Abstract—Cr$^{4+}$-doped saturable absorbers are characterized by long excited state lifetime and appreciable excited state absorption. In this paper, we first solve the three coupled rate equations describing the operation of Cr$^{4+}$-doped saturable-absorber passively Q-switched lasers to obtain the expressions of pulse characteristics such as output energy, peak power, and pulsewidth. We then determine the key parameters of an optimally coupled passively Q-switched laser as functions of two variables concerning the amplifying medium, saturable-absorber medium, and pump level, and generate several design curves. These key parameters include the optimal normalized coupling parameter and the optimal normalized saturable-absorber parameter which maximize the output energy (or maximize the peak power, or minimize the pulsewidth), and the corresponding normalized energy, normalized peak power, and normalized pulsewidth. The results are valid for not only Cr$^{4+}$-doped saturable-absorber Q-switched lasers but also other any lasers passively Q-switched by saturable absorbers with long excited state lifetime and appreciable excited state absorption. Using the expressions and design curves, with the aid of a simple hand calculator, one can predict the pulse characteristics and perform the design of an optimally coupled passively Q-switched laser.

Index Terms—Laser resonators, nonlinear media, optimization methods, solid lasers, Q-switched lasers.

I. INTRODUCTION

In recent years, Cr$^{4+}$-doped crystals have attracted a great deal of attention as passive Q-switches [1]–[8]. These Cr$^{4+}$-doped crystals include Cr$^{4+}$:YAG [1]–[6], Cr$^{4+}$:GSGG [7], Cr$^{4+}$:YSO [8], etc. They have a large absorption cross section and low saturation intensity at the laser wavelength. In comparison with previously used saturable absorbers such as dyes [9] and LiF:F$_2$ color center crystals [10], Cr$^{4+}$-doped crystals are more photochemically and thermally stable and have a higher damage threshold. They can be used as Q-switches for both pulsed lasers [1]–[4], [7], [8] and continuously pumped lasers [5], [6]. Moreover, Cr$^{4+}$ can be codoped with amplifying medium in a monolithic structure to form self-Q-switched lasers [5]. As a result of the above-mentioned advantages, Cr$^{4+}$-doped crystals become the most promising saturable absorbers for passively Q-switched lasers of stability, low cost, reliability, long life, compactness, and simplicity.

The standard tools for analyzing the performance of a Q-switched laser are the rate equations [11], [12]. Cr$^{4+}$-doped crystals have two points which should be considered in the rate equations. One is that Cr$^{4+}$-doped crystals belong to slow saturable absorbers because they have an excited state lifetime much longer than a Q-switched pulsewidth [1]–[3], [8]. The other point is that they have appreciable residual absorption which leads to less than 100% saturated transmission and is imputable to excited state absorption [1]–[3], [8].

The performance of a passively Q-switched laser for each three-way combination of amplifying medium, saturable-absorber medium, and pump level can be optimized through the proper choice of output coupler and initial saturable-absorber transmission [13], [14]. Zhang et al. [13] recently considered the optimization of dye passively Q-switched lasers. In spite of the transcendental nature of the rate equations in which the excited state absorption of the saturable absorber was considered, the analytical expressions for all key parameters of the optimally coupled dye Q-switched laser were obtained. However, those expressions were valid under the condition that the laser intensity inside the resonator was much larger than the saturable intensity of the saturable absorber from the very beginning of the Q-switched laser pulse. This condition can approximately be satisfied in a miniature NAB laser passively Q-switched by BDN dye film [13], but is uncertain for an arbitrary passively Q-switched laser. Thus, those expressions cannot be directly applied to Cr$^{4+}$-doped saturable-absorber passively Q-switched lasers.

More recently, Degnan [14] obtained the key parameters of an energy-maximized passively Q-switched laser as functions of two variables and generated several design curves. These permit the design of an optimum passively Q-switched laser and an estimate of its key performance parameters to be obtained quickly with the aid of a simple hand calculator. However, the excited state absorption of the saturable absorber was not considered in Degnan’s rate equations. Thus, the results obtained by Degnan cannot be directly applied to Cr$^{4+}$-doped saturable-absorber passively Q-switched lasers. Kuo et al. [8] studied the characteristics of a tunable Cr$^{4+}$:YSO Q-switched Cr:LiCAF laser theoretically and experimentally. The excited state absorption of the saturable absorber was considered in their rate equations. However, they solved the rate equations numerically and did not deal with the optimization of the laser.

In this paper, we first solve the three coupled rate equations which describe the operation of lasers passively Q-switched by saturable absorbers with long excited state lifetime and...
appreciable excited state absorption to obtain the expressions of pulse characteristics such as output energy, peak power, and pulselength. We then determine the key parameters of an optimally coupled passively $Q$-switched laser as functions of two variables concerning the amplifying medium, saturable-absorber medium, and pump level, and generate several design curves, which are valid for all slow saturable-absorber passively $Q$-switched lasers. These key parameters include the optimal normalized coupling parameter and the optimal normalized saturable-absorber parameter which maximize the output energy (or maximize the peak power, or minimize the pulselength), and the corresponding normalized energy, normalized peak power, and normalized pulselength. Using the expressions and design curves, with the aid of a simple hand calculator, one can predict the pulse characteristics and perform the design of an optimally coupled passively $Q$-switched laser.

II. FOUR-LEVEL MODEL OF Cr$^{4+}$-DOPED SATURABLE ABSORBERS

The four-level model of Cr$^{4+}$-doped saturable absorbers is shown in Fig. 1 [8]. The transitions that can occur are 1–3, 3–2, 2–1, 4–2, and 2–4. The 3–2 and 4–2 transitions are fast while the 2–1 transition is slow. Both 1–3 and 2–4 transitions give rise to absorption at the laser wavelength and the corresponding absorption cross section are $\sigma_{13}$ and $\sigma_{24}$, respectively. For this model, only level 1 and level 2 are appreciably populated. If a pulse passes through a Cr$^{4+}$-doped saturable absorber, we have [8]

\[
\begin{align*}
\frac{dE}{dz} &= -h\nu n_{s0} \left( 1 - \frac{\sigma_{24}}{\sigma_{13}} \right) \left[ 1 - \exp \left( -\frac{\sigma_{13}E}{h\nu} \right) \right] \\
& - n_{s0} \sigma_{21} E
\end{align*}
\]

where $n_{s1}$ and $n_{s2}$ are the instantaneous population densities in level 1 and level 2, respectively, $n_{s0}$ is the total population density participating in transitions in the pulse duration, $E$ is the pulse energy per unit area, $z$ is the coordinate along the longitudinal direction of the saturable absorber, $h\nu$ is the photon energy, and $dE/dz$ represents the derivative caused by Cr$^{4+}$ absorption alone ($dE/dz$ does not include the nonsaturable absorption caused by the saturable-absorber host crystal which will be involved in the dissipative loss in the coupled rate equations).

When $E$ is small enough, $\exp(-\sigma_{13}E/h\nu) \approx 1 - \sigma_{13}E/h\nu$, and the transmission of the saturable absorber at this situation is called small-signal transmission or initial transmission. When $E$ is large enough, $\exp(-\sigma_{13}E/h\nu) \approx 0$, and the transmission of the saturable absorber at this situation is called saturated transmission. Substituting the simplified results of the two extreme situations into (2) and integrating the results, we obtain

\[
\begin{align*}
2\sigma_{13}n_{s0}I_s &= \ln \left( \frac{1}{T_0^2} \right) \\
2\sigma_{24}n_{s0}I_s &= \ln \left( \frac{1}{T_s^2} \right)
\end{align*}
\]

where $I_s$ is the thickness of the saturable absorber and $T_0$, $T_s$ are the initial and saturated transmissions, respectively.

Dividing (4) by (3), we obtain

\[
\delta = \frac{\sigma_{24}}{\sigma_{13}} = \frac{\ln(T_s)}{\ln(T_0)},
\]

$\delta$ is an important parameter of saturable absorbers. It indicates the degree of the excited state absorption relative to the ground state absorption. It can be obtained by the measurement of $T_0$ and $T_s$ [2].

III. RATE EQUATIONS AND SOLUTIONS

Three coupled rate equations have been used to model a passively $Q$-switched laser [14]. By modifying them to include the excited state absorption of the saturable absorber, and consulting [8], the following coupled rate equations describing the operation of Cr$^{4+}$-doped saturable-absorber $Q$-switched lasers are obtained:

\[
\begin{align*}
\frac{d\phi}{dt} &= \frac{\phi}{t_\phi} \left[ 2\sigma ml - 2\sigma_{13}n_{s1}l_s - 2\sigma_{24}(n_{s0} - n_{s1})l_s \right] \\
&- \ln \left( \frac{1}{R} \right) - L \\
\frac{dn_{s0}}{dt} &= -\gamma \sigma c / n \\
\frac{dn_{s1}}{dt} &= -\sigma_{13}c / n_{s0}
\end{align*}
\]

where $\phi$ is the photon density inside the laser resonator, $n$ is the instantaneous population inversion density, $\sigma$ is the laser stimulated emission cross section, $l$ is the length of amplifying medium, $c$ is the light speed in vacuum, $t_\phi = 2l / c$ is the roundtrip transit time of light in the resonator of optical length $l$, $R$ is the reflectivity of the output mirror, $\gamma$ is the inversion reduction factor, and $L$ is the remaining round-trip dissipative optical loss, which is the sum of the dissipative loss of an actively $Q$-switched laser defined by Degnan [15] and the host crystal absorption loss of the saturable absorber.

In the rate equations of [14], there was a $\gamma_b$ which corresponded to the reduction of the saturable-absorber ground state population density when the saturable absorber absorbed a single photon. If we use a pulse which has about the same width with the $Q$-switched laser pulse to measure $\sigma_{13}$ and $\delta$ as Shimony did [2], and adopt the four-level model of Cr$^{4+}$-doped saturable absorbers in Section II, $\gamma_b$ herein should be unity.
Dividing (7) by (8) and integrating the result, we obtain

$$\frac{n_{s1}}{n_{s0}} = \left( \frac{n}{n_i} \right)^{\alpha}$$

(9)

where \(n_i\) is the initial population inversion density at the start of \(Q\)-switching and \(\alpha\) is a constant concerning the amplifying medium and the saturable-absorber medium

$$\alpha = \frac{\sigma_{13}}{\gamma \sigma}.$$  

(10)

Dividing (6) by (7) and substituting (3), (5), and (9) into the result yield

$$\frac{d\phi}{dn} = \frac{-1}{\gamma l} \left[ 1 - \left( 1 - \delta \right) \ln \left( \frac{1}{R} \right) + \frac{\ln \left( \frac{n}{n_i} \right)}{\Delta} + \frac{\Delta}{\gamma \sigma } \right]$$

$$\left\{ \ln \left( \frac{1}{R} \right) + \delta \ln \left( \frac{1}{T_0^2} \right) + L \right\}$$

(11)

which can be integrated to yield

$$\phi(n) = \frac{1}{\gamma l} \left[ n_i - n - \left( 1 - \delta \right) \ln \left( \frac{1}{R} \right) + \frac{\ln \left( \frac{n}{n_i} \right)}{\Delta} + \frac{\Delta}{\gamma \sigma } \right]$$

$$- \left[ \ln \left( \frac{1}{R} \right) + \delta \ln \left( \frac{1}{T_0^2} \right) + \frac{\Delta}{\gamma \sigma } \right] \cdot \left[ 1 - \left( \frac{n}{n_i} \right)^{\alpha} \right].$$

(12)

Since laser action begins at the moment that the population inversion density crosses the initial threshold value in a passively \(Q\)-switched laser, by setting (6) equal to zero and utilizing \(n_{s0}(t = 0) = n_{s0}\) and \(\alpha_{0}(t = 0) = 0\), we can obtain \(n_i\). Since the maximum photon density occurs when \(d\phi/dn = 0\), by setting (11) equal to zero, we can obtain the expression concerning \(n_i\), the population inversion density at the point of maximum power. By setting (12) equal to zero, we can obtain the expression concerning the final population inversion density \(n_f\):

$$\ln \left( \frac{1}{R} \right) + \frac{\Delta}{\gamma \sigma } \cdot \frac{\Delta}{\gamma \sigma } + L$$

$$= \frac{\ln \left( \frac{1}{R} \right) + \ln \left( \frac{1}{T_0^2} \right) + L}{2\sigma l}$$

(13)

$$\frac{n_f}{n_i} = \frac{\ln \left( \frac{1}{R} \right) + \delta \ln \left( \frac{1}{T_0^2} \right) + L}{2\sigma m_i l} + \frac{\Delta}{\gamma \sigma } \left( \frac{n_i}{n_i} \right)^{\alpha}$$

(14)

$$\frac{n_f}{n_i} = \frac{n_f}{n_i} + \left( \frac{n_{s0}}{n_i} \right) \left( \frac{n_i}{n_f} \right)^{\alpha}$$

(15)

We define a new parameter \(n_{s0}\) as

$$n_{s0} = \frac{\ln \left( \frac{1}{R} \right) + \delta \ln \left( \frac{1}{T_0^2} \right) + L}{2\sigma l}.$$  

(16)

From (14), it can be seen that \(n_{s0}\) actually corresponds to \(n_i\) when \(\alpha\) approaches infinity. Substituting (13) and (16) into (14) and (15) yields the following relations between \(n_i/n_f\), \(n_f/n_i\) and \(n_{s0}/n_i\):

$$\frac{n_f}{n_i} = \frac{n_{s0}}{n_i} + \left( 1 - \frac{n_{s0}}{n_i} \right) \left( \frac{n_i}{n_f} \right)^{\alpha}$$

(17)

$$1 - \frac{n_f}{n_i} + \left( \frac{n_{s0}}{n_i} \right) \ln \left( \frac{n_i}{n_f} \right) - \left( 1 - \frac{n_{s0}}{n_i} \right) \frac{1}{\alpha} \cdot \left[ 1 - \left( \frac{n_f}{n_i} \right)^{\alpha} \right] = 0,$$

(18)

which are shown in Figs. 2 and 3, respectively.
IV. MAXIMIZATION OF OUTPUT ENERGY

When the excited state absorption of the saturable absorber was not considered, Degnan [14] obtained the key parameters of an energy-maximized passively Q-switched laser as functions of two variables, \( \alpha \) and \( \beta \), and generated several design curves. It will now be demonstrated that, when the excited state absorption of the saturable absorber is considered, the key parameters of an optimally coupled passively Q-switched laser can also be expressed as functions of two variables \( \alpha \) and \( \beta \), and similar design curves can be generated.

We begin by defining the new variables:

\[
b = \delta + \frac{1 - \delta}{2\gamma n_1} L \tag{22}
\]
\[
x = \frac{1 - \delta}{2\gamma n_1} \ln \left( \frac{1}{R} \right) \tag{23}
\]
\[
y = \frac{1 - \delta}{2\gamma n_1} \ln \left( \frac{1}{T_0} \right) \tag{24}
\]
\[
e = \frac{1 - \delta}{2\gamma n_1} \frac{E}{\hbar \nu} \tag{25}
\]

\( p = \frac{1 - \delta}{2\gamma n_1} \frac{2\sigma \gamma t_\xi}{\hbar \nu A} P \tag{26} \)
\( w = \frac{2\sigma \gamma n_1^4}{t_\xi} W \tag{27} \)

which, when substituted into (13), (16)–(21), yield

\[
x + y + b = 1 \quad \frac{n_0}{n_i} = x + b \tag{28}
\]
\[
x + b = \frac{n_i}{n_i} - \frac{(n_i) ^\alpha}{1 - \frac{(n_i) ^\alpha}{n_i}} \tag{29}
\]

\[
x + b = \frac{1 - \frac{n_0}{n_i} - \left[ 1 - \left( \frac{n_0}{n_i} \right) ^\alpha \right] \frac{1}{\alpha}}{-\ln \left( \frac{n_i}{n_i} - \left[ 1 - \left( \frac{n_i}{n_i} \right) ^\alpha \right] \frac{1}{\alpha} \right)} \tag{30}
\]
\[
e = -x \ln \left( \frac{n_0}{n_i} \right) \tag{31}
\]
\[
p = x \left( 1 - \frac{n_0}{n_i} + \left( \frac{n_0}{n_i} \right) \ln \left( \frac{n_i}{n_i} \right) \right) \tag{32}
\]
\[
w \approx \frac{c}{p} \tag{33}
\]

What we want to do is to maximize the output pulse energy (or maximize peak power, or minimize pulsewidth) by selecting the optimal \( \ln(1/R) \) and \( \ln(1/T_0^2) \) for a given amplifying medium, a given saturable-absorber medium (\( \delta \) is fixed, \( T_0 \) is changeable), and a given pump level (i.e., a given \( n_0 \)). From (23) to (27), it can be seen that this means maximizing \( c \) (or maximizing \( p \), or minimizing \( w \)) by selecting the optimal \( x \) and \( y \) under the condition of (28). Since \( c \) cannot be expressed as direct function of \( x \) or \( y \), we can not calculate \( dc/dx \) or \( dc/dy \) and set it equal to zero. However, we can treat the problem as follows. First, express \( c \) as the function of \( n_0/n_i \) by substituting (31) into (32). Second, calculate \( dc/d(n_0/n_i) \) and order it equal to zero to obtain the optimal \( n_0/n_i \). Third, determine \( (n_0/n_i)_{\text{opt}} \) according to (30) and (31). Fourth, determine the optimal \( x \) and \( y \), the maximum \( c \), and corresponding \( p \) and \( w \) by substituting \( (n_0/n_i)_{\text{opt}} \) and \( (n_0/n_i)_{\text{opt}} \) into (28), (30), (32)–(34), respectively.

The results are shown in Figs. 4–8, respectively. From the figures, it can be seen that the maximum energy decreases monotonically with \( b = \delta + (1 - \delta)L/(2\gamma n_1) \). This means that the maximum energy from a passively Q-switched laser using a saturable absorber with excited state absorption (\( \delta > 0 \)) is less than that using a saturable absorber which has no excited state absorption (\( \delta = 0 \)) when the other conditions are similar. It can also be seen that the larger is \( \alpha_k \), the larger are \( c_{\text{max}} \) and \( p \), the smaller is \( w \), and when \( \alpha_k \) is larger than 10, the results are very close to those when \( \alpha_k \) approaches infinity.

When \( \alpha_k \) approaches infinity, we can obtain the following analytical expressions of the above-mentioned key parameters...
Fig. 5. Dependence of $y_{\text{opt}}$ on $b$ for different $\alpha$ when the laser is a energy-maximized passively $Q$-switched laser: (a) $\alpha = 1.5$, (b) $\alpha = 2$, (c) $\alpha = 3$, (d) $\alpha = 5$, (e) $\alpha = 10$, and (f) $\alpha = \infty$.

Fig. 6. Dependence of $e_{\text{max}}$ on $b$ for different $\alpha$ when the laser is a energy-maximized passively $Q$-switched laser: (a) $\alpha = 1.5$, (b) $\alpha = 2$, (c) $\alpha = 3$, (d) $\alpha = 5$, (e) $\alpha = 10$, and (f) $\alpha = \infty$.

Fig. 7. Dependence of $p$ on $b$ for different $\alpha$ when the laser is a energy-maximized passively $Q$-switched laser: (a) $\alpha = 1.5$, (b) $\alpha = 2$, (c) $\alpha = 3$, (d) $\alpha = 5$, (e) $\alpha = 10$, and (f) $\alpha = \infty$.

Fig. 8. Dependence of $w$ on $b$ for different $\alpha$ when the laser is a energy-maximized passively $Q$-switched laser: (a) $\alpha = 1.5$, (b) $\alpha = 2$, (c) $\alpha = 3$, (d) $\alpha = 5$, (e) $\alpha = 10$, and (f) $\alpha = \infty$.

When $\delta = 0$, which corresponds to the situation that no excited state absorption exists, the results are consistent with those of [14]. When $\infty \rightarrow \infty$, which means that the saturable absorber is fully saturated immediately after the population inversion density crosses the initial threshold value, the results are the same as those of [13]. When $\delta = 0$ and $\infty \rightarrow \infty$, the
results except \( y_{\text{opt}} \) are consistent with those of an optimally coupled actively Q-switched laser [15], [16].

V. MAXIMIZATION OF PEAK POWER

We use the following method to maximize the peak power. First, express \( P \) as the function of \( n_s/n_i \) for a given \( n_i \) by substituting (29) and (30) into (33). Second, calculate \( \Delta P/d(n_s/n_i) \) and order it equal to zero to obtain the optimal \( n_s/n_i \). Third, determine \( (n_s/n_i)_{\text{opt}} \) according to (30) and (31). Fourth, determine the optimal \( x \) and \( y \), the maximum \( p \) and corresponding \( e \) and \( w \) by substituting \( (n_s/n_i)_{\text{opt}} \) and \( (n_s/n_i)_{\text{opt}} \) into (28), (30), (32)–(34), respectively.

The results are shown in Figs. 9–13, respectively. When \( \delta = 0 \) (which means \( \delta = 0 \) and \( L = 0 \)) and \( \alpha \to \infty \), \( x_{\text{opt}} \) approaches 0.28 and \( P_{\text{max}} \) approaches 0.102, these results are consistent with those obtained by Zayhowski in a peak-power-maximized actively Q-switched laser [16].

VI. MINIMIZATION OF PULSEWIDTH

We still first determine the optimal \( n_s/n_i \) and \( n_s/n_i \) which minimize the pulsewidth by numerical calculation. \( (n_s/n_i)_{\text{opt}} \) and \( (n_s/n_i)_{\text{opt}} \) are independent of \( b \). Substituting \( (n_s/n_i)_{\text{opt}} \) and \( (n_s/n_i)_{\text{opt}} \) into (28), (31), and (34), we obtain \( y_{\text{opt}} \) and \( u_{\text{min}} \), which are also dependent of \( b \). The relations between \( (n_s/n_i)_{\text{opt}}, y_{\text{opt}}, u_{\text{min}} \) and \( \alpha \) are shown in Fig. 14. According to (28), (32), and (33), the other key parameters \( x_{\text{opt}}, e, \) and \( p \), which are related to \( b \), can be obtained from the following expressions:

\[
x_{\text{opt}} = 1 - b - y_{\text{opt}}
\]

\[
e = (b - 1 + y_{\text{opt}}) \ln \left( \frac{n_s}{n_i} \right)_{\text{opt}}
\]

\[
p = \frac{1}{u_{\text{min}}} \left( b - 1 + y_{\text{opt}} \right) \ln \left( \frac{n_s}{n_i} \right)_{\text{opt}}
\]

When \( \delta = 0 \) and \( \alpha \to \infty \), \( x_{\text{opt}} \) becomes \( 0.33 - L/(2\sigma n_i) \) and \( u_{\text{min}} \) becomes 9.39, these results are consistent with those obtained by Zayhowski in a pulsewidth-minimized actively Q-switched laser [16].

VII. CONTINUOUSLY PUMPED Q-SWITCHED LASERS

Because of their photochemical and thermal stability, Cr^4+-doped crystals are highly suitable for continuously pumped Q-switched lasers, especially diode-pumped Q-switched lasers.

For this kind of lasers, when the pump power exceeds a threshold condition, the laser is repetitively Q-switched with
Fig. 12. Dependence of $g$ on $b$ for different $\alpha$ when the laser is a peak-power-maximized passively $Q$-switched laser: (a) $\alpha = 1.5$, (b) $\alpha = 2$, (c) $\alpha = 3$, (d) $\alpha = 5$, (e) $\alpha = 10$, and (f) $\alpha = \infty$.

Fig. 13. Dependence of $w$ on $b$ for different $\alpha$ when the laser is a peak-power-maximized passively $Q$-switched laser: (a) $\alpha = 1.5$, (b) $\alpha = 2$, (c) $\alpha = 3$, (d) $\alpha = 5$, (e) $\alpha = 10$, and (f) $\alpha = \infty$.

A time interval between pulses, $\Delta t$, when $\Delta t \gg \tau_s$, the first excited state lifetime of the saturable absorber, $\Delta t$ can be written as [14]

$$\Delta t = \tau_a \ln \left( \frac{1 - \beta n_i/B \tau_a}{1 - n_i/B \tau_a} \right)$$

$$\beta = 1 - \frac{f_a}{\gamma} \left( 1 - \frac{n_i}{n_i} \right)$$

where $\tau_a$ is the lifetime of the upper laser level, $f_a$ is the Boltzmann occupation factor of the upper laser level, which is the ratio of the upper laser level population to the total manifold population, $B$ is the volumetric pump rate into the upper laser level and is proportional to the CW pump power, and the other parameters have the same meanings as defined in Sections II and III.

Setting $\Delta t \rightarrow \infty$ yields the threshold pump rate $B_{th}$ and the reciprocal of $\Delta t$ gives the pulse repetition rate $f$, i.e.,

$$B_{th} = \frac{n_i}{\tau_a}$$

$$f = (\Delta t)^{-1} = \left( \tau_a \ln \left( \frac{1 - \beta B_{th}/B}{1 - B_{th}/B} \right) \right)^{-1}$$

When $B$ is larger than $2B_{th}$, the repetition rate becomes extremely high (for example, in Nd:YAG, $\tau_a \approx 230 \mu s$, when $\beta = 0.5$ and $B > 2B_{th}$, $f > 2447/\tau_a = 10.7$ kHz) and $f$ is approximately linear with $B$, i.e.,

$$f = \frac{1}{2 \tau_a \ln^{-1} \left[ 1 + \frac{(1 - \beta)B_{th}/B}{1 - B_{th}/B} \right]}$$

$$- \frac{1}{2 \tau_a} \ln^{-1} \left[ 1 + \frac{(1 - \beta)B_{th}/B}{1 - B_{th}/B} \right]$$

$$\approx \frac{1}{2 \tau_a} (1 - \beta) B_{th}/B + \frac{1}{2 \tau_a} (1 - \beta) B_{th}/B$$

$$\approx \frac{1}{\tau_a} B_{th} \frac{1 + \beta}{2}.$$
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Fig. 15. Dependence of \( f(\tau) \) on \( B \).

Fig. 16. Dependencies of \( E \), \( P \), and \( W \) on \( B \).

(13), \( f \) is larger than that determined by (49), \( W \) is larger than that determined by (21), and \( E, P \) are smaller than those determined by (19) and (20). In this situation, the following equations are used to model the operation of continuously pumped \( Q \)-switched lasers:

\[
\frac{d\psi}{dt} = \frac{\phi}{\tau_{\psi}} \left[ 2\sigma_2 (n_a - g_a n_b) l - 2\sigma_{13} n_{1a} l_s - 2\sigma_{14} (n_{1b} - n_{1a}) l_s - \ln \frac{1}{R} - L \right] (50)
\]

\[
\frac{dn_a}{dt} = -f_a \sigma_0 \phi (n_a - g_a n_b) - \frac{n_a}{\tau_a} + B (51)
\]

\[
\frac{dn_b}{dt} = f_b \sigma_0 \phi (n_a - g_a n_b) - \frac{n_b}{\tau_b} (52)
\]

\[
\frac{dn_{1a}}{dt} = -\sigma_{13} \alpha n_{1a} + \frac{n_{1a} - n_{1b}}{\tau_{1a}} (53)
\]

where \( n_a, g_a, f_a, \tau_a \) are the population density, degeneracy, Boltzmann occupation factor, and lifetime of the upper laser level, respectively, four similar parameters of the lower laser level are \( n_b, g_b, f_b, \tau_b \), respectively.

The above equations can only be solved numerically. As an example, we calculate the situation of a diode-pumped \( \text{Cr}^{3+}:\text{YAG} \) \( Q \)-switched \( \text{Nd:YAG} \) laser. The parameter concerning the amplifying medium, the saturable absorber, and the resonator are as follows: \( \sigma = 6.6 \times 10^{-15} \text{ cm}^2, f_a = 0.41, f_b = 0.19, \tau_a = 230 \mu s, \tau_b = 300 \text{ ns}, l = 2 \text{ mm}, \sigma_{13} = 4.3 \times 10^{-18} \text{ cm}^2, \sigma_{21} = 8.2 \times 10^{-19} \text{ cm}^2, n_{1a} = 4.51 \times 10^{17} \text{ cm}^{-3}, l_s = 0.543 \text{ mm}, \tau_{1a} = 4 \mu s, T_0 = 90\%, R = 0.96, L = 0.02, t_s = 0.25 \text{ ns}, \gamma = 0.6, A = 0.15 \text{ mm}^2. \)

The results, along with those when \( \Delta t >> \tau_a \), are shown in Figs. 15 and 16.

VIII. CONCLUSION

In Sections II and III, we have solved the three coupled rate equations describing the operation of passively \( Q \)-switched lasers when the excited state absorption of the saturable absorber is considered and have obtained the expressions of pulse characteristics such as output energy, peak power, and pulsedwidth. Using the relevant parameters of the amplifying medium, the saturable absorber, and the resonator, with the aid of a simple hand calculator, one can easily predict the pulse characteristics according to the following steps. First, calculate \( n_{1a}, n_{1b}, \) and \( \alpha \) according to (10), (13), and (16). Second, obtain \( n_a/n_{1a} \) and \( n_b/n_{1a} \) from Figs. 2 and 3. Third, substitute \( n_{1a}, n_{1b}, n_a/n_{1a}, n_b/n_{1a}, R, \alpha \), etc., into (19)–(21) to obtain \( E, P, W \). This will save a great deal of time in comparison with solving the rate equations numerically by using a computer [8], [9].

In Sections IV–VI, we have determined the key parameters of an optimally coupled passively \( Q \)-switched laser as functions of two variables concerning the amplifying medium, saturable-absorber medium, and pump level, and generated several design curves, which are valid for all slow saturable-absorber passively \( Q \)-switched lasers. These key parameters include the optimal normalized coupling parameter and the optimal normalized saturable-absorber parameter which maximize the output energy (or maximize the peak power, or minimize the pulsedwidth), and the corresponding normalized energy, normalized peak power, and normalized pulsedwidth.

The two variables are \( b = \delta + (1 - \delta)L/(2\sigma_{1a} l) \) and \( \alpha \). These permit the design of an optimally coupled passively \( Q \)-switched laser and the estimate of its pulse characteristics to be obtained quickly by using a simple hand calculator. The steps may be as follows. First, calculate \( b = \delta + (1 - \delta)L/(2\sigma_{1a} l) \) and \( \alpha \). Second, determine the normalized parameters \( x_{\text{OPT}}, y_{\text{OPT}}, z_{\text{OPT}}, \phi, \psi, \) and \( w \) from related figures and (42)–(44). Third, obtain \( R_{\text{OPT}}, T_{\text{OPT}}, E, P, \) and \( W \) according to (23)–(27). In Section VII, we have given the characteristics of continuously pumped \( Q \)-switched lasers.

\( \delta \) is an important parameter of the saturable absorber. It indicates the degree of the excited state absorption relative to the ground state absorption and is the main part of parameter \( b \). Because the output energy and the peak power from a passively \( Q \)-switched laser decrease monotonically with \( b \), the saturable absorber with a smaller \( \delta \) is preferred. \( \alpha \) is a parameter unique to passively \( Q \)-switched lasers. When the other conditions are similar, the larger is \( \alpha \), the larger are the output energy and the peak power, and the smaller is the pulsedwidth. \( \infty \rightarrow \infty \) means that the saturable absorber is fully saturated immediately after the population inversion density crosses the initial threshold value, and when \( \alpha \) is larger than 0.1, the results are very close to those when \( \alpha \) approaches infinity. From the results in this paper, by setting
equal to zero, one can obtain the key parameters of an optimally coupled laser passively $Q$-switched by the saturable absorber which has no excited state absorption. By letting $\delta = 0$ and $\infty \to \infty$, one can obtain the results of an optimally coupled actively $Q$-switched laser.

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