Rope Between Inclines

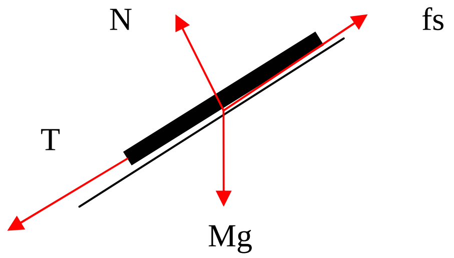
Yaniv Hefetz[1]
Faculty of Applied Mathematics and Theoretical Physics,
University of Cambridge, England

1. Assumptions

We will assume the following:(a) a thin rope; (b) full symmetry and (c) the rope is "differentiable", thus the rope at the cut off point is tangential to the incline.

2. Leaning Rope

On the figure below we sketched the forces that act on the rope:

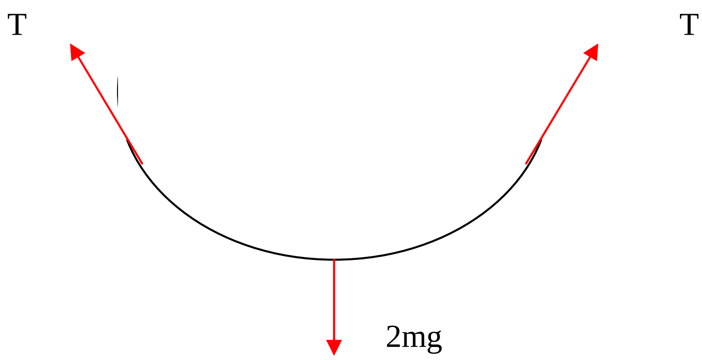


It is easy to see that:

$$N = Mg\cos\theta$$
$$T = f_S - Mg\sin\theta = \mu N - Mg\sin\theta = \mu Mg\cos\theta - Mg\sin\theta$$

hence, $T = Mg(\mu\cos\theta - \sin\theta)$ [1]

3. Hanged Rope



It is easy to see that: $mg = T\sin\theta$ [2]

(1) physics@zahav.net.il

4. The Maximal Angle

From equations [1] and [2] we obtain:

$$\frac{m}{M} = \sin\theta(\mu\cos\theta - \sin\theta)$$

The ratio between the masses is the same as the ratio between the length of hanging rope and the length of the leaning rope.

By differentiation we can get the angle for maximal hanging length (or maximal ratio)

$$\cos\theta(\mu\cos\theta - \sin\theta) + \sin\theta(-\mu\sin\theta - \cos\theta) = 0$$
$$\Rightarrow \mu(\cos^2\theta - \sin^2\theta) - 2\sin\theta\cos\theta = 0$$

for: $\mu = 1$

$$\cos^2\theta - \sin^2\theta - 2\sin\theta\cos\theta = 0$$
$$\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} - 2 = 0$$
$$\frac{1}{Z} - Z - 2 = 0$$
$$1 - Z^2 - Z = 0$$

$$\Rightarrow Z = tg\theta = \sqrt{2} - 1$$

hence, $\theta = \frac{\pi}{8} = 22.5^\circ$

Below is a graph of the length of the hanged rope as function of the length of the leaning rope: