INSTITUTE OF PHYSICS PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **25** (2004) 593–604 PII: S0143-0807(04)76816-6

Study of the conductivity of a metallic tube by analysing the damped fall of a magnet

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Received 26 February 2004
Published 10 June 2004
Online at stacks.iop.org/EJP/25/593
doi:10.1088/0143-0807/25/5/002

Abstract

The fall of a magnet through a hollow conducting tube is described. Although this experiment is well known, a detailed treatment by means of a circuit analysis allows us to relate the conductivity of the tube to the characteristic parameters of the experiment.

1. Introduction

A very common experiment in undergraduate physics laboratories is the study of the damped fall of a magnet through a conductive hollow tube [1]. This experiment is usually done as a qualitative demonstration to show the effects of electromagnetic induction associated with Faraday's law, and not much attention is given to a quantitative analysis. Here we discuss a thorough study of the phenomenon. Although an exact calculation of the currents induced in the tube, which are responsible for the damping of the falling magnet, is very complicated, a reasonable approach is to examine the currents in terms of a single generator of electromotive force (emf) connected to an assembly of resistor circuits that represent the behaviour of the conducting tube. An analysis of this resistor network in terms of circuit theory allows one to relate the parameters involved in the problem (the conductivity and the damping coefficient) to the geometry and electromagnetic characteristics of the apparatus by means of the equivalent circuit concept. The validity of this approach is analysed for several magnets. Good agreement between the experimental results and the conductivity reported in the literature is obtained. This exercise could be proposed as an experimental task for students of an intermediate course on electromagnetism.

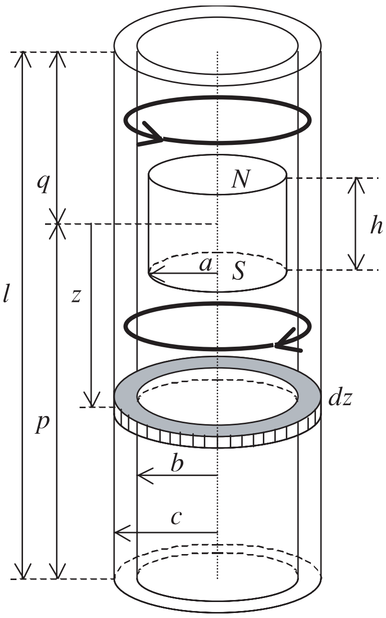

Figure 1. Description and dimensions of the metallic tube. The currents induced above and below the magnet are shown.

2. The description of the problem

Figure 1 shows a diagram of the experimental system. The induced currents above and below the falling magnet are shown. Those above attract the magnet whereas those below repel it, thereby yielding the damping effect that we wish to study. Although a rigorous analysis is very complicated and would require the use of numerical methods, we shall demonstrate how the analysis of a simple circuit lets us relate the falling speed and the geometrical and electromagnetic properties of the system to the conductivity of the tube and the damping coefficient.

The equation of motion is

$$M\,\frac{\mathrm{d}v}{\mathrm{d}t} = Mg - kv, \tag{1}$$

where k (disregarding the aerodynamic friction due to extremely low speed, as we will see later) represents the damping coefficient associated with the induced currents and M is the mass of the magnet. If we integrate equation (1), we obtain

$$v(t) = \frac{Mg}{k}\left(1 - \mathrm{e}^{-\frac{k}{M}t}\right). \tag{2}$$

It is observed that the asymptotic speed (Mg/k) is reached almost instantaneously ($\tau = M/k \ll 1$ s). For this reason, we concluded that, for a tube length l of several decimetres, the speed can be considered practically uniform. For example, for a copper tube with dimensions $l = 1$ m, inner radius $b = 6.5$ mm, outer radius $c = 7.5$ mm (see figure 1) and a SmCo magnet of mass $M = 9.3$ g, height $h = 10$ mm and radius $a = 6$ mm, a falling time of 12.3 s is obtained. We thus find

$$v = \frac{l}{t} = \frac{Mg}{k} \approx 81 \text{ mm s}^{-1}, \qquad k \approx 1.12 \text{ kg s}^{-1}, \qquad \tau = \frac{M}{k} \approx 8 \text{ ms}. \tag{3}$$

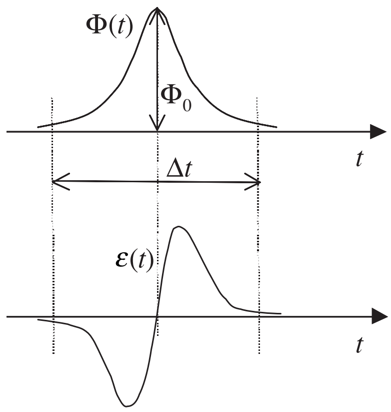

Figure 2. The form of the time dependence of the magnetic flux and induced electromotive force. (See, for example, the third paper quoted in [1].)

It is difficult to relate the damping coefficient k to the geometrical and electromagnetic parameters of the experiment, but we shall do so approximately by using energy conservation. It suffices to say that the variation in the potential energy of the magnet is equivalent to the energy dissipated in the tube through the Joule effect.

When the magnet falls inside the tube, the magnetic flux, $\Phi(t)$, linked by 'the loop' (a tube section Δz in height and resistance R) and the emf, $\varepsilon(t)$, induced in it have the form depicted in figure 2, where Φ_0 is the maximum flux and Δt is the characteristic time of the process. If we neglect the self-inductance of 'the loop' (an issue that will be addressed later), the average of the absolute value of the induced emf is related to the mean value of the modulus of the current as

$$\varepsilon \approx \frac{\Phi_0}{\Delta t/2} \approx IR. \tag{4}$$

It is not easy to evaluate the equivalent resistance, R, that the magnet feels during its fall, that is, the equivalent circuit. This task is difficult because the emf is induced throughout the entire tube, with opposite directions above and below the magnet, although its value diminishes quickly away from the magnet. In fact, the magnetic induction associated with the magnet decreases at least as $1/r^3$, where r is the distance from the magnet [2].

3. The equivalent circuit approximation

The difficulty of the calculation of R and the rapid decrease in the induced emf with the distance to the magnet suggest that the problem may be simplified by considering a single falling generator, instead of the magnet, that feeds the equivalent resistance R corresponding to a length Δz of the tube. To explain this idea, the equivalent circuit is shown in figure 3. In this way, we can model $\varepsilon(z)$ by a simple resistor network. The series resistance, $\alpha\,\mathrm{d}z$, where α is the series resistance per unit length, corresponds to that of a section of the tube, $\mathrm{d}z$, throughout its length, and serves only to feed the loops that actually form the tube. The loops correspond to a parallel conductance, $\beta\,\mathrm{d}z$, where β is the parallel conductance per unit length, which describes the same elementary section ($\mathrm{d}z$) throughout its circumference:

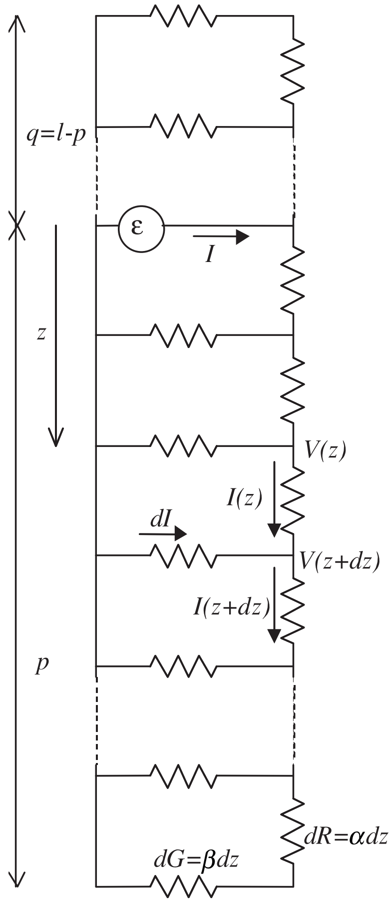

Figure 3. Equivalent circuit used in our circuital approximation. The lengths p, q and z are labelled.

$$\mathrm{d}R = \alpha\,\mathrm{d}z = \frac{1}{\sigma\pi(c^2 - b^2)}\,\mathrm{d}z \tag{5a}$$

$$\mathrm{d}G = \beta\,\mathrm{d}z = \frac{\sigma\,\ln(c/b)}{2\pi}\,\mathrm{d}z, \tag{5b}$$

where σ is the conductivity of the material of the tube.

It is to be remarked that actually the currents above and below the falling magnet flow in opposite directions around the tube, as is depicted in figure 1. The fact that in our equivalent circuit the currents circulate in the same direction (see figure 3) is not a problem because the power balance that we will carry out corresponds to the square of the current (or voltage), all the directional aspects disappearing in this way.

Using circuit analysis, we evaluate the equivalent resistance, R, which the magnet feels. For an elementary section of the tube located at a distance z from the magnet, we have

$$V(z) - V(z + \mathrm{d}z) = I(z)\alpha\,\mathrm{d}z \tag{6a}$$

$$I(z) - I(z + \mathrm{d}z) = -V(z)\beta\,\mathrm{d}z, \tag{6b}$$

that is, current and voltage in the tube satisfy the relations

$$\frac{\mathrm{d}V(z)}{\mathrm{d}z} = -\alpha I(z) \tag{7a}$$

$$\frac{\mathrm{d}I(z)}{\mathrm{d}z} = -\beta V(z). \tag{7b}$$

If we take the derivative of equations (7) with respect to z, we obtain two second-order differential equations. After integration we have

$$V(z) = V_+ \,\mathrm{e}^{-\sqrt{\alpha\beta}z} + V_- \,\mathrm{e}^{+\sqrt{\alpha\beta}z} \tag{8a}$$

$$I(z) = I_+ \,\mathrm{e}^{-\sqrt{\alpha\beta}z} + I_- \,\mathrm{e}^{+\sqrt{\alpha\beta}z}. \tag{8b}$$

If we take the derivative again and use equations (7), the result is

$$\frac{V_+}{I_+} = \sqrt{\alpha/\beta} = -\frac{V_-}{I_-}. \tag{9}$$

Thus, the current and voltage in the tube can be described by

$$V(z) = V_+ \,\mathrm{e}^{-\sqrt{\alpha\beta}z} + V_- \,\mathrm{e}^{+\sqrt{\alpha\beta}z} \tag{10a}$$

$$I(z) = \frac{1}{\sqrt{\alpha/\beta}}(V_+ \,\mathrm{e}^{-\sqrt{\alpha\beta}z} - V_- \,\mathrm{e}^{+\sqrt{\alpha\beta}z}), \tag{10b}$$

where the quantity $R_0 = \sqrt{\alpha/\beta}$ plays the role of a characteristic resistance, because for $z = \infty$, $V_- = 0$ and $V(z)/I(z) = \sqrt{\alpha/\beta}$.

In agreement with figure 3, from the magnet (the generator) we recognize two resistors in parallel, R_p and R_q, associated with the two parts of the tube length $l = p + q$. R_p and R_q obviously correspond to tube sections terminating in an open circuit, $R_L = \infty$, which is why according to the resistance transformation law for transmission lines and distances p and q (see appendix C), we can write [3, 4]

$$R_p = R_0 \frac{R_L + R_0 \tanh(\sqrt{\alpha\beta}p)}{R_0 + R_L \tanh(\sqrt{\alpha\beta}p)} = \frac{R_0}{\tanh(\sqrt{\alpha\beta}p)} \tag{11a}$$

$$R_q = R_0 \frac{R_L + R_0 \tanh(\sqrt{\alpha\beta}q)}{R_0 + R_L \tanh(\sqrt{\alpha\beta}q)} = \frac{R_0}{\tanh(\sqrt{\alpha\beta}q)}, \tag{11b}$$

and therefore,

$$R = \frac{R_p R_q}{R_p + R_q} = \frac{R_0}{\tanh(\sqrt{\alpha\beta}p) + \tanh(\sqrt{\alpha\beta}(l-p))}. \tag{12}$$

Finally,

$$R = \frac{\sqrt{\alpha/\beta}}{\tanh(\sqrt{\alpha\beta}l)[1 + \tanh(\sqrt{\alpha\beta}p) \times \tanh(\sqrt{\alpha\beta}(l-p))]} \tag{13}$$

is the equivalent resistance with which the conductive tube of length l behaves in terms of the location, p, of the magnet (see figure 3).

The actual dependence of the equivalent resistance on p is very simple. In fact, for typical values of α, β, p and l (see the data of our example in section 2 and use equations (5) with a metallic conductivity value for σ), all the expressions in the argument of the hyperbolic tangent are close to unity, which is why the equivalent circuit almost corresponds to a constant resistance given by

$$R \approx \frac{R_0}{2} = \frac{\sqrt{\alpha/\beta}}{2} = \frac{1}{\sigma\sqrt{2(c^2-b^2)\ln(c/b)}}. \tag{14}$$

However, at the ends of the tube ($p = 0$ and $p = l$) the resistance is exactly twice this value.

4. Calculation of the conductivity

According to equation (5*b*) and using the equivalent resistance defined in equation (14), we show that the phenomenon occurs roughly as if the induced emf were operating on a single circular loop, with the current entirely azimuthal, of height Δz given by

$$\frac{2\pi}{\sigma\ln(c/b)}\frac{1}{\Delta z} = \frac{1}{\sigma\sqrt{2(c^2-b^2)\ln(c/b)}}. \tag{15}$$

Thus, we have

$$\Delta z = 2\pi\sqrt{\frac{2(c^2-b^2)}{\ln(c/b)}}. \tag{16}$$

By using equation (16), and according to equation (4), we can write the induced emf in terms of the speed, $v = \Delta z/\Delta t$, of the magnet as

$$\varepsilon \approx \frac{\Phi_0}{\Delta t/2} = \frac{2\Phi_0}{\Delta z/v} = \frac{\Phi_0}{\pi\sqrt{\frac{2(c^2-b^2)}{\ln(c/b)}}}v. \tag{17}$$

We can carry out the energy balance by integrating over the currents in the circular elements $\beta\,\mathrm{d}z$. In fact, in the tube there are only azimuthal induced currents. The series resistances $\alpha\,\mathrm{d}z$ were introduced only to feed these circular loops, so that the complicated dependence in $\varepsilon(z)$ could be modelled using Ohm's law applied to our resistor network. Therefore, power conservation is given by

$$\int_0^p V^2(z)\beta\,\mathrm{d}z + \int_0^q V^2(z)\beta\,\mathrm{d}z = Mgv. \tag{18}$$

In agreement with our analysis and the geometrical and electromagnetic characteristics of the system, we can consider the copper tube as a kind of infinite transmission line ($V_- \approx 0$) and, except when the magnet is close to the ends of the tube, we can write

$$V(z) \approx V_+\,\mathrm{e}^{-\sqrt{\alpha\beta}z} = \varepsilon\,\mathrm{e}^{-\sqrt{\alpha\beta}z}, \tag{19}$$

for $0 \leqslant z \leqslant p$ and $0 \leqslant z \leqslant q$.

We substitute equation (19) in equation (18), integrate, and assume (in accordance with typical values for α, β, p and q) that

$$\mathrm{e}^{-2\sqrt{\alpha\beta}p} \approx \mathrm{e}^{-2\sqrt{\alpha\beta}q} \approx 0, \tag{20}$$

and obtain

$$\frac{\varepsilon^2}{\sqrt{\alpha/\beta}} = Mgv. \tag{21}$$

If we substitute ε, α and β (see equations (17) and (14)) in equation (21), we have

$$\frac{2\Phi_0^2v^2}{\pi^2}\sigma\sqrt{\frac{\ln^3(c/b)}{2(c^2-b^2)}} = Mgv, \tag{22}$$

from which follows the expression that relates the conductivity of the tube and the speed of the falling magnet to the geometrical and electromagnetic characteristics of the experiment:

$$\sigma = \frac{\sqrt{2}}{2}\pi^2 \frac{Mg}{\Phi_0^2}\sqrt{\frac{c^2 - b^2}{\ln^3(c/b)}}\frac{1}{v}. \tag{23}$$

Finally, we use equation (3) to find the expression for the damping coefficient:

$$k = \frac{Mg}{v} = \frac{\sqrt{2}}{\pi^2}\Phi_0^2\sigma\sqrt{\frac{\ln^3(c/b)}{c^2 - b^2}}. \tag{24}$$

5. Experimental evaluation and discussion

Equations (23) and (24) allow us to measure the conductivity of the tube and the damping coefficient by analysing the falling magnet. It is necessary to know the tube geometry, the mass of the magnet, its asymptotic speed and its magnetic flux with great precision.

For the usual dimensions of the magnet (see the data of our example in section 2), an imprecise knowledge of the demagnetizing field and the difficulties involved in evaluating the return flux of the magnetic lines of force suggest that we should ignore the manufacturer's information or numerical methods to evaluate the magnetic flux Φ_0 and instead measure it experimentally. By using a simple integrator and a small coil of radius $(b + c)/2$, we obtained $\Phi_0 = 92\ \mu$Wb for the SmCo magnet. (We used a ballistic galvanometer [5] and results were checked with an integrator based on an operational amplifier.) After carrying out the calculations for this example, we found $\Delta z \approx 88$ mm, $\varepsilon \approx 170\ \mu$V and, using the conductivity of copper ($59.8 \times 10^6\ \Omega^{-1}\ \mathrm{m}^{-1}$) [6], we found $R \approx 8.5\ \mu\Omega$, the equivalent resistance through which a current of the order of 20 A would circulate.

The result for Δz suggests measuring the magnet speed by using two sensing coils located several centimetres away from the ends of the tube. The conductivity we obtained with our calculation was $64 \times 10^6\ \Omega^{-1}\ \mathrm{m}^{-1}$ which differs by about 7% from the published value [6], an error that can be considered reasonable in the context of our analysis.

We conclude by considering the accuracy of our two approximations: the substitution of a distributed emf by a single generator and the use of an equivalent circuit to represent the behaviour of the tube. The discrepancies are essentially associated with these two approximations. Indeed, instead of calculating the emf, $\varepsilon(z)$, created by the magnet during its fall analytically (bearing in mind that the magnet does not correspond to a point dipole, see appendix A), we arranged longitudinal resistances, $\alpha\,\mathrm{d}z$, to feed the circular loops, $\beta\,\mathrm{d}z$. We simply replaced the superposition of the contributions of the form $\sum B_n z^{-n}$ with $n \geqslant 3$ (corresponding to the multipole expansion of the magnet's induction [2]) by an exponential dependence of $V(z)$ (recall the power series expansion of the exponential function [7]). This exponential decay allows us to approximate the induction phenomenon throughout the tube in a simple and reasonable way, although it introduces a small error in the results for the conductivity.

On the other hand, no appreciable error is made when the self-inductance of the equivalent circuit is not considered. The small error introduced by neglecting self-inductance can be readily evaluated by comparing the inductive and Ohmic contributions. The self-inductance of a length Δz of equivalent resistance R is approximately [5]

$$L = \frac{\mu_0\pi(b + c)^2}{4\Delta z}, \tag{25}$$

which, although the problem does not correspond to sinusoidal excitation, allows us to evaluate

Table 1. Summary of the results for three different magnets.

Magnet type	Magnetic flux (μWb)	Mass (g)	Speed (mm s^{-1})	Conductivity ($10^6\ \Omega^{-1}\ m^{-1}$)	Relative error (%)
Sm2Co17	92	9.3	81	64	7.0
NdFeB235	96	8.0	58	70	17.1
Barium ferrite	32	5.3	357	68	13.7

the inductive reactance as the one corresponding to the first harmonic:

$$\omega = \frac{2\pi}{\Delta t} = \frac{2\pi}{\Delta z} v. \tag{26}$$

The ratio $L\omega/R$ is given by

$$\frac{L\omega}{R} = \frac{\frac{\mu_0 \pi^2}{2}\left(\frac{b+c}{\Delta z}\right)^2 v}{\frac{1}{\sigma\sqrt{2(c^2-b^2)\ln(c/b)}}} = \frac{\mu_0 \pi^2 (b+c)^2}{16} \frac{Mg}{\Phi_0^2}, \tag{27}$$

which is of the order of 10^{-3} for our example, fully justifying the approximation.

The error introduced when using the average value of the emf induced during the interval $\Delta t/2$ also could be taken into account. Because a power balance is considered, it would be necessary to work with the rms value of the emf, although due to the form of $\varepsilon(t)$, it is to be hoped that the error will be very small.

There are of course other sources of error, such as accurate determination of the geometry and the measurement of the magnetic flux, Φ_0. In any case, performing the experiment with several conductive tubes of the same dimensions (copper, aluminium and other non-magnetic alloys) readily allows one to compare their conductivities. In fact, within the limit of infinite conductivity (a superconducting loop), one would have an experiment of zero speed or magnetic levitation.

It is also interesting to do experiments for several magnets and tubes with different wall thicknesses. The damping of the fall increases dramatically with the tube thickness. We emphasize that for tubes with very thick walls, the evaluation of the magnetic flux associated with the return of the induction lines of force demands special attention. We repeated the experiment with two different magnets with the same geometry. In table 1 we summarize our results, including those of the example used in our previous calculations.

All these small inaccuracies can be considered by introducing a calibration constant C into equation (22):

$$C\frac{2\Phi_0^2 v^2}{\pi^2}\sigma\sqrt{\frac{\ln^3(c/b)}{2(c^2-b^2)}} = Mgv, \tag{28}$$

where C corresponds to an empirical coefficient, $\sigma_{\mathrm{experimental}}/\sigma_{\mathrm{true}}$, close to unity (1.07 for our example, with the SmCo magnet) that, after experimental measurement, will allow us to take into account such inaccuracies and to obtain a better value for the conductivity.

To obtain satisfactory results, the experiment must be performed using strong magnets. The relative errors associated with the determination of the magnetic flux and the speed when operating with the low-field barium ferrite magnet are greater (see table 1). Nickel-coated magnets (for example, NdFeB magnets) also are inadequate because their exact dimensions are not well known and their rounded shape at the boundaries is a source of distortion in the magnetic lines of force. Their true size, which is smaller than the apparent one, increases the return of magnetic flux, making the precise determination of Φ_0 difficult.

Our expressions can be verified by repeating the experiment with a non-magnetic additional mass on the magnet. (Note that if changes are made by varying the magnet,

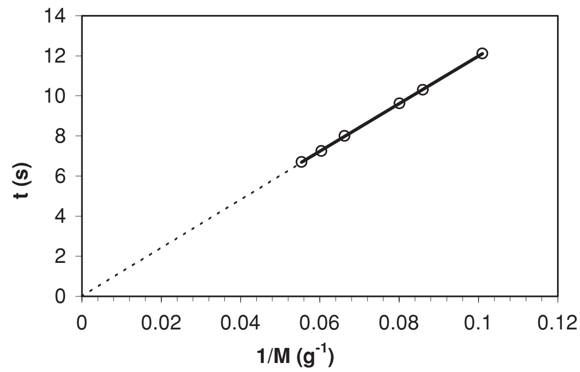

Figure 4. The dependence of the falling time on the mass of the magnet. The fit corresponds to the experimental data obtained by loading the magnet with different amounts of Pb pellets.

the quantity Φ_0 also will be modified due to the variation in the demagnetizing factor.) An example of this experiment can be seen in figure 4 for our SmCo magnet. From equation (23), we can write

$$t = \frac{\sqrt{2}}{\pi^2 g}\sqrt{\frac{\ln^3(c/b)}{c^2 - b^2}}\Phi_0^2 l\sigma\frac{1}{M}, \tag{29}$$

where t represents the time that the magnet takes to fall through a tube of length l. From the slope of the time versus inverse mass (see figure 4), the value of the conductivity is readily obtained. For that purpose, we loaded the magnet with different amounts of Pb pellets up to 10 g.

Acknowledgment

We wish to thank J García (laboratory technician of the Department of Applied Physics) for helping us in the manufacture of the experimental equipment developed for this work.

Appendix A. The dipolar contribution

Although it is very complicated to do a rigorous analysis of the distribution of currents induced in the tube, it is simple to evaluate only the magnetic dipole contribution. In agreement with the usual dimensions of the experimental device, the magnet cannot be considered as a simple point dipole, and it is necessary to include higher order multipole contributions.

According to figure 5, the magnitude of the induced azimuthal electric field, $\boldsymbol{E}$, in a filamentary loop of radius ρ ($b \leqslant \rho \leqslant c$) at a distance z from the magnet (represented by its magnetic moment $\boldsymbol{m}$) in terms of the magnetic vector potential, $\boldsymbol{A}$, is

$$\begin{aligned} E(\rho, z) &= \left|\frac{\partial A(\rho, z, t)}{\partial t}\right| = v\left|\frac{\partial A}{\partial z}\right| = v\left|\frac{\partial}{\partial z}\left(\frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\boldsymbol{r}}{r^3}\right)\right| \\ &= v\frac{\partial}{\partial z}\left[\frac{\mu_0 m\rho}{4\pi(\rho^2+z^2)^{3/2}}\right] = v\frac{3\mu_0 m}{4\pi}\frac{\rho z}{(\rho^2+z^2)^{5/2}}. \end{aligned} \tag{30}$$

The power dissipated in the tube due to the dipole contribution will be

$$P_{\mathrm{dip}} = \int_{\mathrm{tube}} \sigma E^2\,\mathrm{d}v = \sigma v^2\frac{9\mu_0^2 m^2}{16\pi^2}\int_{-q}^{p}\int_{b}^{c}\frac{\rho^2 z^2}{(\rho^2+z^2)^5}2\pi\rho\,\mathrm{d}\rho\,\mathrm{d}z. \tag{31}$$

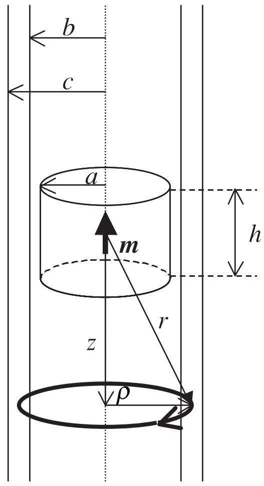

Figure 5. Notation used in the dipolar contribution calculation.

We can do the integral in equation (31) using that b and c are much smaller than p and q, and obtain

$$P_{\mathrm{dip}} \approx \sigma v^2 \frac{15\mu_0^2 m^2}{1024}\left(\frac{1}{b^3}-\frac{1}{c^3}\right). \tag{32}$$

Therefore, the balance of energy, $P_{\mathrm{dip}} = Mgv$, allows us to write

$$\sigma = \frac{1024}{15}\frac{Mg}{\mu_0^2 m^2}\frac{c^3 b^3}{c^3-b^3}\frac{1}{v}, \tag{33}$$

which is to be compared with equation (23).

To evaluate the accuracy that is obtained by including only the magnetic dipole contribution, we determined the magnetic moment of the SmCo magnet experimentally. To do so, we suspended it from a very thin cotton thread and measured its oscillating period in the presence of the Earth's magnetic field. The value of its horizontal component in our laboratory, measured with a tangent compass, is $B_h = 25.2\ \mu\mathrm{T}$, yielding a period $T = 0.447$ s. From the expression [8]

$$T = 2\pi\sqrt{\frac{J}{mB_h}}, \tag{34}$$

where J represents the inertial moment of the magnet ($J = Ma^2/4 + Mh^2/12$), the magnetic moment was found to be 1.26 A m^2. If we substitute this result in equation (33), the conductivity is found to be $24 \times 10^6\ \Omega^{-1}\ \mathrm{m}^{-1}$, which corresponds to approximately 40% of the published value [6], thus illustrating the need to include higher order multipole contributions, or to use the method presented in section 3.

Appendix B. Other approximations

Instead of proceeding to carry out a standard multipole expansion, we would prefer to first try some other options in order to reach better approximation.

The most convenient option corresponds to considering a collection of n equal point dipoles, of magnetic moment $\boldsymbol{m}/n$, uniformly spaced at distances h/n in the symmetry axis of the tube representing the falling magnet. In this way, the calculation of the induced azimuthal electric field, $\boldsymbol{E}$, corresponds to a simple superposition of n contributions as in equation (30). But according to equation (31), the dissipated power would be a complicated polynomial expression with $(n^2+n)/2$ contributions. Except for small values of n it constitutes a tedious problem without major interest.

Another possibility would be the substitution of the magnet by two opposite surface densities of magnetic pole strength at a distance h apart. The calculation is very difficult because this model involves sources out of the symmetry axis, and it does not contribute anything to the better understanding of the problem.

Appendix C. Derivation of expressions (5), (11) and (25)

To obtain the differential longitudinal resistance, dR in equation (5a), we will consider a uniform current distribution in a tube section of length dz and cross-sectional area $\pi(c^2-b^2)$. If σ represents its conductivity, we have

$$\mathrm{d}R = \frac{1}{\sigma\pi(c^2-b^2)}\,\mathrm{d}z = \alpha\,\mathrm{d}z, \tag{35}$$

where α corresponds to the series resistance per unit length of the tube.

The expression for the azimuthal differential conductance, dG in equation (5b), is derived taking into account the circular currents in the loop:

$$\mathrm{d}G = \frac{\mathrm{d}I}{V}, \tag{36}$$

where dI represents the azimuthal current around a section of height dz and V corresponds to the potential difference throughout the loop. According to the symmetry, we can write

$$\mathrm{d}I = \int_b^c \mathbf{J}\cdot\mathrm{d}\mathbf{S} = \sigma\int_b^c \mathbf{E}\cdot\mathrm{d}\mathbf{S} = \frac{\sigma V}{2\pi}\,\mathrm{d}z\int_b^c \frac{\mathrm{d}r}{r} = \frac{\sigma V}{2\pi}\ln\left(\frac{c}{b}\right)\mathrm{d}z. \tag{37}$$

And therefore

$$\mathrm{d}G = \frac{\sigma\ln(c/b)}{2\pi}\,\mathrm{d}z = \beta\,\mathrm{d}z, \tag{38}$$

β being the parallel conductance per unit length.

Equations (11) are obtained from expressions (10) according to circuit theory of transmission lines, considered as a distributed parameter network [3–5]. For that purpose, we start writing equations (10) in terms of the distance $p-z$ or $q+z$ from the ends (the load R_L) of the tube (see figure 1). For the bottom end, at a distance p from the magnet, we have

$$\begin{aligned} V(p) &= V_+\,\mathrm{e}^{-\sqrt{\alpha\beta}p} + V_-\,\mathrm{e}^{+\sqrt{\alpha\beta}p} = R_L I(p)\\ V_+\,\mathrm{e}^{-\sqrt{\alpha\beta}p} &- V_-\,\mathrm{e}^{+\sqrt{\alpha\beta}p} = R_0 I(p), \end{aligned} \tag{39}$$

and in a similar way we can find the expressions for the other end of the tube at a distance q from the magnet.

Combining both expressions, one has

$$\begin{aligned} V_+ &= \tfrac{1}{2}(R_L+R_0)\,\mathrm{e}^{+\sqrt{\alpha\beta}p} I(p)\\ V_- &= \tfrac{1}{2}(R_L-R_0)\,\mathrm{e}^{-\sqrt{\alpha\beta}p} I(p), \end{aligned} \tag{40}$$

and equations (10) can be rewritten as

$$\begin{aligned} V(z) &= \frac{I(p)}{2}[(R_L + R_0)\,\mathrm{e}^{+\sqrt{\alpha\beta}(p-z)} + (R_L - R_0)\,\mathrm{e}^{-\sqrt{\alpha\beta}(p-z)}] \\ I(z) &= \frac{I(p)}{2R_0}[(R_L + R_0)\,\mathrm{e}^{+\sqrt{\alpha\beta}(p-z)} - (R_L - R_0)\,\mathrm{e}^{-\sqrt{\alpha\beta}(p-z)}]. \end{aligned} \tag{41}$$

It helps us to define a new quantity $\zeta = p - z$ which allows us to find the resistance along the tube (looking towards the terminating impedance R_L):

$$R(\zeta) = \frac{V(\zeta)}{I(\zeta)} = R_0 \frac{(R_L + R_0)\,\mathrm{e}^{+\sqrt{\alpha\beta}\zeta} + (R_L - R_0)\,\mathrm{e}^{-\sqrt{\alpha\beta}\zeta}}{(R_L + R_0)\,\mathrm{e}^{+\sqrt{\alpha\beta}\zeta} - (R_L - R_0)\,\mathrm{e}^{-\sqrt{\alpha\beta}\zeta}}. \tag{42}$$

Finally, using the hyperbolic functions, the resistance along the tube is

$$R(\zeta) = R_0 \frac{R_L + R_0 \tanh(\sqrt{\alpha\beta}\zeta)}{R_0 + R_L \tanh(\sqrt{\alpha\beta}\zeta)}. \tag{43}$$

For distances $\zeta = p$ or $\zeta = q$, and taking into account that $R_L = \infty$, we find the desired equations:

$$\begin{aligned} R_p &= \frac{R_0}{\tanh(\sqrt{\alpha\beta}p)} \\ R_q &= \frac{R_0}{\tanh(\sqrt{\alpha\beta}q)}. \end{aligned} \tag{44}$$

Expression (25) can be easily justified using the self-inductance per unit length of an infinite coil of average radius $(b + c)/2$. Assuming a uniform current distribution, J, and neglecting the edge effects, the magnetic induction inside the tube is

$$B \approx \mu_0 \frac{\Delta I}{\Delta z} = \mu_0 J(c - b), \tag{45}$$

where $\Delta I/\Delta z$ represents the azimuthal current density per unit length in the conductive tube. For a height Δz, the self-induction will be

$$L = \frac{\Phi}{\Delta I} = \frac{\mathbf{B}\cdot\mathbf{S}}{\Delta I} \approx \frac{\mu_0}{\Delta z}\pi\left(\frac{b + c}{2}\right)^2. \tag{46}$$

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