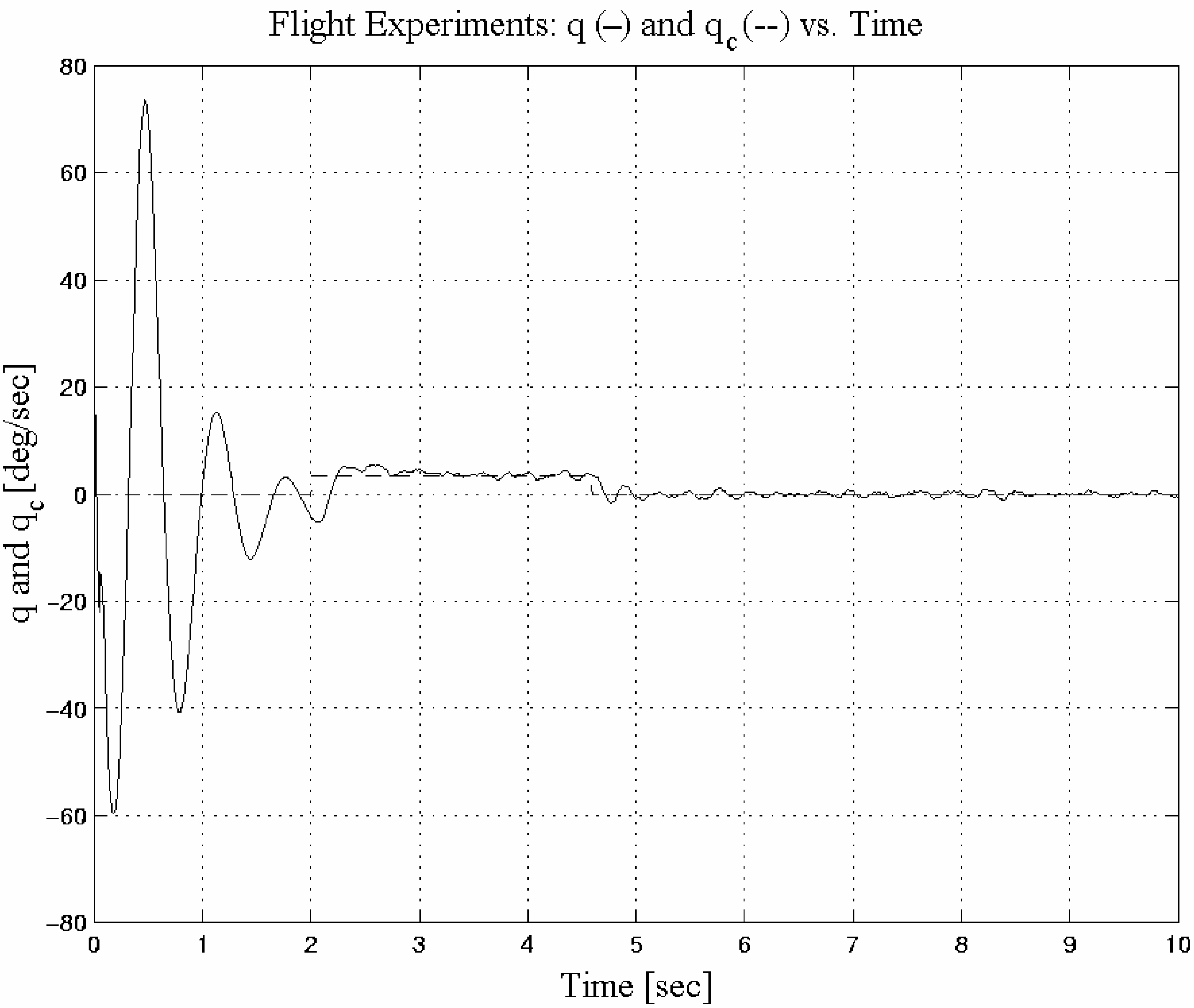

Fig. 7 Flight test: q and q_c versus time