Solar sail

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Consider a cosmic sail-boat moving under the influence of the pressure of sun-light at a distance from the Sun similar to that of Earth. What kind of accelerations you can expect? How do those light-pressure forces compare with the gravitational pull of the Sun?

A sail gains momentum through either reflection or absorption of photons. The most efficient sail is a high reflector, because each photon loses momentum $p = 2h\nu/c$ in reflection compared to only $h\nu/c$ in absorption.

The Sun is a blackbody emitter with radius $R_S = 6.96 \times 10^8$ m and effective temperature $T \approx 6000$ K. Therefore, at any distance r from the Sun, the total radiation intensity (in W/m^2) is:

$$I(r) = \sigma T^4 \frac{R_S^2}{r^2}, \tag{1}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^2/K^4 is the Stefan-Boltzmann constant. From Planck's formula, the spectral intensity at particular frequency ν is:

$$I_\nu(r,\nu) = \frac{2\pi h \nu^3}{c^2} \frac{1}{\exp(\frac{h\nu}{kT}) - 1} \frac{R_S^2}{r^2}, \tag{2}$$

where $h = 6.626 \times 10^{-34}$ J s is the Planck's constant and $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann's constant. From Eq.(2) one can obtain the number of photons ΔN of frequency ν passing through area A during a time interval Δt:

$$\frac{\Delta N}{A \Delta t} = \frac{I_\nu(r,\nu)}{h\nu} \tag{3}$$

The force exerted by these photons (each losing momentum $p = 2h\nu/c$) is:

$$F_\nu(r,\nu) = \frac{\Delta N}{\Delta t} p = \frac{2A}{c} I_\nu(r,\nu) \tag{4}$$

The total force is then obtained after integration over the whole spectrum of the Solar radiation:

$$F_S(r) = \int_0^\infty F_\nu(r,\nu) d\nu = \frac{2A}{c} \int_0^\infty I_\nu(r,\nu) = \frac{2A}{c} I(r)$$

$$F_S(r) = \frac{2\sigma T^4 R_S^2}{c} \frac{A}{r^2} \approx 2 \times 10^{17} \frac{A}{r^2} \tag{5}$$

As an example, let's consider a large sail of area $A = 1$ km^2 situated at Earth's orbit $r = 1.496 \times 10^{11}$ m. The force exerted by the solar radiation, according to Eq.(5) is $F_S = 9$ N.

Let's compare the radiation pressure force Eq.(5) with the gravitational force, given by Newton's formula:

$$F_G(r) = G M_S \frac{m}{r^2} = 1.33 \times 10^{20} \frac{m}{r^2}, \tag{6}$$

where $G = 6.67 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} is Newton's gravitational constant, $M_S = 1.99 \times 10^{30}$ kg is the solar mass, and m is the mass of the sail.

It is clear from Eqs.(5,6) that the radiation pressure on the sail can compensate for the gravitational pull at any distance from the Sun only if the following relation between sail's area and mass is satisfied:

$$A = \frac{c G M_S}{2\sigma T^4 R_S^2} m \approx 663.3m. \tag{7}$$

A sail made of aluminum (with density $\rho_{\mathrm{Al}} = 2643$ kg/m^3) can be in equilibrium with the gravitational force only if it has thickness:

$$d = \frac{2\sigma T^4 R_S^2}{c G M_S \rho_{\mathrm{Al}}} \approx \frac{1}{663.3\rho_{\mathrm{Al}}} \approx 570 \text{ nm} \tag{8}$$