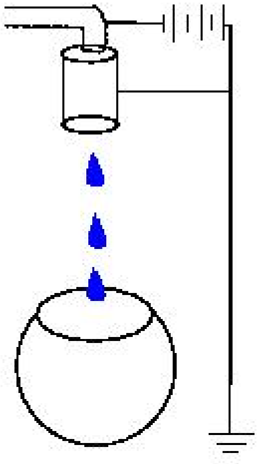

Electrostatic machine depicted in the figure consists of an insulated metallic spherical container into which drops charged with a certain potential are dripping from a faucet above. Determine the dependence of the maximal potential of the spherical container on the height from which the drops are falling.

Solution:

The drops, once arrived in the container, will transfer their charges to the conducting walls. The charged wall will have a repellant action on the following falling drops. Eventually, there will be a critical situation in which a drop will have zero velocity at the container entrance. In that situation the container will have the maximum potential.

Note: Anyway, if the container is not large enough (or if the drops are sufficiently big), then it will be filled up before any drop will reach its top with zero velocity. In this case the potential of the full container will be $V = \frac{Q}{4\pi\varepsilon_0 R}$, where I assumed a spherical container, as that one in the picture above, its radius being *R*. Its charge is *Q*=*Nq*, where *N* is the number of drops that fill the container and *q* is the charge of each drop. Assuming that each drop has the potential V_0, then $V = NV_0 \frac{r}{R}$, *r* being the radius of a drop. From mass conservation it follows that $R^3 = Nr^3$, which, combined with the above eq. will give $V = V_0\left(\frac{R}{r}\right)^2$. So, in this case, the maximum potential of the container will not depend on the height from which the drops are falling.

Assuming now that the container is large enough, the energy conservation written at the faucet exit and at the top of the container reeds (h > 2R):

$$mgh + \frac{qq_1}{4\pi\varepsilon_0 (h-R)} = \frac{mv^2}{2} + mg \cdot 2R + \frac{qq_1}{4\pi\varepsilon_0 R},$$

where the height h is measured from the container bottom to the faucet exit and q_1 is the charge of the container at an arbitrary moment. From this it follows that

$$\frac{mv^2}{2} = mg(h-2R) - \frac{qq_1}{4\pi\varepsilon_0}\frac{h-2R}{R(h-R)}$$

It can be seen that as q_1 increases, the drops velocity at the container top decreases. The container charging will stop when the velocity will be zero, as mentioned above. In that critical situation, the container charge will be

$$q_1 = \frac{mgR(h-R)}{rV_0},$$

V_0 being, as above, the potential of a drop.

If the density of the liquid is ρ, then the critical potential of the container is

$$V = \frac{\rho g}{3\varepsilon_0}\frac{r^2 (h-R)}{V_0}.$$

<u>*Note:*</u> This set-up is currently known as the Kelvin electrostatic generator.