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About this book

- Latest research on Sliding Mode Control
- Constituted from the invited and plenary talks of the 12th IEEE International Workshop Variable Structure Systems (VSS), held 12-14 Jan. 2012 in Mumbai, India
- Written by leading experts in the field

The sliding mode control paradigm has become a mature technique for the design of robust controllers for a wide class of systems including nonlinear, uncertain and time-delayed systems. This book is a collection of plenary and invited talks delivered at the *12th IEEE*

International Workshop on Variable Structure System held at the Indian Institute of Technology, Mumbai, India in January 2012. After the workshop, these researchers were invited to develop book chapters for this edited collection in order to reflect the latest results and open research questions in the area.

The contributed chapters have been organized by the editors to reflect the various themes of sliding mode control which are the current areas of theoretical research and applications focus; namely articulation of the fundamental underpinning theory of the sliding mode design paradigm, sliding modes for decentralized system representations, control of time-delay systems, the higher order sliding mode concept, results applicable to nonlinear and underactuated systems, sliding mode observers, discrete sliding mode control together with cutting edge research contributions in the application of the sliding mode concept to real world problems.

This book provides the reader with a clear and complete picture of the current trends in Variable Structure Systems and Sliding Mode Control Theory.

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Related subjects » Applications - Control Engineering

TABLE OF CONTENTS

sliding- mode control.- Higher Order Sliding Mode Based Accurate Tracking of Unmatched Perturbed Outputs.- On the Second Order Sliding Mode Approach to Distributed and Boundary Control of Uncertain Parabolic PDEs.- Practical relative degree approach in sliding- mode control.- Higher Order Sliding Mode Based Accurate Tracking of Unmatched Perturbed Outputs.

Contents

Practical relative degree approach in sliding-mode control

Arie Levant

Abstract The high-order sliding-mode approach offers a robust way to solve numerous output-regulation problems when the system relative degree is known. Still the difficult cases remain when the relative degree does not exist, is very high, or the mathematical model is not reliable. The notion of practical relative degree is proposed, which generalizes the standard relative-degree notion for the cases of uncertain systems lacking certain mathematical model. Practical output regulation is ensured. Computer simulation and practical results confirm the theoretical approach.

1 Introduction

Control under heavy uncertainty conditions is one of the main subjects of the modern control theory, and the main idea to deal with such problems is to single out and keep properly chosen constraints successively lowering the system dimension. Sliding-mode (SM) or high-gain control are the corresponding methods [12, 20, 42]. The simplest problem of such kind is to make the output σ of a single-input single-output (SISO) "black-box" system vanish.

Sliding mode is accurate and insensitive to disturbances [12, 42]. The informal definition of the relative degree (RD) [18] is the least order of the total output derivative, which explicitly contains the control. While standard SMs are applicable to make a sliding variable vanish, if its RD is 1 [12, 40, 42], high-order sliding modes (HOSMs) [2, 3, 8, 9, 11, 14, 22, 24, 36, 37, 39, 41, 44] are capable of keeping constraints of higher RDs. One of the main reasons for their application is the possibility [2, 22, 28] to effectively attenuate the so-called chattering effect [7, 15, 16] caused by the high control-switching frequency.

Establishing the needed constraint $\sigma = 0$ requires the stabilization of the sliding variable σ at zero. The corresponding dynamics of σ is of the order of the RD and is

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typically uncertain. Theoretically it also allows feedback linearization [18], though the uncertainty prevents its direct application. Finite-time stabilization is preferable, since it provides for higher robustness, simpler overall performance analysis, and higher accuracy in the presence of small sampling noises and delays. With the RD 1 such finite-time stabilization is easily obtained by means of the relay control, which is widely used in the standard sliding-mode control. With higher RDs the problem is much more complicated and corresponds to the HOSM approach [22, 24].

HOSM actually is a motion on the discontinuity set of a dynamic system understood in Filippov's sense [13]. The sliding order characterizes the dynamics smoothness degree in the vicinity of the mode. Let the task be to make some smooth scalar function σ vanish, keeping it at zero afterwards. Then successively differentiating σ along trajectories, a discontinuity will be encountered sooner or later in the general case. Thus, a sliding mode $\sigma \equiv 0$ may be classified by the number r of the first successive total time derivative $\sigma^{(r)}$ which is not a continuous function of the state space variables or does not exist due to some reason, like trajectory nonuniqueness. That number is called the sliding order [22, 24], and the motion $\sigma \equiv 0$ is said to be in rth-order sliding (r-sliding) mode. If σ is a vector, also the sliding order is a vector.

Thus, with the RD r a discontinuous control providing for $\sigma \equiv 0$, inevitably generates an r-sliding mode. In order to attenuate the chattering, the control derivative is used as a new discontinuous control [22, 2, 28]. The RD with respect to the new control turns to be $r+1$ and an $(r+1)$-SM is to be established.

The main result of the HOSM control theory probably is the list of universal SISO controllers corresponding to each RD [24, 26, 33]. Only a few parameters (usually the control magnitude and one differentiator parameter) are to be adjusted to make the "black-box" output exactly vanish in finite time. Thus, the RD turns to be the main needed system information, and it is typically assumed to be known.

Recent results [31, 28] show that fast stable actuators [31] and sensors [28], which can be considered as singular perturbations [20], as well as any small system perturbations affecting the RD [27], only partially destroy the performance of homogeneous SM controllers. Thus, the system RD actually becomes a design parameter. When developing a mathematical model of a controlled process, one can deliberately neglect some dynamics, in order to simplify the model and the corresponding controller.

Unfortunately, the available model can be very complicated, and sometimes it is difficult to present it as a simple low-order system with negligible slow and stable singular disturbances. Moreover, the very existence of the system RD is rather restrictive. It requires the system to be described by a smooth ordinary differential equation linear in control. Hence, a designed controller critically depends on the chosen model form, even if the model is considered unreliable.

The question arises, whether it is possible to treat a system as a "black box", avoiding any dependence on the model. Some recent practical results [17] show that it is possible. An attempt is made in this paper to mathematically justify such approach. The corresponding notion, called practical relative degree (PRD), is formally introduced. It also admits experimental identification by simulation or real life tests.

The PRD is informally defined as the order of the output derivative, which is explicitly affected by control step-function. It can involve system delays and output noises, and it does not require a system mathematical description. In particular, if the system RD exists, it is also a system PRD, but the system can also have a few lower PRDs. A homogeneous SM control designed for a certain RD is shown to be applicable also with the same PRD. The accuracy of the corresponding output regulation is determined by the output reaction delay value.

Even if the RD exists and is known, while theoretically providing for the exact output regulation, a corresponding HOSM controller can be practically impossible to realize due to high information or energy demand. In such a case the PRD approach turns to be a real alternative. Moreover, one can try to apply a number of simple controllers corresponding to lower RDs, without real detection of a PRD. The natural application of such approach lies in the field of artificial-intelligence research.

Computer simulation and recent practical results of blood glucose control [17] demonstrate the feasibility of the suggested approach.

2 Homogeneous sliding mode control.

2.1 Standard SISO Regulation Problem

Definition 1. Consider a discontinuous differential equation $\dot{x} = f(x)$ (Filippov differential inclusion $\dot{x} \in F(x)$ [13, 25]) with a smooth output function $\sigma = \sigma(x)$, and let it be understood in the Filippov sense. Then, provided that

1. successive total time derivatives σ, $\dot{\sigma}$, ..., $\sigma^{(r-1)}$ are continuous functions of x,
2. the set
$$\sigma = \dot{\sigma} = \ldots = \sigma^{(r-1)} = 0 \qquad (1)$$
is a non-empty integral set,
3. the Filippov set of admissible velocities at the r-sliding points (1) contains more than one vector,

the motion on set (1) is said to exist in r-sliding (rth-order sliding) mode [22, 24]. In the non-autonomous case the additional coordinate t is formally added, $\dot{t} = 1$.

Consider a dynamic system of the form

$$\dot{x} = a(t,x) + b(t,x)u, \ \sigma = \sigma(t,x), \qquad (2)$$

where $x \in \boldsymbol{R}^n$, a, b and σ: $\boldsymbol{R}^{n+1} \to \boldsymbol{R}$ are unknown smooth functions, $u \in \boldsymbol{R}$, the dimension n might be also uncertain. Only measurements of σ are available in real time. The task is to provide in finite time for exactly keeping $\sigma \equiv 0$.

The relative degree r of the system is assumed to be constant and known. In other words, for the first time the control explicitly appears in the rth total time derivative of σ and

$$\sigma^{(r)} = h(t,x) + g(t,x)u, \tag{3}$$

where $h(t,x) = \sigma^{(r)}\Big|_{u=0}$, $g(t,x) = \frac{\partial}{\partial u}\sigma^{(r)} \neq 0$. It is supposed that

$$0 < K_m \leq \frac{\partial}{\partial u}\sigma^{(r)} \leq K_M, \ \left|\sigma^{(r)}\Big|_{u=0}\right| \leq C \tag{4}$$

holds for some K_m, K_M, $C > 0$. It is always true at least in compact operation regions. Trajectories of (2) are assumed infinitely extendible in time for any Lebesgue-measurable bounded control $u(t, x)$.

Finite-time stabilization of smooth systems at an equilibrium point by means of continuous control is considered in [1, 6]. In our case any continuous control

$$u = \varphi(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}), \tag{5}$$

providing for $\sigma \equiv 0$, should satisfy the equality $\varphi(0,0,\ldots,0) = -h(t,x)/g(t,x)$, whenever (1) holds. Since the problem uncertainty prevents it, *the control has to be discontinuous at least on the set* (1). Hence, the r-sliding mode $\sigma = 0$ is to be established.

As follows from (3), (4)

$$\sigma^{(r)} \in [-C, C] + [K_m, K_M]u. \tag{6}$$

The obtained inclusion does not "remember" anything on system (2) except the constants r, C, K_m, K_M. Thus, provided (4) holds, the finite-time stabilization of (6) at the origin simultaneously solves the stated problem for all systems (2).

Note that the realization of this plan requires real-time differentiation of the output. The controllers, which are further designed, are *r-sliding homogeneous* [25]. The corresponding notion is introduced below.

2.2 Homogeneous sliding modes

Definition 2. A function f: $\mathbf{R}^n \to \mathbf{R}$ (respectively a vector-set field $F(x) \subset \mathbf{R}^n$ (see [25]), $x \in \mathbf{R}^n$, or a vector field f: $\mathbf{R}^n \to \mathbf{R}^n$) is called *homogeneous of the degree* $q \in \mathbf{R}$ *with the dilation* [1]

$$d_\kappa : (x_1, x_2, ..., x_n) \mapsto (\kappa^{m_1} x_1, \kappa^{m_2} x_2, ..., \kappa^{m_n} x_n),$$

where m_1, ..., m_n are some positive numbers (*weights*), if for any $\kappa > 0$ the identity $f(x) = \kappa^{-q} f(d_\kappa x)$ holds (respectively $F(x) = \kappa^{-q} d_\kappa^{-1} F(d_\kappa x)$, or $f(x) =$

$\kappa^{-q}d_\kappa^{-1}f(d_\kappa x)$). The non-zero homogeneity degree q of a vector field can always be scaled to ± 1 by an appropriate proportional change of the weights m_1, ..., m_n.

The homogeneity of a vector field $f(x)$ (a vector-set field $F(x)$) can equivalently be defined as the invariance of the differential equation $\dot{x}=f(x)$ (differential inclusion $\dot{x}\in F(x)$) with respect to the combined time-coordinate transformation

$$G_\kappa : (t, x) \mapsto (\kappa^p t, d_\kappa x),$$

where p, p = - q, might naturally be considered as the weight of t. Indeed, the homogeneity condition can be rewritten as

$$\dot{x} \in F(x) \Leftrightarrow \frac{d(d_\kappa x)}{d(\kappa^p t)} \in F(d_\kappa x).$$

It was proved in [25] that if $\dot{x} \in F(x)$ is a homogeneous Filippov inclusion with a negative homogeneous degree -p, then uniform finite-time stability, uniform asymptotic stability and the contractivity feature [25] are equivalent and the maximal settling time is a continuous homogeneous function of the initial conditions of the degree p. Furthermore it was proved there that in the presence of variable delays of the order τ^p, and sampling noises of x_i of the order τ^{m_i} the trajectories converge in finite-time into a region featuring $x_i = O(\tau^{m_i})$. Finite-time stability of homogeneous discontinuous differential equations was also considered in [38].

Suppose that feedback (5) imparts homogeneity properties to the closed-loop inclusion (5), (6). Due to the term [-C, C], the right-hand side of (5) can only have the homogeneity degree 0 with $C \neq 0$. Thus, the homogeneity degree of $\sigma^{(r-1)}$ is to be opposite to the degree of the whole system, i.e. $\deg \sigma^{(r-1)} = \deg t = -q$.

Scaling the system homogeneity degree to -1, achieve that the homogeneity weights of t, σ , $\dot{\sigma}$, ..., $\sigma^{(r-1)}$ are 1, r, r - 1, ..., 1 respectively. This homogeneity is further called the *r-sliding homogeneity*. The inclusion (5), (6) is called *r-sliding homogeneous* if for any $\kappa > 0$ the combined time-coordinate transformation

$$G_\kappa : (t, \sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}) \mapsto (\kappa t, \kappa^r \sigma, \kappa^{r-1}\dot{\sigma}, \ldots, \kappa\sigma^{(r-1)}) \qquad (7)$$

preserves the closed-loop inclusion (5), (6).

Transformation (7) transfers (5), (6) into

$$\frac{d^r(\kappa^r\sigma)}{d(\kappa t)^r} \in [-C, C] + [K_m, K_M]\varphi(\kappa^r\sigma, \kappa^{r-1}\dot{\sigma}, \ldots, \kappa\sigma^{(r-1)}).$$

Obviously, (5), (6) is r-sliding homogeneous if $\deg \varphi = 0$, i.e.

$$\varphi(\kappa^r\sigma, \kappa^{r-1}\dot{\sigma}, \ldots, \kappa\sigma^{(r-1)}) \equiv \varphi(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}). \qquad (8)$$

Definition 3. Controller (5) is called r-sliding homogeneous (rth order sliding homogeneous) if (8) holds for any $(\sigma, \dot{\sigma}, ..., \sigma^{(r-1)})$ and $\kappa > 0$. The corresponding sliding mode is also called homogeneous (if exists).

Such a homogeneous controller is inevitably discontinuous at the origin (0, ..., 0), unless φ is a constant function. Most known r-sliding controllers, $r \geq 2$, are based on r-sliding homogeneous controllers. An important exception is the terminal 2-sliding controller maintaining 1-sliding mode $\dot{\sigma} + \beta \sigma^{\rho} \equiv 0$, where $\rho = (2k+1)/(2m+1)$, $\beta > 0$, $k < m$, and k, m are natural numbers [36, 44]. Indeed, the homogeneity requires $\rho = 1/2$ and $\sigma \geq 0$.

2.3 Arbitrary order sliding mode controllers

Following is one of the most known r-sliding controller families [24, 26, 33] called quasi-continuous. The controllers of the form

$$u = -\alpha \Psi_{r-1,r}(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}), \tag{9}$$

are defined by recursive procedures, have the magnitude $\alpha > 0$, and solve the general output regulation problem from Section 2.1 with the relative degree r. Quasi-continuous r-sliding controller is a feedback function of σ, $\dot{\sigma}$, ..., $\sigma^{(r-1)}$ being continuous everywhere except the manifold $\sigma = \dot{\sigma} = \ldots = \sigma^{(r-1)} = 0$ of the r-sliding mode. In the presence of errors in evaluation of σ and its derivatives, these equalities never take place simultaneously with $r > 1$. Therefore, control practically turns to be a continuous function of time.

The parameters of the controllers can be chosen in advance for each r. Only the magnitude α is to be adjusted for any fixed C, K_m, K_M, most conveniently by computer simulation, avoiding complicated and redundantly large estimations. Obviously, α is to be taken negative with $(\partial/\partial u)\sigma^{(r)} < 0$. In the following $\beta_1, \ldots, \beta_{r-1} > 0$ are the controller parameters, and i = 1,..., r-1.The following procedure defines a family of such controllers [26]:

$$\varphi_{0,r} = \sigma,\ N_{0,r} = |\sigma|,\ \Psi_{0,r} = \varphi_{0,r}/N_{0,r} = \operatorname{sign}\sigma,$$
$$\varphi_{i,r} = \sigma^{(i)} + \beta_i N_{i-1,r}^{(r-i)/(r-i+1)} \Psi_{i-1,r},$$
$$N_{i,r} = |\sigma^{(i)}| + \beta_i N_{i-1,r}^{(r-i)/(r-i+1)},\ \Psi_{i,r} = \varphi_{i,r}/N_{i,r}, u = -\alpha \Psi_{r-1,r}.$$

Note that while enlarging α increases the class (4) of systems, to which the controller is applicable, parameters β_i are tuned to provide for the needed convergence rate [32]. Asymptotic accuracies of these controllers are readily obtained from their homogeneity properties. In particular $\sigma^{(i)} = O(\tau^{r-i})$, i = 0, 1, ..., r-1, if the measurements are performed with the sampling interval τ.

A controller providing for the time-optimal stabilization of the inclusion (6) under the restriction $|u| \leq \alpha$ was proposed in [8]. Such controllers are also r-sliding homogeneous providing for the same asymptotic accuracy. Unfortunately, in practice they are only available for $r \leq 3$.

Controller adjustment. The magnitude of the controllers [24, 26] can be increased without loss of the convergence. The corresponding controller gets the form

$$u = -\alpha\Phi(t,x)\Psi_{r-1,r}(\sigma,\dot{\sigma},\ldots,\sigma^{(r-1)}), \tag{10}$$

where $\alpha > 0$, and $\Psi_{r-1,r}$ were introduced above. Note that controller (10) is not homogeneous. While the function Φ can be chosen large to control exploding systems, it is also reasonable to make the function Φ decrease and even vanish, when approaching the system operational point, therefore reducing the chattering [32].

It follows from [33] that the parameters $\beta_1,\ldots,\beta_{r-1}$ can be chosen one-by-one by means of relatively simple simulation of concrete differential equations $\varphi_{i,r} = 0$, all of which are to be finite-time stable. The controller with the resulting parameters formally provides for the universal solution of the stated problem. Nevertheless, in practice one often needs to adjust the convergence rate, either to slow it down relaxing the requirements to actuators, or to accelerate it in order to meet some system requirements. In that context note that redundantly enlarging the magnitude $\alpha\Phi$ of controller (9) does not accelerate the convergence, but only increases the chattering, while its reduction may lead to the convergence loss.

The main procedure is to take the controller

$$u = -\gamma^r\alpha\Psi_{r-1,r}(\sigma,\dot{\sigma}/\gamma,\ldots,\sigma^{(r-1)}/\gamma^{r-1}),\ \gamma > 0,$$

instead of (9), providing for the approximately γ times reduction of the convergence time [32]. With $0 < \gamma < 1$ the convergence is slowed down.

In the case of quasi-continuous controllers the form of the controller is preserved. The new parameters $\tilde{\beta}_1, \ldots, \tilde{\beta}_{r-1}$, $\tilde{\alpha}$ are calculated according to the formulas $\tilde{\beta}_1 = \gamma\beta_1$, $\tilde{\beta}_2 = \gamma^{r/(r-1)}\beta_2$, $\ldots$, $\tilde{\beta}_{r-1} = \gamma^{r/2}\beta_{r-1}$, $\tilde{\alpha} = \gamma^r\alpha$. The larger γ the faster the convergence. Following are the resulting quasi-continuous controllers with $r \le 4$, simulation-tested β_i and a general gain function Φ:

1. $u = -\gamma\alpha\Phi\,\mathrm{sign}\,\sigma$,
2. $u = -\gamma^2\alpha\Phi\,(\dot{\sigma} + \gamma|\sigma|^{1/2}\,\mathrm{sign}\,\sigma)/(|\dot{\sigma}| + \gamma|\sigma|^{1/2})$,
3. $u = -\gamma^3\alpha\Phi\,[\ddot{\sigma} + 2\gamma^{3/2}(|\dot{\sigma}| + \gamma|\sigma|^{2/3})^{-1/2}(\dot{\sigma} + \gamma|\sigma|^{2/3}\,\mathrm{sign}\,\sigma)]/$
$[|\ddot{\sigma}| + 2\gamma^{3/2}(|\dot{\sigma}| + \gamma|\sigma|^{2/3})^{1/2})]$,
4. $\varphi_{3,4} = \dddot{\sigma} + 3\gamma^2[\ddot{\sigma} + \gamma^{4/3}(|\dot{\sigma}| + 0.5\gamma|\sigma|^{3/4})^{-1/3}(\dot{\sigma} + 0.5\gamma)|\sigma|^{3/4}\,\mathrm{sign}\,\sigma)]$
$[|\ddot{\sigma}| + \gamma^{4/3}(|\dot{\sigma}| + 0.5\gamma|\sigma|^{3/4})^{2/3}]^{-1/2}$,
$N_{3,4} = |\dddot{\sigma}| + 3\gamma^2[|\ddot{\sigma}| + \gamma^{4/3}(|\dot{\sigma}| + 0.5\gamma|\sigma|^{3/4})^{2/3}]^{1/2}$,
$u = -\gamma^4\alpha\Phi\,\varphi_{3,4}/N_{3,4}$.

Chattering attenuation. The standard chattering attenuation procedure is to consider the control derivative as a new control input, increasing the relative degree and the sliding order by one [22, 1, 28]. It was many times successfully applied in practice (for example see [3]), though formally the convergence is only locally ensured in some vicinity of the (r + 1)-sliding mode $\sigma \equiv 0$. Global convergence can be easily obtained in the case of the transition from the relative degree 1 to 2 [22]; semi-global convergence can be assured with higher relative degrees using integral (r + 1)-sliding modes [30].

3 Arbitrary Order Robust Exact Differentiation.

Any r-sliding homogeneous controller can be complemented by an (r-1)th order differentiator [4, 41, 43] producing an output-feedback controller. In order to preserve the demonstrated exactness, finite-time stability and the corresponding asymptotic properties, the natural way is to calculate $\dot{\sigma}$, ..., $\sigma^{(r-1)}$ in real time by means of a robust finite-time convergent exact *homogeneous* differentiator [23, 24]. Its application is possible due to the boundedness of $\sigma^{(r)}$ provided by the boundedness of the feedback function φ in (5).

3.1 Standard arbitrary-order robust exact differentiator

Let the input signal $f(t)$ be a function defined on [0, ∞) and consisting of a bounded Lebesgue-measurable noise with unknown features, and of an unknown base signal $f_0(t)$, whose kth derivative has a known Lipschitz constant $L > 0$. The problem of finding real-time robust estimations of $\dot{f}_0(t)$, $\ddot{f}_0(t)$, ... ,$f_0^{(k)}(t)$ being exact in the absence of measurement noises is solved by the differentiator [24]

$$\begin{aligned}
&\dot{z}_0 = v_0, v_0 = -\lambda_k L^{1/(k+1)}|z_0 - f(t)|^{k/(k+1)}\text{sign}(z_0 - f(t)) + z_1,\\
&\dot{z}_1 = v_1, v_1 = -\lambda_{k-1} L^{1/k}|z_1 - v_0|^{(k-1)/k}\text{sign}(z_1 - v_0) + z_2,\\
&\dots\\
&\dot{z}_{k-1} = v_{k-1}, v_{k-1} = -\lambda_1 L^{1/2}|z_{k-1} - v_{k-2}|^{1/2}\text{sign}(z_{k-1} - v_{k-2}) + z_k,\\
&\dot{z}_k = -\lambda_0 L\,\text{sign}(z_k - v_{k-1}).
\end{aligned} \tag{11}$$

The parameters $\lambda_0, \lambda_1, \dots, \lambda_k > 0$ being properly chosen, the following equalities are true in the absence of input noises after a finite time of the transient process:

$$z_0 = f_0(t); \quad z_i = v_{i-1} = f_0^{(i)}, \ \ i = 1, \dots, k.$$

Note that the differentiator has a recursive structure. Once the parameters λ_0, λ_1, $\dots$, λ_{k-1} are properly chosen for the (k - 1)th order differentiator with the Lipschitz constant L, only one parameter λ_k is needed to be tuned for the kth order differentiator with the same Lipschitz constant. The parameter λ_k is just to be taken sufficiently large. Any $\lambda_0 > 1$ can be used to start this process. Such differentiator can be used in any output feedback.

Thus an infinite sequence of parameters λ_n can be built, valid for all k. In particular, one can choose $\lambda_0 = 1.1$, $\lambda_1 = 1.5$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 5$, $\lambda_5 = 8$, which is enough for $k \leq 5$. Another possible choice of the differentiator parameters with $k \leq 5$ is $\lambda_0 = 1.1$, $\lambda_1 = 1.5$, $\lambda_2 = 3$, $\lambda_3 = 5$, $\lambda_4 = 8$, $\lambda_5 = 12$ [25].

The homogeneity features imply the asymptotic accuracy of the differentiator [25]. Let the measurement noise be any Lebesgue-measurable function with the magnitude not exceeding ε. Then the accuracy $|z_i(t) - f_0{}^{(i)}(t)| = O(\varepsilon^{(k+1-i)/(k+1)})$ is obtained. That accuracy is shown to be the best possible [21, 23]. Differentiators (11) with constant and variable parameters L have been already proved useful in practical and theoretical observation [3, 5].

Due to the specific homogeneity features of the differentiator (11), its output-feedback combination with controller (9) produces an r-sliding homogeneous controller: when used in the feedback closure of (6) an r-sliding homogeneous differential inclusion is produced. Thus the asymptotic accuracy of the output-feedback controller remains the same as of controller (9) with *direct* measurements.

3.2 Homogeneous tracking differentiator

The following construction is called a *homogeneous tracking differentiator* [29] of the order k. As previously let the input be a function $f(t) = f_0(t) + \eta(t)$, $|f_0^{(k+1)}| \leq L$, $|\eta| \leq \varepsilon$. Construct an auxiliary dynamic system $w^{(k+1)} = v$ rewritten as

$$\begin{array}{l} \dot{w}_0 = w_1, \cdots, \dot{w}_{k-1} = w_k, \\ \dot{w}_k = v \end{array} \tag{12}$$

with the input v, output w_0 and the measured signal $f(t)$ to be tracked. For the further use rewrite differentiator (11) symbolically by the formula $z = D_{k,\lambda,L}(f)$. Now close system (12) by the feedback

$$\begin{array}{l} v = -\varpi L \Psi_{k,k+1}(z), \\ z = D_{k,\lambda,\varpi_1 L}(w_0 - f) \end{array}. \tag{13}$$

where $\varpi_1 \geq \varpi > 1$. Here $\Psi_{k,k+1}$ is the quasicontinuous controller introduced in Section 2.3, but also any other (k+1)-sliding homogeneous controller can be used. With sufficiently large ϖ obtain a system which starts to track the function $f_0(t)$ in finite time. That implies the following simple theorem.

Theorem 1. *With sufficiently large $\varpi > 1$ and any $\varpi_1 \geq \varpi$ tracking differentiator (12), (13) provides for the finite-time convergence of w_i to $f_0^{(i)}$, $i = 0, 1, \ldots, k$. The asymptotic accuracy of the tracking differentiator (12), (13) is exactly the same as of the standard differentiator [24] (11). In particular, with continuous-time sampling* *the tracking accuracies* $|w_i - f_0^{(i)}| \leq \mu_i L^{i/(k+1)} \varepsilon^{(k+1-i)/(k+1)}$ *are obtained. The coefficients* μ_i *only depend on the tracking differentiator parameters.*

The parameter $\varpi > 1$ can be chosen once and forever. The value $\varpi_1 \geq \varpi$ is adjusted according to the circumstances, in particular, larger values are to be considered in the presence of significant noises. It can be shown that also the digital-implementation asymptotic accuracy of (12), (13) is exactly the same as of the standard differentiator (11) [24]. Also its output-feedback combination with controller (9) produces an r-sliding homogeneous controller which can be used for the solution of the problem from Section 2.1. Also here the asymptotic accuracy of the output-feedback controller remains the same as of controller (9) with *direct* measurements.

The main advantage of the tracking differentiator is that its estimations w_i of the input derivatives $f_0^{(i)}$, $i = 0, 1, \ldots, k-1$, are successive integrals of the kth-order derivative estimation w_k.

4 Practical relative degree concept.

4.1 Practical relative degree (PRD) definition

Consider a SISO system with a scalar input $u \in \mathbf{R}$ (the control), and output $\tilde{\sigma} \in \mathbf{R}$. The output depends on the internal state of the system, which changes in time. The control influences the state in some way. The nature of the state remains unknown. The task is to keep the output $\tilde{\sigma}$ close to zero.

The input belongs to a certain class. For example, it should be Lebesgue-measurable, or continuous, etc. It is assumed in the following that the system accepts Lebesgue-measurable inputs, but the results do not change if inputs are required to have any predefined smoothness.

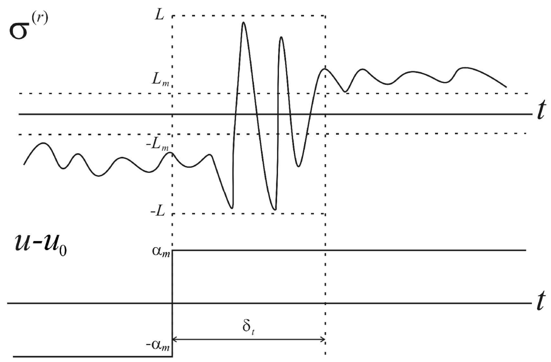

Fig. 1 Reaction of a system with PRD r to a step function.

Definition 4. A natural number r is called a *practical relative degree* (PRD) of the SISO system with the input (control) $u \in \mathbf{R}$, and output $\tilde{\sigma} \in \mathbf{R}$, if there exist positive ε, δ_t, α_m, α_M, L, L_m, $\alpha_m \le \alpha_M$, $L \le L_m$, and $u_0 \in \mathbf{R}$, $\lambda_\sigma = \pm 1$ such that

1. The system accepts any bounded input $u(t)$, $|u - u_0| \le \alpha_M$. The corresponding output can be always represented as a sum of two components, $\tilde{\sigma}(t) = \sigma(t) + \eta(t)$, where $|h| \le \varepsilon$. With $r > 0$ the function σ is assumed r-1 times differentiable with $\sigma^{(r-1)}$ having a uniform Lipschitz constant L. Respectively $\sigma^{(r)}$ exists almost everywhere, $\sigma^{(r)} \le L$.
2. Let $w = \lambda_s \sigma$. For any time moment t_0, if starting from t_0 the inequality $\alpha_M \ge u(t) - u_0 \ge \alpha_m$ $(-\alpha_M \le u(t) - u_0 \le -\alpha_m)$ is kept, then starting from the moment $t_0 + \delta_t$ the output satisfies $w^{(r)} \ge L_m$ (respectively $w^{(r)} \le -L_m$).

Parameters u_0, λ_s, δ_t and ε are respectively called the *reference input*, the *input influence direction*, the *delay* and the *approximation* parameters.

In the following $u_0 = 0$, $\lambda_s = 1$ are assumed. The characteristic reaction of $\sigma^{(r)}$ to the input step function is shown in Fig. 1.

Definition 5. A natural number r is called a *local practical relative degree*, if there exist three time values t_1, t_2, T, $t_1 < t_2$, $\delta_t < T$, such that requirement 2 of Definition 4 is fulfilled for each $t_0 \in [t_1, t_2]$ over the time interval of the length T, and the first requirement is true over the time interval $[t_1, t_2 + T]$.

Obviously, in general one cannot hope to keep $\sigma = 0$ exactly. It should be stressed that the function σ does not necessarily has to have some real meaning. It can be simply an output of some smoothing filter, in particular, of a tracking differentiator. Though ε and δ_t are naturally assumed small with respect to T, t_2 - t_1 , it is not formally required. Local practical relative degree is a temporary feature of a system, usable for temporarily controlling its output.

The following Proposition is obvious.

Proposition 1. *If system (1) satisfies the assumptions of Section 2.1 it also has the practical relative degree r with $\varepsilon = \delta_t = 0$, $u_0 = 0$, $\lambda_\sigma = 1$. If condition (4) is not globally satisfied, but the relative degree still is r, then it has the local practical relative degree r over any compact region of the extended state space t, x.*

Indeed, one can choose any values $\alpha_M > \alpha_m > C/K_m$, $L > C + K_M \alpha_M$, $L_m = L - C - K_M \alpha_M$. If condition (4) is only locally satisfied $\lambda_\sigma = \operatorname{sign} g$ is taken (recall that since the RD is r, the function $g = \frac{\partial}{\partial u}\sigma^{(r)}$ does never vanish).

Choose a controller for a system with the PRD r. The following construction is valid for any r-sliding homogeneous controller [24, 25, 26, 33] of the form (10). Let

$$u = u_0 - \alpha_m \lambda_s \Phi(z_0, z_1, \ldots, z_{r-1}) \Psi_{r-1,r}(z_0, z_1/\gamma, \ldots, z_{r-1}/\gamma^{r-1}), \tag{14}$$

$$z = D_{r-1,\lambda,\tilde{L}}(z_0 - \tilde{\sigma}). \tag{15}$$

Here functions Φ, $\Psi_{r-1,r}$ are described above, and meantime $\Phi \equiv 1$, $D_{r-1,\lambda,\tilde{L}}$ is the homogeneous (r-1)th order differentiator (11). Choose α so that control (10) provides for the finite-time stabilization of the simple system $\sigma^{(r)} = u$; and choose $\gamma > 0$ so that $\gamma^r \alpha \leq L_m$. The differentiator parameter $\tilde{L} > L$ is taken.

Theorem 2. *With the parameters chosen as above, controller (14), (15) provides in finite time for the accuracy* $\sigma^{(i)} \leq \eta_i \max[\delta_t^{r-i}, \varepsilon^{(r-i)/r}]$, $|\tilde{\sigma}| \leq \tilde{\eta}_0 \max[\delta_t^r, \varepsilon]$, $\tilde{\eta}_0$, η_i *being constants only depending on the parameters* L, α_m, L_m *of definition 4 and the choice of the controller parameters* $\tilde{L}$, γ.

Note that with $\varepsilon = \delta_t = 0$ exact output regulation is obtained, which, in particular, extends the known HOSM control results to systems of any nature, for example to possibly discontinuous systems nonlinear in control.

Idea of the proof. After the differentiator converges the arguments of the controller (12) are close to the corresponding derivatives of σ with the errors described in Section 3.1. Thus, in the additional time δ_t the state space with coordinates $(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)})$ is divided in the two regions. The first region corresponds to the points where respectively to the PRD definition the trajectories satisfy the differential inclusion

$$\sigma^{(r)} \in -[L_m, L]\Psi_{r-1,r}(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}),$$

while in the second one any other values of $\sigma^{(r)}$, $\sigma^{(r)} \in -[L, L]$, are possible. Consider the homogeneity transformation

$$G_\kappa : (t, \delta_t, \varepsilon, \sigma, \dot{\sigma}, ..., \sigma^{(r-1)}) \mapsto (\kappa t, \kappa\delta_t, \kappa^r \varepsilon, \kappa^r \sigma, \kappa^{r-1}\dot{\sigma}, ..., \kappa\sigma^{(r-1)}). \quad (16)$$

The second region is proved to be described by homogeneous inequalities, which are preserved by transformation (16). As the result the trajectories satisfy some homogeneous differential inclusion invariant with respect to (16). The further proof follows the standard homogeneity technique [25]. □

Since under the conditions of the PRD definition exact keeping $\sigma \equiv 0$ is impossible with $\varepsilon > 0$, one can use continuous control without compromising the system accuracy. Indeed, it is enough to take the gain function Φ equal to a homogeneous norm of $(\sigma/\gamma_1^r, \dot{\sigma}/\gamma_1^{r-1}, ..., \sigma^{(r-1)}/\gamma_1)$ saturated at 1 with a sufficiently large $\gamma_1 > 0$ in (14).

4.2 PRD identification

One can identify a PRD using analytical methods developed in the sequel. Another way is to experimentally identify a PRD by simulation or even by a real-life test. According to definition 4 the measured system output $\tilde{\sigma}(t)$ is to be the sum of a smooth component and a bounded (preferably small) additional term, $\tilde{\sigma}(t) = \sigma(t) + \eta(t)$. Suppose that in fact also the function $\sigma^{(r)}$ is absolutely continuous with its derivative almost everywhere bounded by some number $\tilde{L}$. Such additional smoothness is usual due to the presence of sensors. Respectively call the PRD *strong*. Apply a tracking differentiator to single out the smooth component $\sigma(t)$:

$$\begin{aligned}
&\dot{w}_0 = w_1, \cdots, \dot{w}_{r-1} = w_r, \\
&\dot{w}_r = -\varpi \tilde{L} \Psi_{r,r+1}(z), \\
&z = D_{r,\lambda,\varpi_1 \tilde{L}}(w_0 - \tilde{\sigma})
\end{aligned} \tag{17}$$

Here $\varpi > 1$ is chosen in advance, and any $\varpi_1 \geq \varpi$ fits. The following proposition is an easy consequence of Theorem 1.

Proposition 2. *Let the system have a strong PRD r, $|\sigma^{(r+1)}| \leq \tilde{L}$. Then observer (17) provides for the accuracies $|w_0^{(i)} - \sigma^{(i)}| \leq \mu_i \tilde{L}^{i/(r+1)} \varepsilon^{(r-i+1)/(r+1)}$, $i = 0, 1, \ldots, r$, established in uniformly bounded time with $\mu_i > 1$ being constants only depending on the observer parameters. Thus, with $L_m > \mu_r \tilde{L}^{r/(r+1)} \varepsilon^{1/(r+1)}$ the function $\sigma(t)$ can be redefined as w_0 in definition 4 with the corresponding change of other parameters. On the other hand, if after some transient the output w_0 of the differentiator differs from the system output $\tilde{\sigma}$ by not more than some constant, and requirements 1, 2 of Definition 4 are satisfied for $\sigma = w_0$, then the system has the strong PRD r.*

Note that, as follows from Theorem 1, if the component σ is already known from the simulation context, the PRD itself can be identified by a bit simpler standard differentiator of the form (11).

4.3 PRD features

In this subsection general examples and properties of the practical relative degree are demonstrated. Consider a SISO system

$$\mu_z \dot{z} = f_z(z, u_z), v_z = v_{z0}(z) + \eta_z(t), \tag{18}$$

where $z \in \mathbf{R}^m$, $u_z \in \mathbf{R}$, $|u_z| \leq U_z$, is the control and the input of the actuator, v_{z0} is a continuous output function, $\mu_z > 0$ is a time-constant parameter, and $\eta_z(t)$ is some deterministic Lebesgue-measurable noise of the magnitude ε_z. The system is understood in Filippov's sense [13] and features the Bounded Input - Bounded State property. The initial values $z(0)$ are assumed belonging to a compact. Thus, z forever belongs to a larger compact W_z. The function $f_z(z, u_z)$ is assumed piece-wise continuous in the region $z \in W_z$, $|u_z| \leq U_z$ with a finite number of compact continuity regions and continuity components extendible up to the region boundaries.

System (6) is further called *a transmission unit*, if the above conditions are satisfied, and there is such $k \neq 0$ that with $\mu_z = 1$ and any $u_z = \text{const}$, the output v_{z0} converges to ku_z uniformly in u_z and initial values of z. That means that for any t_0, $\delta > 0$ there exists $T > 0$ such that with any $z(t_0)$, u_z, $u_z = \text{const}$, the inequality $|v_{z0} - ku_z| \leq \delta$ is kept, starting from the moment T.

Examples. Any LTI stable system with the transfer function $P(\mu_z w)/Q(\mu_z w)$ is a transmission unit, provided $\deg Q - \deg P > 0$, Q is a Hurwitz polynomial, $P(0)/Q(0) = k$. With infinitesimally small μ_z traditional models of actuators and sensors are produced. Another example: $\ddot{v}_z = -\alpha \operatorname{sign}(v_z - ku_z) - \beta \dot{v}_z$, $a, b > 0$, $z = (v_z, \dot{v}_z)$.

Proposition 3. *Transmission units have PRD equal to 0 with the approximation parameter* ε_z.

See [28, 31] for the proof. Note that the time constants of the units are not required to be infinitesimal. Low-pass filters also have zero PRD. In the following Propositions the cascade connections are assumed to satisfy the obvious compliance conditions of input and output bounds.

Proposition 4. *Irrespectively of the connection order, a cascade connection of a SISO system of the PRD r and another SISO system of the PRD 0 has the PRD equal to r with the delay parameter being the sum of two delay parameters. The new approximation parameter equals that parameter of the last system, i.e. of the system at the output.*

A cascade system considered in [28] with actuator and sensor also depending on the middle-system internal state, has the PRD equal to the RD of the system in the middle.

Proposition 5. *A cascade connection of a SISO system of the practical relative degree* r_1 *with the zero approximation parameter and a system of the form (1)-(3) of the relative degree* r_2 *with some bounded output noise forms a new SISO system of the practical relative degree* $r_1 + r_2$.

Note that putting a system with regular RD before a system with PRD may even lead to the loss of PRD. The above propositions allow constructing a lot of systems with PRD. Similar results are also true with respect to the local PRD.

Obviously a system can have a few PRDs. For example, consider a cascade system of successively connected smooth transmission units with RDs r_1, r_2, and a SISO system (1)-(3) with the RD r between them. Let all approximation parameters be zero. Then the resulting system has PRDs $r_1 + r + r_2$, $r_1 + r$, $r + r_2$, and r.

5 Simulation and applications

5.1 Disturbed-kinematic-car-model control

Consider a simple kinematic model of car control

$$\dot{x} = V\cos\varphi,\ \dot{y} = V\sin\varphi,\ \dot{\varphi} = \frac{V}{\Delta}\tan\theta,\ \dot{\theta} = u,$$

where x and y are Cartesian coordinates of the rear-axle middle point, φ is the orientation angle, V is the longitudinal velocity, Δ is the length between the two axles and θ is the steering angle (i.e. the real input), u is the system input (control). The task is to steer the car from a given initial position to the trajectory $y = g(x)$, where

$g(x)$ and y are assumed to be available in real time. The relative degree of the system is 3.

Now consider a disturbed system.

$$\dot{x} = V\cos\phi,\ \dot{y} = V\sin\phi,\ \ddot{\phi} = -4\,\mathrm{sign}(\phi-\varphi) - 6\dot{\phi},$$
$$\dot{\varphi} = \frac{V}{\Delta}\tan\theta,\ \dot{\theta} = \zeta_1.$$

Let $V = \mathrm{const} = 10m/s$, $\Delta = 5m$, $x = y = \varphi = \theta = 0$ at $t = 0$, $g(x) = 10\sin(0.05x) + 5$. Introduce the actuator transmission unit

$$\ddot{\zeta}_1 = -100(2(\zeta_1 - u) + 0.01\dot{\zeta}_1)^3 - 100(\zeta_1 - u) - 2\dot{\zeta}_1,$$

and the sensor transmission unit

$$\dddot{\zeta}_2 = -100(\zeta_2 - y) - 2\dot{\zeta}_2 - 0.02\ddot{\zeta}_2,\ \sigma = \zeta_2 + 0.01\dot{\zeta}_2 - g(x),\ \tilde{\sigma} = \sigma + \eta(t),$$

which produces the noisy output $\tilde{\sigma}$ with σ being a smooth component. Here η is a noise, $|\eta| \leq 0.01m$; $\zeta_2 = -10$, $\dot{\zeta}_2 = 2000$, $\ddot{\zeta}_2 = -80000$, $\zeta_1 = \dot{\zeta}_1 = \phi = \dot{\phi} = 0$ at $t = 0$.

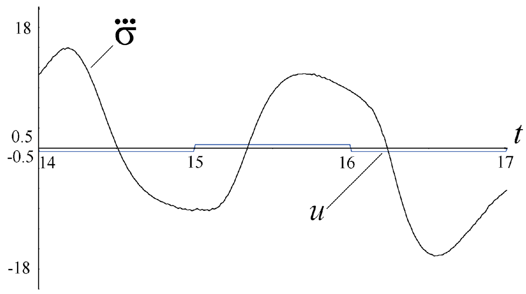

Fig. 2 PRD identification. PRD equals 3.

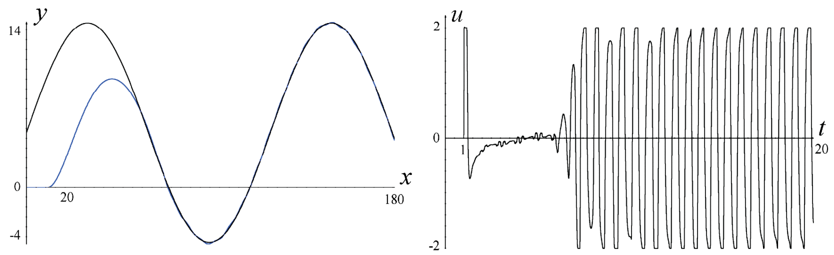

Fig. 3 Performance of the "disturbed-car" control.

Note that the disturbed system does not have a relative degree, for it is not smooth. With $\phi \equiv \varphi$, $\eta = 0$ the RD would be equal to 8. Propositions 2-5 show that it has a local strong PRD equal to 3. The PRD identification results obtained by means of the third-order differentiator $z = D_{3,\{1.1,1.5,2,3\},100}(\tilde{\sigma})$

$$
\begin{aligned}
&\dot{z}_0 = v_0,\, v_0 = -9.49|z_0 - \tilde{\sigma}(t)|^{3/4}\,\mathrm{sign}(z_0 - \tilde{\sigma}(t)) + z_1,\\
&\dot{z}_1 = v_1,\, v_1 = -9.28|z_1 - v_0|^{2/3}\,\mathrm{sign}(z_1 - v_0) + z_2,\\
&\dot{z}_2 = v_2,\, v_2 = -15|z_2 - v_1|^{1/2}\,\mathrm{sign}(z_2 - v_1) + z_3,\\
&\dot{z}_3 = -110\,\mathrm{sign}(z_3 - v_2).
\end{aligned}
$$

of the form (11) are demonstrated in Fig. 2.

The applied control consists of the quasi-continuous 3-sliding controller

$$
\begin{aligned}
&u = 0,\ \ 0 \le t < 1,\\
&u = -2[s_2 + 2(|s_1| + |s_0|^{2/3})^{-1/2}(s_1 + |s_0|^{2/3}\,\mathrm{sign}\, s_0)]/[|s_2| + 2(|s_1| + |s_0|^{2/3})^{1/2}]\\
&\hfill \text{with } t \ge 1;
\end{aligned}
$$

and the second-order differentiator

$$
\begin{aligned}
&\dot{s}_0 = v_1,\, v_1 = -9.28|s_0 - \tilde{\sigma}(t)|^{2/3}\,\mathrm{sign}(s_0 - \tilde{\sigma}(t)) + s_1,\\
&\dot{s}_1 = v_1,\, v_1 = -15|s_1 - v_0|^{1/2}\,\mathrm{sign}(s_1 - v_0) + s_2,\\
&\dot{s}_2 = -110\,\mathrm{sign}(s_2 - v_1).
\end{aligned}
$$

The tracking results are shown in the coordinates x, y of the "car" in Fig. 3. The obtained accuracy is $|y - g(x)| \le 0.16m$.

Model	RD	No. States
Bergman	3	3
Candas-Radziuk	3	4
Cobelli	3	7
Hovorka	5	8
Dalla Man	5	8
Sorensen	5	18

- Output: blood glucose
    - Input: insulin

Fig. 4 Summary of different models for the dynamics of the blood glucose concentration.

5.2 Practical control of glucose blood concentration

Consider now the practical problem of controlling the glucose concentration in the human blood [10, 17, 19, 35]. The concentration is measured in real time once per minute, and a pump injects insulin, when it is needed. A number of models are available with the relative degrees changing from 3 to 5 and the number of variables changing from 3 to 18 (Fig. 4, courtesy to A.G. Gallardo-Hernandez, [17]).

Computer simulation shows that most models have the PRD 3 [17] (not all models were checked). A controller of the same form, as for the above "car" control, was applied. Negative values of the insulin injections were simply zeroed. The recent experimental results on **live rats** with the same control are shown in Fig. 5 (courtesy to A.G. Gallardo-Hernandez, [17]).

6 Conclusions

A new concept of practical relative degree is introduced, which generalizes the standard relative degree notion to systems of arbitrary nature, not necessarily described by ordinary differential equations. The results are new already for smooth dynamic systems nonlinear in control.

A new homogeneous tracking differentiator is proposed featuring the same asymptotic accuracy as the standard homogeneous differentiator [24], but at the same time producing smooth estimations of the input derivatives, higher-order estimations being exact derivatives of the lower order ones. It has found natural application in identification of the practical relative degree.

Propositions 1, 2-5 provide many examples of systems with practical relative degrees. One system can have a few practical relative degrees, which means that controllers developed for different relative degrees can prove to be efficient for the same system. The lowest practical relative degree is not always the best choice: a lot

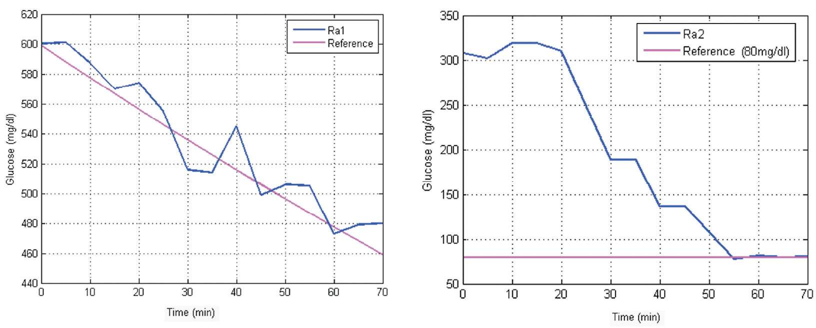

Fig. 5 Control of the glucose concentration in the blood of live rats. PRD equals 3.

depends also on on the corresponding delay and approximation parameters. The new concept significantly generalizes the previous results showing that one can neglect fast stable actuators [31], fast stable sensors [28] and small perturbations changing the relative degree [27].

Thus, actually a new class of systems is singled out. The further theoretical research is to find various practical examples of systems with practical relative degrees and estimation of their delay and approximation parameters, which actually can be of no resemblance to real noises and delays. The notion can be probably extended to multi-input multi-output case.

Another natural application is in the artificial intelligence research. In such a case one can successively try universal HOSM controllers corresponding to lower practical relative degrees 1, 2, 3, even without performing an attempt of the practical relative degree identification.

References

1. Bacciotti, A.,Rosier, L.: Liapunov functions and stability in control theory. Springer Verlag, London (2005)
2. Bartolini, G., Ferrara, A., Usai, E.: Chattering avoidance by second-order sliding mode control, IEEE Trans. Automat. Control 43(2), 241-246 (1998)
3. Bartolini, G., Pisano, A., Punta, E., Usai, E.: A survey of applications of second-order sliding mode control to mechanical systems. International Journal of Control 76(9/10), 875-892 (2003)
4. Bartolini G., Pisano A., Usai E.: First and second derivative estimation by sliding mode technique. Journal of Signal Processing 4(2), 167-176 (2000)
5. Bejarano, F.J., Fridman, L.: High order sliding mode observer for linear systems with unbounded unknown inputs. International Journal of Control 83(9), 1920-1929 (2010)
6. Bhat, S.P., Bernstein, D.S.: Finite time stability of continuous autonomous systems, SIAM J. Control Optim. 38(3), 751-766 (2000)
7. Boiko, I., Fridman, L.: Analysis of chattering in continuous sliding-mode controllers. IEEE Trans. Automatic Control 50(9), 1442-1446 (2005)
8. Dinuzzo, F., Ferrara, A.: Higher order sliding mode controllers with optimal reaching. IEEE Trans. Automatic Control 54(9), 2126-2136 (2009)
9. Defoort, M., Floquet, T., Kokosy, A., Perruquetti, W.: A novel higher order sliding mode control scheme. Systems & Control Letters 58, 102-108 (2009)
10. Dorel, L.: Glucose level regulation via integral high-order sliding modes. Mathematical Biosciences and Engineering 8(2), 549-60 (2011)
11. Evangelista, C., Puleston, P., Valenciaga, F.: Wind turbine efficiency optimization. Comparative study of controllers based on second order sliding modes, International Journal of Hydrogen Energy (2010), available online 8 January 2010.
12. Edwards, C., Spurgeon, S.K.: Sliding mode control: theory and applications. Taylor & Francis (1998)
13. Filippov, A.F.: Differential equations with discontinuous right-hand side. Kluwer, Dordrecht, the Netherlands (1988)
14. Floquet, T., Barbot, J.-P., Perruquetti, W.: Higher-order sliding mode stabilization for a class of nonholonomic perturbed systems. Automatica 39, 1077-1083 (2003)
15. Fridman, L.: Chattering analysis in sliding mode systems with inertial sensors. International Journal of Control 76(9/10), 906-912 (2003)

16. Furuta, K., Pan, Y.: Variable structure control with sliding sector. Automatica 36, 211-228 (2000)
17. Gallardo-Hernandez, A.G., Fridman, L., Levant, A., Shtessel, Y., Leder, R., Islas-Andrade, S., Revilla-Monsalve, C.: High-order sliding-mode control of blood glucose concentration via practical relative degree identification. In Proc. IEEE CDC 2011, Orlando, FL, USA, Dec 12-15, (2011).
18. Isidori, A.: Nonlinear control systems, second edition. Springer Verlag, New York (1989)
19. Kaveh, P., Shtessel, Y.B.: Blood glucose regulation using higher-order sliding mode control. International Journal of Robust and Nonlinear Control 18(4-5), 557-569 (2008)
20. Kokotovic, P.V., Khalil, H.K., O'Reilly, J.: Singular perturbation methods in control: analysis and design. SIAM, (1999)
21. Kolmogoroff, A.N.: On inequalities between upper bounds of consecutive deriva-tives of an arbitrary function defined on an infinite interval. Amer. Math. Soc. Transl 2, 233-242 (1962)
22. Levant, A. (Levantovsky, L.V.): Sliding order and sliding accuracy in sliding mode control, International Journal of Control 58(6), 1247-1263 (1993)
23. Levant, A.: Robust exact differentiation via sliding mode technique. Automatica 34(3), 379-384 (1998)
24. Levant, A.: Higher-order sliding modes, differentiation and output-feedback control. International Journal of Control 76 (9/10), 924-941 (2003)
25. Levant, A.: Homogeneity approach to high-order sliding mode design. Automatica 41(5), 823-830 (2005)
26. Levant, A.: Quasi-continuous high-order sliding-mode controllers. IEEE Trans. Automat. Control 50(11), 1812-1816 (2006)
27. Levant, A.: Robustness of homogeneous sliding modes to relative degree fluctuations. In: Proc. of 6th IFAC Symposium on Robust Control Design, June 16-18, 2009, Haifa, Israel (2009)
28. Levant, A.: Chattering analysis. IEEE Transactions on Automatic Control 55(6), 1380-1389 (2010)
29. Levant, A.: Digital sliding-mode-based differentiation. In Proc. 11th Scientific Workshop VSS'12, Mumbay (India), January 12-14 (2012)
30. Levant, A., Alelishvili, L.: Integral high-order sliding modes. IEEE Trans. Automat. Control 52(7), 1278-1282 (2007)
31. Levant, A., Fridman, L. : Accuracy of homogeneous sliding modes in the presence of fast actuators. IEEE Transactions on Automatic Control 55(3), 810-814 (2010)
32. Levant, A., Michael, A.: Adjustment of high-order sliding-mode controllers. International Journal of Robust and Nonlinear Control 19(15), 1657-1672 (2009)
33. Levant, A. and Y. Pavlov, (2008) Generalized homogeneous quasi-continuous controllers, *International Journal of Robust and Nonlinear Control*, 18(4-5), 385–398.
34. Levant, A., Pridor, A., Gitizadeh, R., Yaesh, I., Ben-Asher, J.Z.: Aircraft pitch control via second order sliding technique. J. of Guidance, Control and Dynamics 23(4), 586-594 (2000)
35. Kaveh, P., Shtessel, Y.B.: Blood glucose regulation using higher-order sliding mode control. International Journal of Robust and Nonlinear Control 18(4-5), 557–569 (2008)
36. Man, Z., Paplinski, A.P., Wu, H.R.: A robust MIMO terminal sliding mode control for rigid robotic manipulators. IEEE Trans. Automat. Control 39(12), 2464-2468 (1994)
37. Massey, T., Shtessel, Y.: Continuous traditional and high order sliding modes for satellite formation control. AIAA J. Guidance, Control, and Dynamics 28(4), 826-831 (2005)
38. Orlov, Y.: Finite time stability and robust control synthesis of uncertain switched systems. SIAM J. Cont. Optim. 43(4), 1253-1271 (2005)
39. Pisano, A., Davila, J., Fridman, L., Usai, E.: Cascade control of PM DC drives via second-order sliding-mode technique. IEEE Transactions on Industrial Electronics 55(11), 3846-3854 (2008)
40. Sira-Ramirez, H.: On the dynamical sliding mode control of nonlinear systems. International Journal of Control 57(5), 1039-1061 (1993)
41. Shtessel, Y.B., Shkolnikov, I.A.: Aeronautical and space vehicle control in dynamic sliding manifolds. International Journal of Control 76(9/10), 1000-1017 (2003)

42. Utkin, V.I.: Sliding modes in optimization and control problems. Springer Verlag, New York (1992)
43. Yu, X., Xu, J.X.: An adaptive signal derivative estimator. Electronic Letters 32(16) (1996)
44. Yu, S., Yu, X., Shirinzadeh, B., Man,Z.: Continuous finite-time control for robotic manipulators with terminal sliding mode, Automatica 41(11), 1957-1964 (2005)