Problem "Isotropic Universe" 01/04

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Abstract

Observer O is located in an infinite space and is bombarded by particles coming from infinity and moving along straight lines in random directions. This "rain" of particles is isotropic in its directions. At large distance from the observer there is a region of scatterers S. The region contains many scatterers of the particles. The scatterers are not isotropic. However, their orientation is random. Can the observer O detect the presence of the region S in the space by simply observing the distribution of particles arriving at O from different directions? Note: You may assume that the scatterers are "dilute", i.e. a particle is not scattered more than once.

1 General Scattering

We first consider the most general case of scattering. A light particle scattered by a heavy scatterer has a differential cross section $d\sigma/d\Omega = bb'/\sin\theta$, where $b = b(\theta)$ the scattering parameter as a function of the scattering angle. The probability that a particle be scattered inside $d\Omega$ at an angle θ is $p(\theta)d\Omega = (1/\sigma)(d\sigma/d\Omega)d\Omega$. Therefore, the number of particles in a ray being scattered at an angle θ from a small volume of scatterers of known density ρ is $dN = N\rho dx(bb'/\sin\theta)d\Omega$, where dx is the linear length of the volume. If the scattering angle is random, meaning it is distributed uniformly (with the same probability) to all its possible values, we have $b(\theta) = b_0 \cos\frac{\theta}{2}$ and the probability of scattering at an angle θ is $p_{out}(\theta)d\Omega = (1/4\pi)d\Omega$, independent from the incoming angle. It is also obvious that the probability a particle enters the small volume of scatterers at an angle θ_{in} is $p_{in}(\theta_{in})d\Omega$. We now apply the above concepts to the problem at hand.

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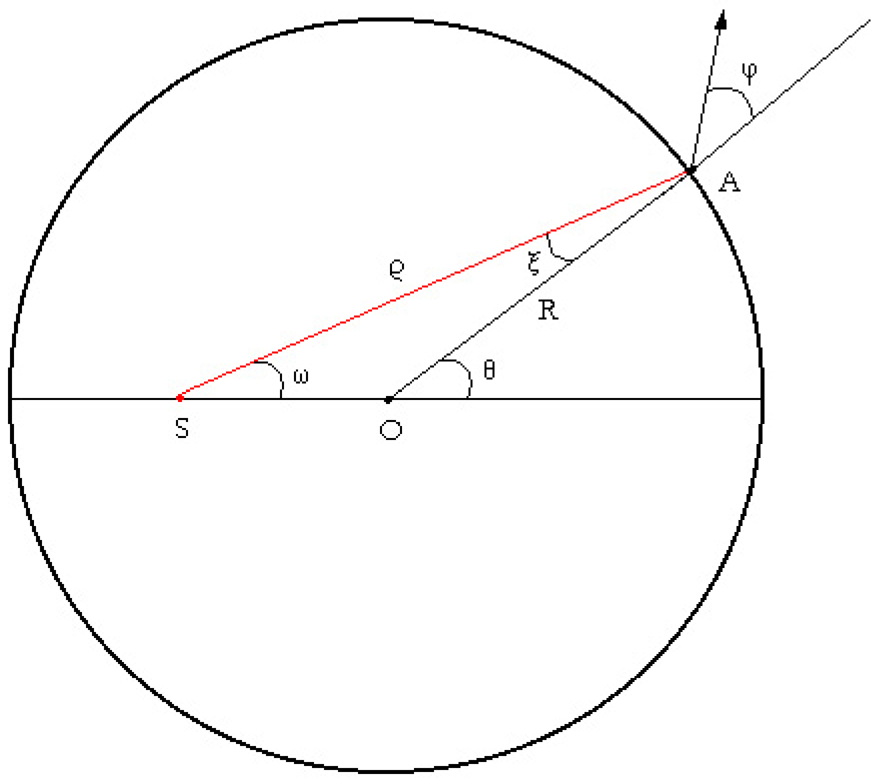

Figure 1: Model for the isotropic universe.

2 Analysis

We begin with a model for the isotropic universe. Imagine a sphere of radius R with the observer O at its center (Figure 1). Each point in the sphere represents an isotropic emitter of particles. Point S represents the small region of scatterers, positioned at a distance $R/2$ from the observer. In the limit $R \to \infty$, the symmetric choice $R/2$ will have no restrictive effect to our investigation.

Let us first assume there are no scatterers in S at all. Each emitter emits N_0 particles at once. The probability of emitting a single one at an angle ϕ is $(1/4\pi)d\Omega$, independent of the angle. Thus, the number of particles it emits at the correct angle π to reach the observer is $dN = (N_0/4\pi)d\Omega$. The total number of particles arriving at the observer from all the emitters in the sphere is $N = (N_0/4\pi)\int d\Omega = N_0$.

If there are scatterers in S, the emitter at point A will radiate particles at angle π that will reach the observer directly and also particles that will reach the observer after they are scattered at S. The latter particles must be emitted at angle $\pi - \xi$, where $\cos\xi = \frac{1}{2R}\frac{R^2+2R^2\cos\theta}{R^2+(R/2)^2+2R^2\cos\theta}$. Thus, the total number of particles reaching the scatterers are $N_{in} = N_0\int p(\pi-\xi)d\Omega = (N_0/4\pi)\int d\Omega = N_0$. The ones arriving at S from an angle $\omega = \theta - \xi$ are $dN_{in}(\omega) = (N_0/4\pi)d\Omega$. If there are ρSdx scatterers[1] in a small volume of S, then the probability that one particle is scattered from one of the scatterers at the right angle to reach O, is $(\rho Sdx/4\pi)d\Omega$. Therefore, the number of particles reaching O from S after being emitted from A is $dN = dN_{in}(\omega)(\rho Sdx)(p_{out}(0)d\Omega) = (N_0\rho Sdx/\left(4\pi\right)^2)d\Omega(\xi)d\Omega(0)$.

The contribution of the scatterers to the particles observed at O is (sum over all incoming directions) $dN_s = (N_0\rho Sdx/4\pi)d\Omega(0)$. The probability that a particle coming from the emitter at $\theta = \pi$ is not scattered at all from the volume of scatterers is $1-\rho Sdx$. This corresponds to another $(N_0/4\pi)(1-\rho Sdx)d\Omega$ particles reaching the observer.

In the end, the number of particles reaching the observer from the direction of the scatterers is $(N_0/4\pi)(1-\rho Sdx)d\Omega(0) + (N_0\rho Sdx/4\pi)d\Omega(0) = (N_0/4\pi)d\Omega(0)$, which is exactly the number of particles entering from any other direction! This means that in first approximation, where the region S is very small (considered a point) compared to its distance from the observer, no difference should be found in the study of the intensity of rays of particles in the various directions: The region of scatterers is hidden by the isotropy of the particles.

[1] S here is the surface of the volume.