Question 12/02

CHARGED DROP

The shape of a freely suspended liquid drop is kept spherical (with radius *R*) by the surface tension *g*. [For simplicity we assume weightlessness.] Assume that the liquid is conducting and it is being gradually charged. What will happen as the charge *Q* increases?

Solution

When the drop is charged it appears a surface charge density, $\sigma = Q/4\pi R^2$, and consequently an electrostatic pressure, $p = \sigma^2/2\varepsilon_0$, which tends to increase the drop radius. If we were considering a soap bubble this pressure will actually increase the drop size, but for a liquid drop we can only wait for its breaking-off and separation by the repulsive electrostatic forces. By considering the energy balance between surface and electrostatic energies, we can easily find the electric charge necessary to break the drop.

Obviously the most energetically favourable drop breaking corresponds to two equal drops with one half in charge ($Q/2$) and radius $R' = R/2^{1/3}$. To simplify the calculations we will consider the drops at a great distance apart, and therefore we neglect the electrostatic energy due to Coulomb interaction.

The energy increment has two contributions (superficial and electrostatic), and must have a negative value to attain the drop breaking:

$$\Delta W = \Delta W_s + \Delta W_e \leq 0, \quad (1)$$

when:

$$\Delta W_s = g\Delta S = g\left[2\left(4\pi R'^2\right) - 4\pi R^2\right] = 4\pi g R^2\left(2^{1/3} - 1\right) \quad (2)$$

$$\Delta W_e = 2\left(\frac{1}{2}\frac{\left(Q/2\right)^2}{4\pi\varepsilon_0 R'}\right) - \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}\left(2^{-2/3} - 1\right).$$

Finally:

$$Q \geq \pi\sqrt{32\varepsilon_0\frac{\left(2^{1/3} - 1\right)}{\left(1 - 2^{-2/3}\right)}}\sqrt{gR^3}\,. \quad (3)$$

For a water drop ($g = 72.8\times10^{-3}$ N/m), 2 mm radius, we find:

$$Q \geq 1.2\times10^{-5}\sqrt{R^3} \approx 1nC\,, \quad (4)$$

that corresponds to a potential

$$V = \frac{Q}{4\pi\varepsilon_0 R} \approx 4800V\,. \quad (5)$$

It is very easy to make the experiment in a physics laboratory. A few kV (e. g. the power supply of a CRT) are enough to break a water jet in small drops.