BOOLEAN PRAGMATICS

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As is well-known, Lesniewsky developed mereology as an alternative to set theory. Its promise as an alternative foundational theory was reduced when Tarski showed (see Tarski 1927, 1935) that mereologies are identical to Boolean algebras minus the bottom element 0 (and helped developing the theory of Boolean algebras in set theory). After all, if that's the only difference, why quibble about a 0 element that is easily deleted and easily regained?

I will argue in this paper that the 0 element is not as easily deleted in the nominal domain, and this shows up in Boolean semantics, the semantics of noun phrases. And I will argue that the 0 element is indeed easily deleted and regained in the verbal domain, which is to its advantage, because we can use it as a basis for Boolean pragmatics.

1. BOOLEAN SEMANTICS: A BIRDS-EYE VIEW

1.1. Complete Atomic Boolean algebras

A *complete lattice* is a set B ordered by a partial order of *part-of*, $\sqsubseteq$, and a *sum* operation $\sqcup$, which maps every subset X of B onto $\sqcup$X, the smallest element of B that all elements of X are part of.

Every complete lattice has a minimal element, $0 = \sqcup(\emptyset)$. (Trivially all elements of Ø are part of everything, hence the smallest element of B that all elements of Ø are part of is just the smallest element of B, null.)

A *complete atomic Boolean algebra* is a complete lattice satisfying*:*

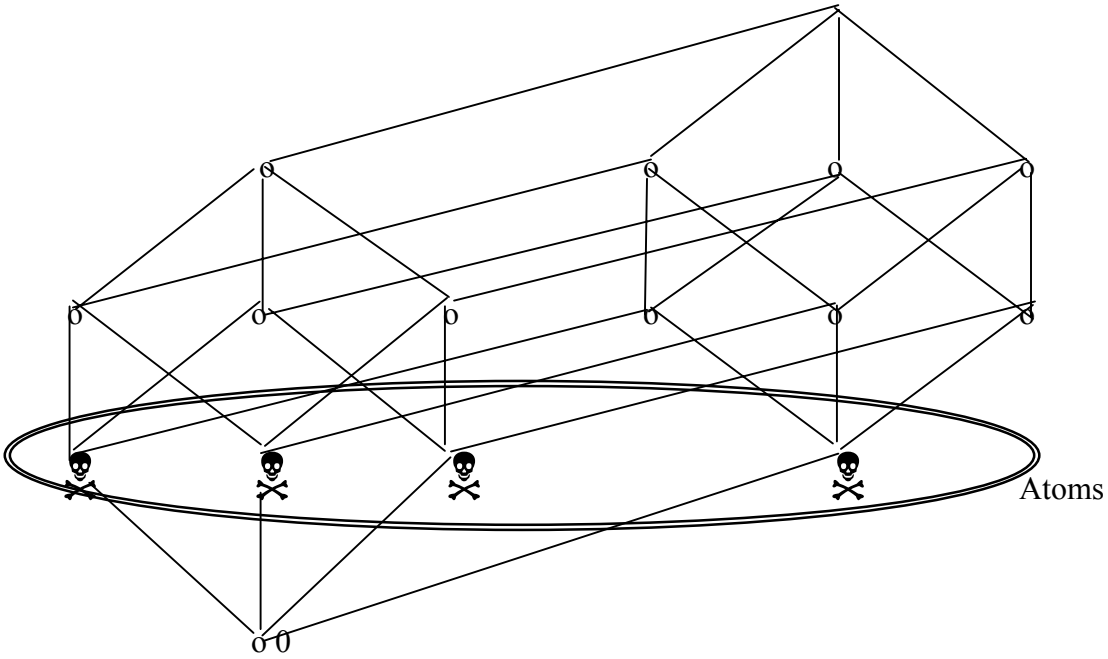
-*Distributivity:* For any b in B: if you cut b into b_1 and b_2, then any part of b ends up as either part of b_1, or of b_2, or split over b_1 and b_2.

-*Remainder:* If c is a non-null proper part of b, and you cut c out of b, then what is left is itself a non-null proper part of b that has no non-null part in common with c: the remainder of c in b.

Atomicity: Every non-null element has a part which has only itself as non-null part: an *atom* in B.

Cardinality is counting of atomic parts: $|b| = |\{a \in ATOM_B\ a \sqsubseteq b\}|$.

Below is a picture of a sixteen element complete atomic Boolean algebra with four atoms.



The two central ideas of the Boolean semantics of noun phrases are due to Link 1983 and Sharvy 1980.

1.2. Semantic pluralization.

Link 1983 proposes that the domain of individuals forms a complete atomic Boolean algebra, and he lets singular and plural count nouns denote sets of Boolean elements. He assumes that *singular predicates* denote sets of atoms, and that *semantic pluralization* is closure under sum:

> Let P be a subset of B:
> $*P = \{b \in B\text{: for some } X \subseteq P\text{: } b = \sqcup X\}$
> The set of all sums of P-elements.

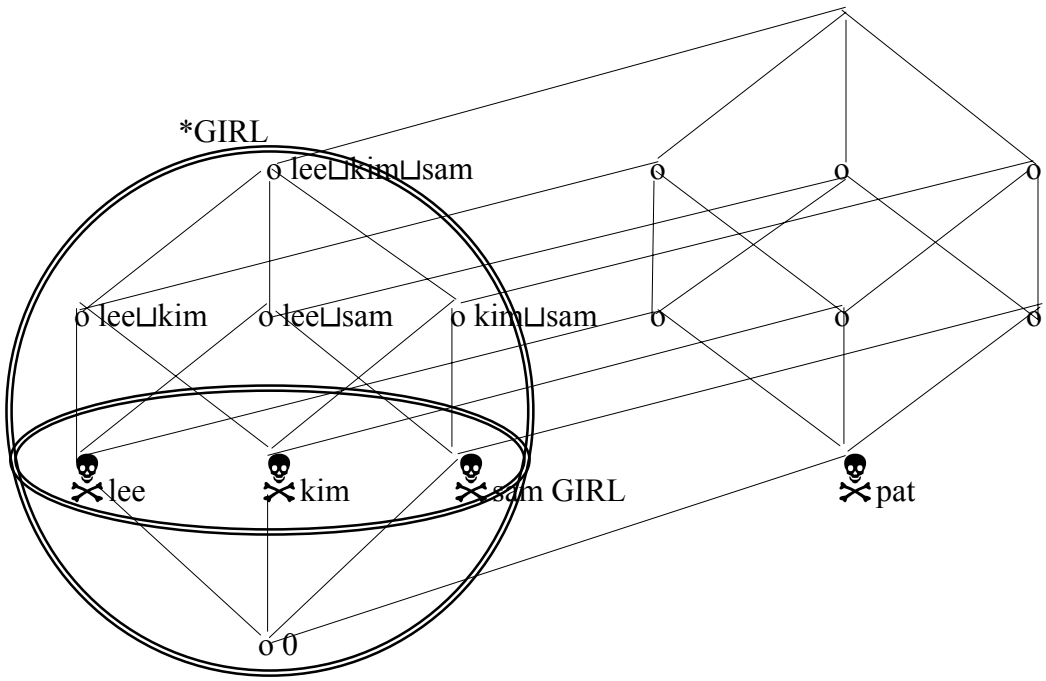
For example, let the singular noun *girl* denote GIRL, where

$$GIRL = \{lee, kim, sam\}.$$

Then the plural noun *girls* denotes *GIRL, where

$$*GIRL = \{0, lee\sqcup kim, lee\sqcup sam, kim\sqcup sam, lee\sqcup kim\sqcup sam\}$$

For our present purposes, it is important to note that $0 \in *GIRL$.
The reason is that $\emptyset \subseteq GIRL$, hence $\sqcup\emptyset \in *GIRL$ by definition of * and, as we saw, $\sqcup\emptyset = 0$.



1.3. Definiteness.

Sharvy 1980 generalizes Russell's definite description operation to a presuppositional sum operation, the sigma operation: if the noun *nomen* denotes N, then the definite noun phrase *the nomen* denotes σ(N). And the semantics for σ is specified as follows:

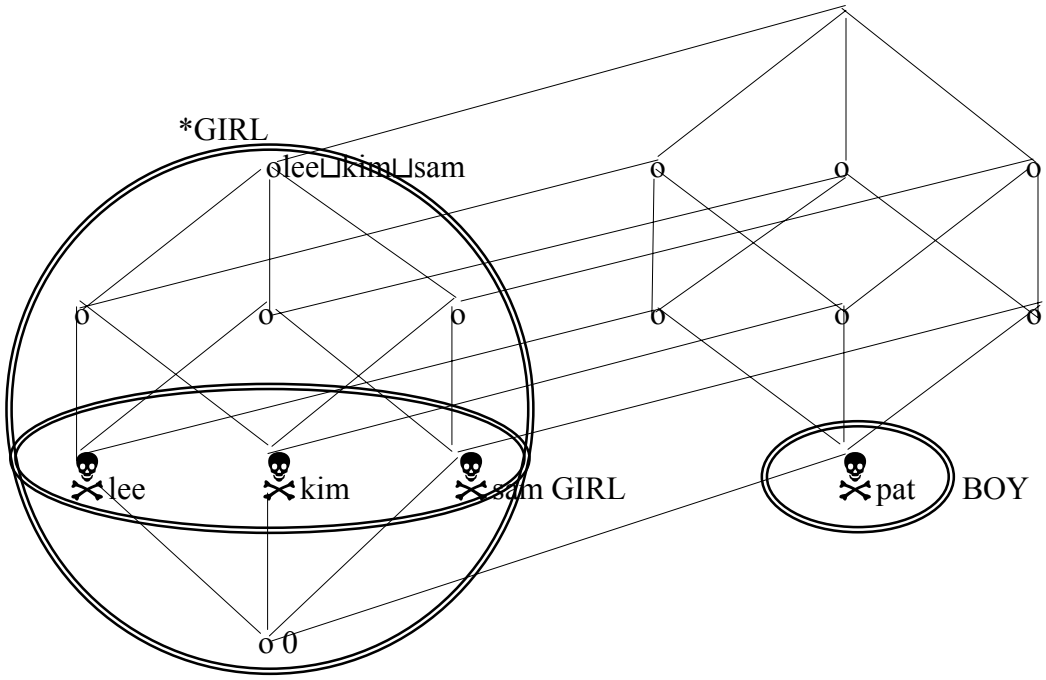
$$\sigma(N) = \begin{cases} \sqcup N \text{ if } \sqcup N \in N \\ \perp \quad \text{otherwise} \qquad \text{(where } \perp \text{ stands for undefined)} \end{cases}$$

We assume that *girl* denotes GIRL and GIRL = {lee, kim, sam}.
We assume that *boy* denotes BOY and BOY = {pat}.
Then the following sums are given:

⊔GIRL = ⊔*GIRL = lee⊔kim⊔sam
⊔BOY = pat

With this we get the following noun phrase interpretations:

the girl	is interpreted as σ(GIRL)	
	σ(GIRL) = ⊥	because lee⊔kim⊔sam ∉ GIRL
the girls	is interpreted as σ(*GIRL)	
	σ(*GIRL) = lee⊔kim⊔sam	because lee⊔kim⊔sam ∈ *GIRL
the boy	is interpreted as σ(BOY)	
	σ(BOY) = pat	because pat ∈ BOY



1.4. Numerical noun phrases.

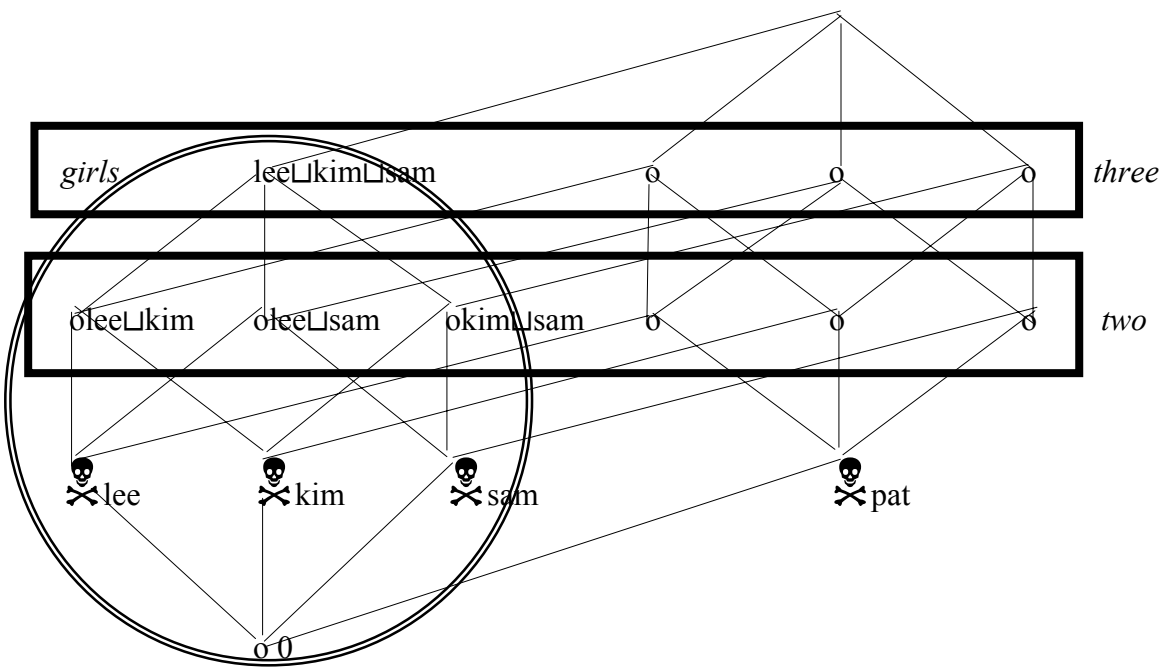
Landman 2004 proposes that numericals have the semantics of intersective adjectives:

three is interpreted as:	$\lambda x.\|x\|=3$, the set of all sums of three atoms.

The intersective semantics gives the following noun phrase interpretations:

three girls is interpreted as	$\lambda x.{*}GIRL(x) \wedge \|x\|=3$ = {lee⊔kim⊔sam}
two girls is interpreted as	$\lambda x.{*}GIRL(x) \wedge \|x\|=2$ = {lee⊔kim, lee⊔sam, kim⊔sam}

The picture shows that these noun phrase denotations are not closed upward or downward within *GIRL:

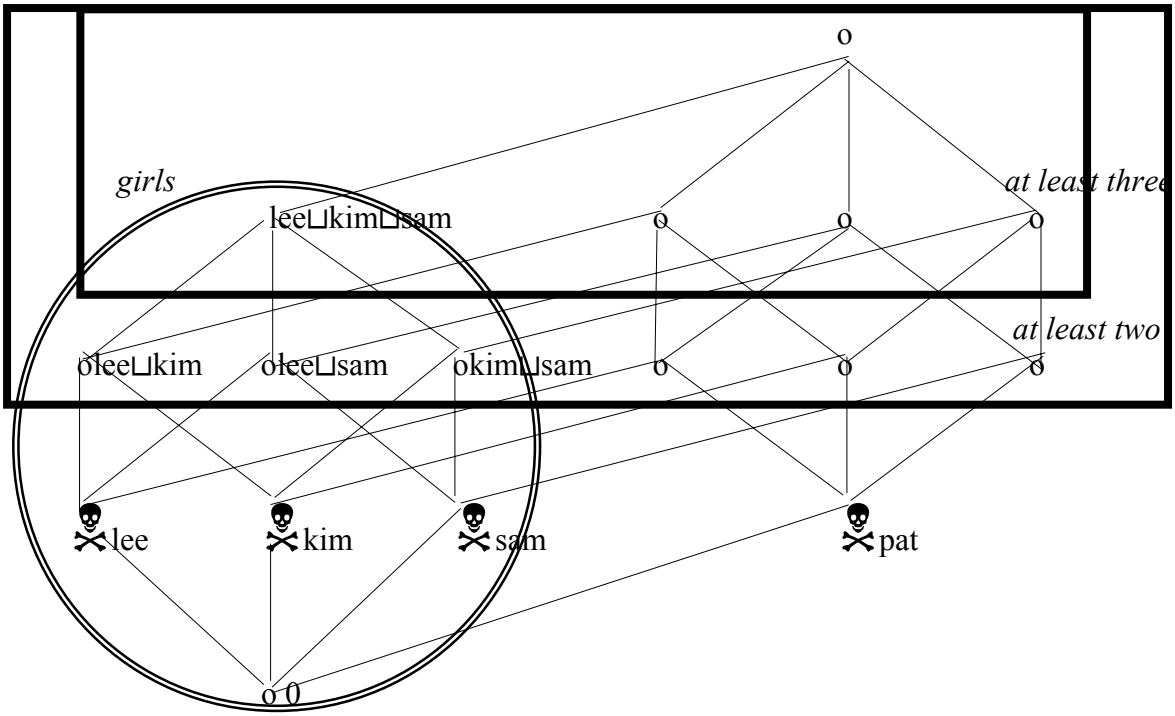


at least three is interpreted as: $\lambda x.|x|\geq 3$,
the set of sums of at least three atoms.

The intersective semantics gives the following noun phrase interpretations:

at least three girls is interpreted as $\lambda x.{}^*GIRL(x) \wedge |x|\geq 3$
$= \{lee\sqcup kim\sqcup sam\}$
at least two girls is interpreted as $\lambda x.{}^*GIRL(x) \wedge |x|\geq 2$
$= \{lee\sqcup kim, lee\sqcup sam, kim\sqcup sam, lee\sqcup kim\sqcup sam\}$

In this case the noun phrase denotations are closed upward within *GIRL:

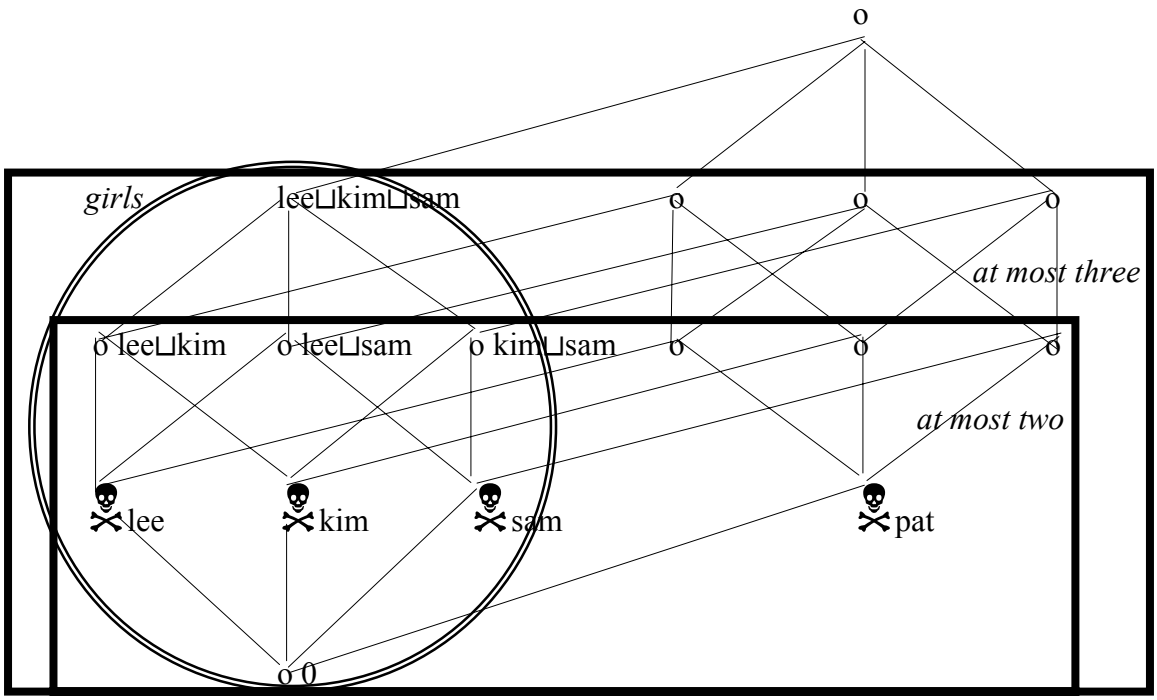


at most three is interpreted as: $\lambda x.|x| \leq 3$,
the set of sums of at most three atoms.

The intersective semantics gives the following noun phrase interpretations:

at most three girls is interpreted as $\lambda x.{}^*GIRL(x) \wedge |x| \leq 3$
$= {}^*GIRL$
at most two girls is interpreted as $\lambda x.{}^*GIRL(x) \wedge |x| \leq 2$
$= \{lee \sqcup kim, lee \sqcup sam, kim \sqcup sam, lee, kim, sam, 0\}$

In this case the noun phrase denotations are closed downward within *GIRL:



This semantics gives the correct interpretations for the definite noun phrases:

the girls is interpreted as $\sigma(*GIRL)$
$\sigma(*GIRL) = lee \sqcup kim \sqcup sam$

the three girls is interpreted as $\sigma(\lambda x.*GIRL(x) \wedge |x|=3)$
$\sigma(\lambda x.*GIRL(x) \wedge |x|=3) = lee \sqcup kim \sqcup sam$

the two girls is interpreted as $\sigma(\lambda x.*GIRL(x) \wedge |x|=2)$
$\sigma(\lambda x.*GIRL(x) \wedge |x|=2) = \perp$

Namely: $\sqcup(\lambda x.*GIRL(x) \wedge |x|=2) = lee \sqcup kim \sqcup sam$, and
$lee \sqcup kim \sqcup sam \notin \lambda x.*GIRL(x) \wedge |x|=2$

the at least two girls is interpreted as $\sigma(\lambda x.*GIRL(x) \wedge |x|\geq 2)$
$\sigma(\lambda x.*GIRL(x) \wedge |x|\geq 2) = lee \sqcup kim \sqcup sam$

the boy is interpreted as $\sigma(BOY)$
$\sigma(BOY) = pat$

the at most three girls is interpreted as $\sigma(\lambda x.*GIRL(x) \wedge |x|\leq 3)$
$\sigma(\lambda x.*GIRL(x) \wedge |x|\leq 3) = lee \sqcup kim \sqcup sam$

the at most two girls is interpreted as $\sigma(\lambda x.*GIRL(x) \wedge |x|\leq 2)$
$\sigma(\lambda x.*GIRL(x) \wedge |x|\leq 2) = \perp$

Namely: $\sqcup(\lambda x.*GIRL(x) \wedge |x|\leq 2) = lee \sqcup kim \sqcup sam$, and
$lee \sqcup kim \sqcup sam \notin \lambda x.*GIRL(x) \wedge |x|\leq 2$

2. BOOLEAN PRAGMATICS

2.1. Undefined and null-denoting definites

We now come to the null object. For this reason we introduce the dog. There isn't any. We interpret *dog* as DOG, and assume that DOG = $\emptyset$.

Here are the predictions of the theory for the definite noun phrases with noun *dog*:

the dog is interpreted as $\sigma(\mathrm{DOG})$
$\sigma(\mathrm{DOG}) = \perp$

Namely: $\mathrm{DOG} = \emptyset$, $\sqcup\emptyset = 0$.
$\sigma(\mathrm{DOG})$ is defined if $0 \notin \emptyset$, which is not the case,
hence $\sigma(\mathrm{DOG}) = \perp$

the dogs is interpreted as $\sigma(*\mathrm{DOG})$
$\sigma(*\mathrm{DOG}) = 0$

Namely: $\mathrm{DOG}=\emptyset$, $\sqcup\emptyset = 0$.
$\emptyset \subseteq \emptyset$, $*\emptyset = \{x: \exists X \subseteq \emptyset: x = \sqcup X\} = \{x: x = \sqcup\emptyset\} = \{\sqcup\emptyset\} = 0$,.
Hence $*\mathrm{DOG} = \{0\}$, and thus $\sigma(*\mathrm{DOG}) = 0$

the at least two dogs is interpreted as $\sigma(\lambda x.*\mathrm{DOG}(x) \wedge |x|\geq 2)$
$\sigma(\lambda x.*\mathrm{DOG}(x) \wedge |x|\geq 2) = \perp$

Namely: $*\mathrm{DOG} = \{0\}$, $|0|<2$,
hence $\lambda x.*\mathrm{DOG}(x) \wedge |x|\geq 2 = \emptyset$
and $\sigma(\emptyset) = \perp$

the at most two dogs is interpreted as $\sigma(\lambda x.*\mathrm{DOG}(x) \wedge |x|\leq 3)$
$\sigma(\lambda x.*\mathrm{DOG}(x) \wedge |x|\leq 3) = 0$

Namely: $*\mathrm{DOG} = \{0\}$, $|0|\leq 2$,
hence $\lambda x.*\mathrm{DOG}(x) \wedge |x|\geq 2 = \{0\}$
and $\sigma(\{0\}) = 0$

The *central observation* is: When the denotation of the singular noun is empty, some definites are undefined, others denote 0.

2.2. Presuppositions and implicatures.

I follow the standard assumption about the undefinedness that relates to the Sharvy condition for definedness: undefinedness leads to presuppositionality:

> *Presuppositionality:*
> My use of a definite in a context k presupposes that the definite is not undefined in k and that means that its Sharvy condition for definedness is satisfied in k.

I am not interested here at all in adding to the theories of presuppositionality. I am solely concerned with the standard diagnostics: presuppositions cannot be cancelled by direct denial, and in this they differ from conversational implicatures.

I accept this diagnostics, and with it the argument that we teach in introductory classes that universal noun phrases like *every dog* do not have a presupposition that there is a dog or there are dogs, but only an implicature to that effect.

This is, of course, what the standard semantics of *every* as a determiner relating sets of individuals predicts:

> *every* is interpreted as EVERY
> EVERY = $\lambda Q \lambda P.\ Q \subseteq (ATOM \cap P)$
> The relation that holds between Q and P if Q is a set of atoms (singularities) and Q is a subset of P.

The semantics of *every dog* is:

> *every dog* is interpreted as $\lambda P.DOG \subseteq P$
> $\lambda P.DOG \subseteq P$ is the set of all properties that every dog has.

If DOG = Ø, $\lambda P.DOG \subseteq P$ = pow(B), the set of all properties.
This means, of course, that when the denotation of the noun *dog* is empty, the interpretation of *every dog verbum* is *trivially true* for all predicates *verbum*.

Grice's Maxims of Quantity and Quantity instruct us to *tell the truth*, but to *avoid triviality*. This brings in a conversational implicature:

> *Conversational implicature:*
> If I use *every dog* in a context in accordance with the informativeness maxims, there is an implicature that DOG≠Ø, i.e. *there are dogs.*

The standard semantics does not produce a presupposition that there are dogs, but does produce an implicature. And, of course, as we show in our introduction classes, this is correct, because the implicature can be cancelled by direct denial:

Suppose I stand trial for fraud, and I say (1a) to the judge, but add *sotto voce* (1b) to you:

(1) a. Your honor, ***every person who has come to me during 2004 with a winning lottery ticket*** has gotten a prize.
b. ✓Fortunately, I was on a polar expedition the whole year.

The judgement is that this continuation is felicitous. What I am trying to do here is mislead the judge, *without* perjuring myself. I am making a statement that is *trivially true*, hoping that the judge will be naïve enough to take it for a non-trivially true statement.

If the existence statement were a presupposition, the continuation would be infelicitous. The fact that the *sotto voce* comment is felicitous, and the analysis of what the effect is that I am trying to reach, suggest that the classical semantic analysis of *every* as not presuppositional is correct.

2.3. Presuppositional and non-presuppositional definites.

We now come back to the definites, and we check in the same context the felicity of the *sotto voce* continuation for different definites. We find the following. In the cases in (2) - (4), the continuation is felicitous:

The persons who…
(2) a. Your honor, ***the persons who have come to me during 2004 with a winning lottery ticket*** have gotten a prize.
b. ✓Fortunately, I was on a polar expedition the whole year.

The at most three persons who…
(3) a. Your honor, ***the at most three persons who have come to me during 2004 with a winning lottery ticket*** have gotten a prize.
b. ✓Fortunately, I was on a polar expedition the whole year.

The less than three persons who…
(4) a. Your honor, ***the less than three persons who have come to me during 2004 with a winning lottery ticket*** have gotten a prize.
b. ✓Fortunately, I was on a polar expedition the whole year.

In the cases in (5)-(8), the continuation is not felicitous:

The three persons who…
(5) a. Your honor, ***the three persons who have come to me during 2004 with a winning lottery ticket*** have gotten a prize.
b. #Fortunately, I was on a polar expedition the whole year.

The at least three persons who…

(6) a. Your honor, ***the at least three persons who have come to me during 2004 with a winning lottery ticket*** have gotten a prize.

b. #Fortunately, I was on a polar expedition the whole year.

The more than three persons who…

(7) a. Your honor, ***the more than three persons who have come to me during 2004 with a winning lottery ticket*** have gotten a prize.

b. #Fortunately, I was on a polar expedition the whole year.

The many persons who…

(8) a. Your honor, ***the many persons who have come to me during 2004 with a winning lottery ticket*** have gotten a prize.

b. #Fortunately, I was on a polar expedition the whole year.

(Note that I didn't put in an example with *the few persons….* parallel to *the many people.* The reason is that it is difficult to get clear judgements here. The question is whether *few* in *the few...* means *a number less than a small number* or *a* ***positive*** *number less than a small number*.)

Also infelicitous is the singular:

The person who… $\perp$

(9) a. Your honor, ***the person who has come to me during 2004 with a winning lottery ticket*** has gotten a prize.

b. #Fortunately, I was on a polar expedition the whole year.

We must make sure in the latter case that we do not get interference with a generic reading. The contrast shows up clearly in following non-generic case:

(10) a. Of the ten students in my class I would say that *the* ***students*** *that studied for the test* got a good grade (✓but nobody did).

b. Of the ten students in my class I would say that *the* ***student*** *that studied for the test* got a good grade (#but nobody did).

We note a contrast, a remarkably robust contrast, between the cases in (2)-(4) and (10a), where the continuation is felicitous, and the cases in (5)-(9) and (10b), where the continuation is infelicitous. And we observe that the cases where the continuation is felicitous are *exactly the cases* that the Boolean semantics predicts to be *0-denoting* if the denotation of the noun phrase complement of the determiner *the* is empty, while the cases where the continuation is infelicitous, are *exactly the cases* that the Boolean semantics predicts to be *undefined* if the denotation of the noun phrase complement of the determiner *the* is empty.

We observe, then, that the Boolean semantics, plus the standard assumption about the connection between presuppositions and undefinedness, makes the correct prediction

for the cases in (5)-(9) and (10b): the theory predicts that these cases have an existence presupposition, and they do.

Also, the theory makes the correct prediction about the *absence* of a presupposition in the cases (2)-(4) and (10a): the theory predicts that the semantics of these cases does not involve undefinedness, so a presupposition is not expected. And indeed, what we find is an implicature rather than a presupposition.

Finally, at a broad theoretical level, we observe that if we had chosen our domains to be mereologies, rather than Boolean algebras, we would not have predicted a difference at all, because mereologies do not have a 0-object, and hence subsume the 0-denoting case under the $\perp$-case. With that move you would expect presuppositions in all the cases discussed here, and you would not expect to find the contrast we *do* find.

So the 0 object shows up in noun denotations and in the data: it affects the presupposition *versus* implicature status of the existence statement associated with the noun complement of the definite determiner.

2.4. The pragmatics of the null event.

I will assume a neo-Davidsonian event semantics along the lines of Landman 2000 in which verbs denote sets of events and are linked to arguments through roles, which are – with one exception – partial functions from events to objects of the argument type. The domain of events is itself a complete atomic Boolean algebra, and hence contains a null object, 0_e, the null event. The one exception concerns the null-event. I make the following two assumptions about events, roles and the null-event.

> *Assumption 1:* If for some role R: $R(e)=0$ then $e=0_e$
> If the value for one of the roles defined on event e is specified as 0,
> then e is the null event.

> *Assumption 2:* For every object $b \in B$ and role R: $R(0_e) = b$
> The null event is an event for which anything in B fills every role.

Thus, roles are *functions* on normal events, but *relations* on 0_e: anything is the Agent of 0_e, and the Theme, and… As we will see, this makes 0_e a *trivial event*.

In Landman 2000 I assumed that verbal predicates are by default pluralized. I will continue to assume that, but in a slightly modified form. We have seen that the semantics of nouns makes predictions about which noun phrases include the nominal null object in their denotation. I assume that verbal predicates do not semantically constrain their denotation in this way: for verbal predicates we can in essence *choose* whether to include 0_e or not.

$$*\text{VERB} = \{e \in E\text{: for some } X \subseteq \text{VERB: } e = \sqcup X\}$$
$$^{\circledast}\text{VERB} = *\text{VERB} - \{0_e\}$$
$$= \{e \in E\text{: for some } X \subseteq \text{VERB: } X \neq \emptyset \text{ and } e = \sqcup X\}$$

Both *VERB and $^{\circledast}$VERB are plural, but *VERB is a Boolean denotation, while $^{\circledast}$VERB a mereological denotation. I propose:

Assumption 3: In verbal denotations we can shift between plural interpretations *VERB and ☼VERB.

Thus, the inclusion or exclusion of 0_e in a verbal denotation is not semantically constrained. 0_e is the only event for which roles can specify 0 as the value. 0_e itself takes everything as its value for every role.

Now suppose we have a verbal predicate *verbum*. In a world we associate with it a set of atomic events VERB and assume that the denotation of *verbum* is *VERB or ☼VERB.

Now we look at the statement $0_e \in verbum$. Obviously, this is true if we choose the denotation to be *VERB and false if we choose it to be ☼VERB.

What is important for our purposes is that the truth value of $0_e \in verbum$ doesn't depend on anything *but the choice between * and ☼*. And this means that the truth value of $0_e \in verbum$ is *not contingent* at all: $0_e \in verbum$ is *trivially* true if the denotation of *verbum* is chosen to be *VERB, and *trivially false* if the denotation of *verbum* is chosen to be ☼VERB.

The intuition is the following: for normal events e, it is a matter of the semantics of the verb *verbum*, the facts in the world, and the context whether or not in a particular situation and context event e is in the denotation of the verb. This means that it is, in general, a contingent matter whether or not $e \in verbum$ is true.

But this is not the case for the statement $0_e \in verbum$: it is a trivially true statement or a trivially false statement, depending on our choice of denotation for *verbum* *VERB or ☼VERB.

What does our choice depend on? That is specified in the next assumption:

Assumption 4: Pragmatics of the choice of *VERB *versus* ☼VERB:
Trivialities are either semantically harmless or innocent. Choose in context whichever denotation of *verbum*, *VERB or ☼VERB, is semantically innocent.

The choice is driven pragmatically by the informativeness maxims, which I will give for our purposes the following specific form:

1. *Quality: Speak the truth.*
   - A truth is better than a falsehood
   - A tautology is better than a contradiction.
2. *Quantity: Avoid triviality.*
   - A contingent statement is better than a trivial one.

With this, we follow Leibniz and calculate.

2.5. The 0-object at work.

Let us look at a normal contingent sentence: (11a) and its alternative choices (11b) and (11c):

(11) a. Ronya meowed .
b. $\exists e[e \in {}^{*}\text{MEOWED} \wedge \text{Agent}(e)=\text{Ronya}]$
c. $\exists e[e \in {}^{\text{☼}}\text{MEOWED} \wedge \text{Agent}(e)=\text{Ronya}]$

By definition of * and ☼, $0_e \in {}^{*}\text{MEOW}$ and $0_e \notin {}^{\text{☼}}\text{MEOW}$, and by definition of 0_e; $\text{Agent}(0_e) = \text{Ronya}$. This means that (11b) is trivially true, while (11c) is a contingent statement. By assumption (3), *avoid triviality,* the speaker intends to utter the contingent statement (11c), and that is how the hearer understands (11a). We choose interpretation (11c).

(Note that am here assuming that both alternatives are considered in the grammatical derivation. Alternatively, we can assume that one is chosen in the grammar, but we allow back-tracking to change it, when necessary. I do not, at this point, feel the need to choose between these two alternatives.)

Let us look at the statements (2) – (4), in the *sotto voce* context. We assume that the denotation of the singular noun phrase *person who has come to me during 2004 with a winning lottery ticket* is the empty set. This means that in all cases in (2) – (4), the statement made is (12a) or (12b):

(12) a. $\exists e[e \in {}^{*}\text{GOT} \wedge \text{Receiver}(e)=0 \wedge \text{Theme}(e) \in \text{PRIZE}]$
b. $\exists e[e \in {}^{\text{☼}}\text{GOT} \wedge \text{Receiver}(e)=0 \wedge \text{Theme}(e) \in \text{PRIZE}]$
There is a sum of getting events with 0 as Receiver and a prize as Theme.

Assumption 1 tells us that if Receiver(e)=0 then $e=0_e$. So (12) is equivalent to (13):

(13) a. $0_e \in {}^{*}\text{GOT} \wedge \text{Receiver}(0_e)=0 \wedge \text{Theme}(0_e) \in \text{PRIZE}$
b. $0_e \in {}^{\text{☼}}\text{GOT} \wedge \text{Receiver}(0_e)=0 \wedge \text{Theme}(0_e) \in \text{PRIZE}$

Assumption 2 tells us that $\text{Receiver}(0_e)=0$ and $\text{Theme}(0_e) \in \text{PRIZE}$ are trivially true of 0_e, hence (13) is equivalent to (14):

(14) a. $0_e \in {}^{*}\text{GOT}$
b. $0_e \in {}^{\text{☼}}\text{GOT}$

This means that in this situation, the statement made by (14) is not a contingent statement at all, it is either (14b), a contradiction, or (14a), a tautology. By the maxim of quality, a tautology is better than a contradiction, and we choose to interpret the cases in (2) – (4) as (14a), a tautology.

What have we achieved? We have given a semantics in which under the assumption that the relative noun denotation is empty, the cases in (2) – (5) turn out to be *trivial*, while the case in (6) – (9) are undefined.

Undefinedness brings in a presupposition, that is why the continuations in (6) - (9) are infelicitous. But the cases in (2) – (5) are not infelicitous, they are tautologies, provide no information, hence provide no information that the judge can show to be false or incriminating, which was what I was after.

Now that we have linked the semantics and pragmatics of the cases in (2) – (5) formally to triviality, the pragmatic explanation of why the use of the universal noun phrase *every nomen* conversationally implicates that the denotation of *nomen* is not empty carries over straight away to the cases in (2) – (5): *avoid triviality* brings in an *implicature* that the denotation of the noun phrase complement of the definite article is not empty.

We conclude:

-The inclusion-exclusion of the 0-object in the denotation of noun phrases (in languages that realize plurality on nouns) is semantically encoded: the Boolean semantics of nouns tells us which noun phrase denotations contain 0 and which don't.

-The inclusion/exclusion of the 0_e in the denotation of verb phrases is open to pragmatic manipulation, I call it Boolean pragmatics.

-Boolean semantics and Boolean pragmatics together account for the observed differences in presuppositions and implicatures for the definite noun phrases discussed.

2.6. Boolean pragmatics and presupposition accommodation.

We now look at the examples in (15).

(15) a. In every family, ***the three boys*** go into the army.
b. In every family, ***the boys*** go into the army

We observe a contrast that is similar to what we have observed above:
`

(15a) presupposes that *in every family there are three boys*.
(15b) does not presuppose that *in every family there are boys.*

Mechanisms have been proposed in the literature to account for the fact that (15a) has this presupposition. And given these mechanisms, mechanisms (like local accommodation) have been proposed to account for the fact that (15b) does not have this presupposition.

My claim is that we do not need any mechanism here: the basic facts fall out of the Boolean semantics and pragmatics.

We look at (15a) first. As we will see, in this case, the choice between * and ☼ is not at issue, so for ease we choose *. The interpretation of (15a) is (16):

(16) $\forall f[\mathrm{FAMILY}(f) \rightarrow \exists e[{}^*\mathrm{GA}(e) \wedge \mathrm{Agent}(e) = \sigma(\lambda x.{}^*\mathrm{BOY}(x) \wedge |x|=3 \wedge \mathrm{In}(x,f))]$
For every family f, there is a sum of going into the army (GA) events with *the three boys in family f* as agent
(It is, of course, debatable whether the subject here is an agent or a theme.)

We assume that there are families, $f_1,..,f_n$.
The statement in (16) is equivalent to the conjunction in (17):

(17) $\exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge |x|=3 \wedge IN(x,f_1))] \wedge \ldots$
$\wedge \exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge |x|=3 \wedge IN(x,f_n))]$

For each family f_k, the definite $\sigma(\lambda x.{}^*BOY(x) \wedge |x|=3 \wedge IN(x,f_k))$ denotes a sum of three boys, if there are three boys in f_k, and is undefined if there aren't.

So if in family f_k there aren't three boys, $\sigma(\lambda x.{}^*BOY(x) \wedge |x|=3 \wedge IN(x,f_k))$ is undefined. We take this to mean that the conjunct of (17) corresponding to family f_k, $\exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge |x|=3 \wedge IN(x,f_k))]$, presupposes that there are three boys in f_k.

Since the same holds for each conjunct in (17), we take this to mean that (15a) indeed presupposes that *in every family, there are three boys.*

(There are, obviously, questions here about *how* the notion of presupposition exactly links to the undefinedness semantics, and how projection takes place. There is a whole tradition of different answers to this question, but I am not concerned here with presupposition projection at that level of detail. In other words, I am happy to do it your way.)

We look at the alternative interpretations for (15b) in (18):

(18) a. $\forall f[FAMILY(f) \rightarrow \exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge In(x,f))]$
b. $\forall f[FAMILY(f) \rightarrow \exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge In(x,f))]$
For every family f, there is a sum of going into the army (GA) events with *the boys in family f* as agent.

For families $f_1,\ldots,f_n$, the statements in (18) are equivalent to the conjunctions in (19):

(19) a. $\exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_1))] \wedge \ldots$
$\wedge \exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_n))]$

b. $\exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_1))] \wedge \ldots$
$\wedge \exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_n))]$

For each family f_k, the definite $\sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_k))$ denotes a sum of the boys in that family. If there are no boys in a family f_k, then the definite $\sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_k))$ denotes 0.

Let us assume again that in family f_k there are no boys. The options for the relevant conjunct are given in (20):

(20) a. $\exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_k))]$
b. $\exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*BOY(x) \wedge IN(x,f_k))]$

Given that $\sigma(\lambda x.{*}BOY(x) \wedge IN(x,f_k)) = 0$, for any event e satisfying this, Agent(e)=0, and hence $e=0_e$. The statement $Agent(0_e) = 0$ holds by assumption 2. Hence these options are equivalent to the options in (21):

(21) a. $0_e \in {*}GA$
b. $0_e \in {}^{\circledast}GA$

(21a) is a tautology, while (21b) is a contradiction. With this in mind, we look back at the conjunction in (19). We note:

If T is a tautology, $(\varphi \wedge T)$ is logically equivalent to φ
If C is a contradiction, $(\varphi \wedge C)$ is logically equivalent to C

Obviously, we want (15b) to be an informative statement, and this means that we must choose interpretation (18a) and not (18b):

(18) a. $\forall f[FAMILY(f) \rightarrow \exists e[{*}GA(e) \wedge Agent(e) = \sigma(\lambda x.{*}BOY(x) \wedge In(x,f))]$
(19) a. $\exists e[{*}GA(e) \wedge Agent(e) = \sigma(\lambda x.{*}BOY(x) \wedge IN(x,f_1))] \wedge \ldots$
$\wedge\ \exists e[{*}GA(e) \wedge Agent(e) = \sigma(\lambda x.{*}BOY(x) \wedge IN(x,f_n))]$

In that case, the conjunct in question is a tautology, *and it drops out of the conjunction.* This means, *de facto*, that (15a) is equivalent to (22):

(22) a. In every family ***that has boys in it***, the boys go into the army.
b. $\forall f[FAMILY(f) \wedge \exists x[BOY(x) \wedge IN(f,x)]$
$\rightarrow \exists e[{*}GA(e) \wedge Agent(e) = \sigma(\lambda x.{*}BOY(x) \wedge IN(x,f))]$

This means, of course, that (15b) does not presuppose that every family has boys in it, on the contrary: (15b) has nothing to say about families without boys.

This means that the facts in (15) follow, *pace* an articulated notion of presupposition, from the Boolean semantics and Boolean pragmatics, and don't need any further assumptions about presupposition accommodation.

2.7. More Boolean pragmatics

Here too, there *is* space for pragmatic 0_e manipulation. Look at (23):

(23) In no family do ***the girls*** go into the army.

(23) does *not* presuppose that there are girls in *any* family, let alone in *every* family. We take the two options for (23) to be the options in (24):

(24) a. $\neg\exists f[FAMILY(f) \wedge \exists e[{*}GA(e) \wedge Agent(e) = \sigma(\lambda x.{*}GIRL(x) \wedge IN(x,f))]$
b. $\neg\exists f[FAMILY(f) \wedge \exists e[{}^{\circledast}GA(e) \wedge Agent(e) = \sigma(\lambda x.{*}GIRL(x) \wedge IN(x,f))]$

The options in (24) are equivalent to the conjunctions in (25):

(25) a $\neg\exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_1))] \wedge \ldots$
$\wedge \neg\exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_n))]$

b. $\neg\exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_1))] \wedge \ldots$
$\wedge \neg\exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_n))]$

We look at family f_k, where there are no girls. The relevant options for the conjunct are given in (26):

(26) a $\neg\exists e[{}^*GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_k))]$
b. $\neg\exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_k))]$

In this case, the options for the conjunct reduce to those in (27):

(27) a $\neg(0_e \in {}^*GA)$
b. $\neg(0_e \in {}^{\text{☼}}GA)$

In this case, it is (27b) which is the tautology and (27a) which is the contradiction, hence in this case, we choose as interpretation (24b):

(24) b. $\neg\exists f[FAMILY(f) \wedge \exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f))]$
(25) b. $\neg\exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_1))] \wedge \ldots$
$\wedge \neg\exists e[{}^{\text{☼}}GA(e) \wedge Agent(e) = \sigma(\lambda x.{}^*GIRL(x) \wedge IN(x,f_n))]$

In (25b), the conjunct corresponding to family f_k is a tautology and drops out of the conjunction: (23) is not about families in which there are no girls: it says of families in which there are girls, that the girls don't go into the army.

Again, the facts follow directly from the Boolean semantics and pragmatics.

3. CONCLUSION

The contrasts discussed in this paper (which to my knowledge were first noted in Landman 2004) are surprisingly robust, and form a rather impressive confirmation of the Boolean semantics presented above.

To me it comes rather as a surprise that the choice between Boolean algebras and mereologies makes a difference after all, and too that the structures best suited for natural language semantics turn out to be the mathematically most simple and general ones.

By interpreting the null event as we did, we managed to link statements with 0-denoting definites directly to triviality, unifying these cases with the classical approach to universal quantification. Again, to me, this is confirmation from a rather surprising direction that the classical approach to the universal quantifier is the right one.

Finally, Boolean pragmatics – null event manipulation triggered by triviality concerns – is a very minimal and local operation, and the Gricean argumentations it

manages to avoid are rather impressive. By this I mean that, while the Boolean pragmatic operation is completely local, the Gricean rational for choosing one interpretation over another is itself of a very global nature. It is nice to be able to keep those rationales out of the manipulative system. It seems interesting to see how far this system can be pushed, how much more can be done by very little.

ACKNOWLEDGEMENTS

This paper is a reworked version of material that appears in Landman 2004 (spread over several sections). In its present form it formed part of a talk I gave at the Summer Conference of Abralin, the Brazilian Association for Linguistics, in February 2011 in Curatiba. Thanks to Roberta Pires de Oliveira and Susan Rothstein for convincing me to extract and rework this material and present it.

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