

Mach-Zehnder interferometer temperature sensor simulation

March 19, 2014

1 Introduction

1.1 Temperature profile

We first assume a time-dependent fiber temperature profile. If Newton's law of cooling holds, we may write

$$\frac{dT}{dt} = -\kappa(T - T_s) \quad (1)$$

where $T(t)$ is the fiber temperature, κ is the heating rate and T_s is the surrounding temperature.

Hence,

$$T(t) = T_s + (T_0 - T_s) e^{-\kappa t} \quad (2)$$

where $T(0) = T_0$.

1.2 Gaussian beams and interference

The fibers consisting the Mach-Zehnder interferometer are single-mode fibers, so we assume a Gaussian beam profile at the output of the fiber of diameter $W_0 = 4.3 \mu\text{m}$ [2]. For a wavelength $\lambda = 633 \text{ nm}$ we know from Gaussian optics [1, p.83] the Rayleigh range z_0 and we therefore have the complex amplitude of the beam $U(\rho, z)$. We will use this expression to calculate the amplitude of the beam inside the beam-splitter. The important contribution that allows the formation of interference fringes is the $\exp\left(-jk\frac{\rho^2}{2R(z)}\right)$ part. Let's assume that both the fibers' tips are located at exactly the same distance from the beam-splitter, then both would have the exact same shape of the wavefront and the whole Gaussian shape would appear and disappear together. However, if the tips are at different distances from the beam-splitter, they would have different curvatures at the beam splitter, and their combination in the beam-splitter would result in interference fringes, as seen on Figure 1.

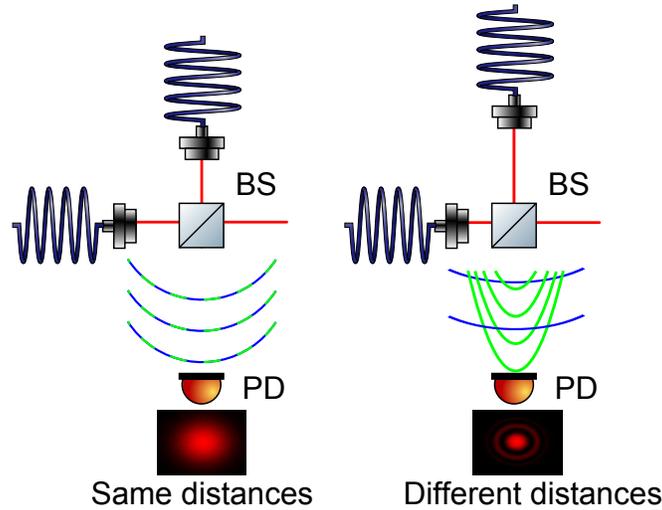


Figure 1: Mach-Zehnder interferometer schematic when the fiber tips are at the same distance (left) and at different distances (right) from the beam-splitter.

2 Simulation

- Define the following constants (please use my notation to establish a common language):
 - W_0 the mode diameter inside the fiber
 - λ the wavelength
 - $k = 2\pi/\lambda$
 - $z_{bs,1}, z_{bs,2}$ the fiber tip-beamsplitter distance of both the tips.
 - x the coordinate vector in the transversal direction
- Create the complex amplitudes $U1(\rho, z_{bs,1})$ and $U2(\rho, z_{bs,2})$, where $U2$ has a global phase $\Phi(T(t))$ according to the temperature profile you chose, or use Eq. 2. **IMPORTANT: Add a random noise, with varying frequency around 0.6 Hz to simulate a real experiment.**
- Sum the amplitudes and take the absolute value.
- **Technical tip:** to make a red colormap like the beams in Figure 1, type the following line: `red_map = [linspace(0,1,256)', zeros(256,2)];` , and after the `imagesc()` function, type `colormap(red_map);`

3 Possible uses

- Run a movie (A link to a movie I made: [3]).

- Record the value of a pixel/ average of several central pixels as a function of time.
- Compare with experimental results / try to fit the temperature profile to fit to the experimental results and report what κ you got or what profile you got if you have a custom profile.
- Fit the distance and thickness of the fringes to a photo of the fringes you take (you need a ruler or other scale in the photo) to experimentally find $z_{bs,1}, z_{bs,2}$.

References

- [1] Teich, M. C., & Saleh, B. E. A. (1991). Fundamentals of photonics. Canada, Wiley Interscience, 3.
- [2] Thorlabs Catalog: SM600 single mode fiber.
<http://www.thorlabs.de/thorcat/12600/SM600-SpecSheet.pdf>
- [3] <https://www.dropbox.com/s/06ddx7urlpdihx/TempSensor1.avi>