REINSURANCE CONTRACTS

A Utility Approach vs. Insurance Capacity Considerations

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Present explanations for the existence of reinsurance are based on utility analysis and assume
some sort of risk aversion in the insurer-reinsurer interface. This analogy to the insured-insurer
interface stands in contrast to capital market equilibrium theory, since, unlike the insured, the
insurer cannot gain from further diversification of a risk insured at the market price of risk. The
existence of reinsurers must, therefore, be explained by market imperfections.

The present study explains reinsurance as an outcome of the solvency regulation and capacity
considerations of insurers and reinsurers. This approach enables the handling of various types of
reinsurance contracts (and therefore offers also a wider perspective than earlier studies, which
focused only on proportional, quota share, agreements). Interesting relationships between
reinsurance pricing, solvency and capacity are observed within our framework.

1. Introduction

1.1. Reinsurance and risk sharing according to utility theory

There are two basic approaches to the analysis of reinsurance. One views
the reinsurance as a loss-sharing agreement between parties predestined to be
insurers and reinsurers (in hierarchical chains). The other views the reinsur-
ance as part of a symmetric risk-sharing pool (risk exchange), where parties
redistribute the risks with which they are endowed. Since both approaches
deal with risks, it is only natural for the analysis of reinsurance arrangements
to be based on the conventional tools of economics of uncertainty, and
especially on the utility analysis of the two parties (in analogy to the
insured-insurer relationship).

In his seminal works on risk exchanges Borch (1960, 1961, 1962) used the
economic concept of utility to justify the use of Pareto-optimal risk
exchange, and characterized the optimal contracts in the (re)insurance

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market. These ideas were developed further by Buhlmann (1970), Buhlmann and Gerber (1978), Buhlmann and Jewell (1979) and others.

Analytic solutions to the risk-sharing problem are greatly simplified by explicit assumptions concerning the underlying distributions (risks), and the utility functions of the parties involved. Under the particular assumption of exponential utility functions – which represent constant absolute risk aversion – the optimal sharing rule is always proportional, and depends only on the respective risk-aversion parameters [Borch (1962, 1985a), Baton and Lemaire (1981a, p. 59), Barnea et al. (1985, p. 29), Lemaire and Quairiere (1986)]. It is, therefore, not surprising that studies of risk exchanges [e.g., Gerber (1978), Buhlmann and Gerber (1978), Buhlmann and Jewell (1979)], and of hierarchical chains of reinsurance [e.g., Gerber (1984)], which are based on the restrictive assumptions that all companies evaluate their portfolio by means of specific (exponential) utility functions and that the claims are normally distributed, allow only for proportional reinsurance. A similar result was given by Pressacco (1979) for the quadratic utility case.

Using the tools of game theory, Buhlmann and Jewell (1979), Lemaire (1979) and Baton and Lemaire (1981a) characterized the core of the game, proved that it always exists, and showed that other solutions may be the result of collective rationality, where subcoalitions of companies are formed, and side payments are transferred. In a later paper Baton and Lemaire (1981b) analyze a more dynamic situation, in a multi-criteria situation in which players attempt to maximize their payoffs but also try to enter a 'stable' coalition. Other types of side payments are considered by Buhlmann and Jewell (1979) and by Gerber (1978), and lead to optimality of other forms of reinsurance such as stop loss insurance [see Gerber (1978) and Pesonen (1967)] or deductible policies [Raviv (1979)].

The form of reinsurance affects the pricing formula, and is simultaneously also affected by it. Pricing according to the expected value of the loss plus a loading leads to stop loss arrangements, or deductible policies. Using a variance-dependent ratemaking formula leads to quota share reinsurance [Buhlmann and Gerber (1978)].

1.2. Reinsurance in a competitive capital market

These approaches to reinsurance theory suffer from two deficiencies. First, in practice, there is no mechanism in the reinsurance market to enforce such a classical proportional pooling solution, and the models fail to explain the frequent use of mixed coverage or non-proportional reinsurance arrangements [Helten and Beck (1983)].

The second, and more important, problem with the traditional analysis of reinsurance lies in the use of utility analysis, and the assumed risk aversion on the parts of the insurer and reinsurer. As pointed out by Doherty and
Tinic (1981) and, more recently, by Blazenko (1986), the insurer–reinsurer interface should not be treated in analogy to the insured–insurer interface. The insurer, unlike the insured, is not endowed with the risk; if he cannot cope with it, or if the price is inappropriate, he does not have to underwrite it.\footnote{The insurer, though, may sometimes be willing to underwrite a part of the risk which fits into his portfolio (and cede the rest to the reinsurers).}

When the risk is underwritten by the insurer, at the right price – which is determined in the capital market [see Biger and Kahane (1978), Kahane (1979) and later Fairley (1979) and Hill (1979)] – the market value of the insurer cannot be increased by further buying of reinsurance. Therefore, reinsurance is redundant under capital market equilibrium [Doherty and Tinic (1981)]. Diversified corporations with large numbers of shareholders must demonstrate risk neutrality: non-neutral utility considerations should not be employed, therefore, to explain the insurer–reinsurer interface.

1.3. The coexistence of insurers and reinsurers

The above statement calls for further explanation; under capital market equilibrium the maximum diversification – which could be reached by holding the market portfolio – is considered in the pricing formula [see the argumentation leading to the well-known Capital Assets Pricing Model, and its implications for insurance – Biger and Kahane (1978), Kahane (1979)]. Thus the simple statistical argument, that a larger portfolio is typically less risky, should not be used to explain the coexistence of (re)insurers in the market. The market portfolio (say, a global portfolio, including all the risks in the world) could be insured by a global ‘super-insurer’, and this would be the most efficient portfolio in terms of risk reduction. The question of the existence of multiple insurers and reinsurers then arises: splitting the portfolio among many insurers can, at best, leave the risk level unchanged, but would normally increase the risk. The objective of this paper – to explain the coexistence of many insurers and their observed hierarchy – is thus not trivial, and cannot simply be handled by the commonly used statistical effect of risk-pooling.

Recent studies have attempted to explain the common use of reinsurance in practice in other ways: Eden (1977) explains the existence of reinsurance by leaving the single-period framework, and showing the possible benefits resulting from the use of insurance for the allocation of uncertainty over time. Mack (1983) argues that the gap between theory and practice could be explained by additional assumptions concerning differences in cost structure, the uncertainty concerning the relevant loss distribution and utility function, and the concentration on single-period models. Gerber (1984) adds the
sequencing of the bargaining as an important element (the reinsurer first declaring his loading policy, the ceding company reacting to it). Borch (1985b) explains the willingness of (risk neutral) insurers to reinsurance by the effect of limited liability and capacity problems. Blazenko (1986) applied micro-economic analysis of monopolistic and monopsonistic situations to explain the reinsurance mechanism (but, unfortunately, focused only on quota-share reinsurance and ignored other forms). Eden and Kahane (1986) tried another approach – based on differences in the effectiveness of monitoring moral hazard – to explain the insurance market structure, and especially the coexistence of the reinsurer alongside the insured and insurer.

1.4. Reinsurance resulting from solvency regulation

The present paper focuses on the insurer–reinsurer interface, in an attempt to explain the coexistence of various forms of reinsurance (proportional and non-proportional). The explanation is based on an institutional factor – the capital adequacy (solvency) regulation – and is strongly related to the idea of insurance capacity. A somewhat related concept has been offered in the important papers by Doherty (1980), Doherty and Tinic (1981) and Borch (1985b), which explain the possibility of extending market capacity by introducing non-proportional reinsurance. The idea is based on the ability of reinsurance to improve the solvency of the insurer – thus enabling him to collect higher premiums from the insureds (premiums being determined within the limited mean-variance framework). We replace this price mechanism by the use of solvency regulation.

Popular forms of reinsurance are examined in order to understand the considerations of both the insurer and reinsurer, and to find the simultaneous optimum, if any exists.2 We seek a reinsurance solution which guarantees that both the insurer and reinsurer maintain adequate profitability, while remaining solvent. Each form of reinsurance splits a given risk differently between the insurer and reinsurer, each party bearing a specific share of the profit and the risk, and ties up some of its capacity. The optimum is thus reached by simultaneously balancing these offsetting variables. The reinsurance agreement is viewed as an incentive contract dealing with the tradeoff between profitability and the probability of ruin. In that respect this paper could be regarded as a continuation and extension of Stone’s (1973) seminal work on the capacity problem.

1.5. Reinsurance and market capacity

We define capacity as the maximum premiums which can be underwritten

2 In this article we examine separately proportional and non-proportional reinsurance. We shall not deal with combinations of the two types of reinsurance.
for a given capital while fulfilling the solvency constraint. Alternatively it could be measured by the minimum capital required to underwrite a given portfolio. An increase in capacity is thus characterized by a reduction of the required equity.

The question whether reinsurance may increase the market capacity has given rise to an interesting and controversial discussion in the literature. Doherty (1980) has used a portfolio approach to prove that proportional contracts do not increase the market capacity and to question whether non-proportional contracts may do so. Borch (1985b) has shown that if the portfolios of the ceder and the reinsurer are stochastically independent then additional capacity is usually gained by reinsurance.

We suggest that reinsurance has a dual effect on the required capacity:

(a) The 'risk-pooling effect', which is the risk reduction resulting from statistical considerations when all the risks are pooled together into a single global portfolio ('super global insurer').
(b) 'Redistribution effect' – the effect resulting from the splitting of the world portfolio among insurers and reinsurers.

Capacity may be gained by pooling stochastically independent portfolios. But the mere allocation of the combined portfolio usually decreases the capacity. In other words, the risk pooling effect is positive while the redistribution effect is negative (or zero).

Given the loss distribution associated with the risk, and given the solvency regulating one can calculate the amount of capital required in order to insure the risk. Sharing given risks among the insurer and reinsurer (and the insured) allocates the cash flows, profit and capacity among the parties. In the case of full information, redistribution in itself does not create a synergistic effect and, therefore, it is unreasonable to expect that one way of sharing will be preferable to the other. At most, a study of the motives of all parties could lead to a better understanding of the equilibrium pricing mechanism.

Section 2 develops the general framework, which focuses on two common reinsurance contracts: proportional (quota share) reinsurance and non-proportional excess of loss (hereinafter, XL) reinsurance. The analysis is confined to two parties – the insurer and reinsurer – which are subject to some sort of external regulation in the form of a probabilistic constraint on the permissible probability of ruin. The sharing agreement between the parties is then examined under three alternative scenarios: full and symmetric information, imperfect but symmetric information, and an environment with asymmetric information.

Section 3 deals with the case of full and symmetric information. The dual effect of reinsurance is examined. It is shown that the price of the reinsurance
contract should reflect the capacity which is tied up in the business. (While it is obvious to reinsurance practitioners, this effect has occasionally been overlooked in the literature.)

Section 4 deals with the case of imperfect but symmetric information. It is assumed that the parameters of the loss distributions must be estimated so that uncertainty is inevitable [see a summary of the empirical problems in Norberg (1982)]. Earlier contributions on the problem of parameter uncertainty [Houston (1964), Venezian (1983)] are used in the analysis.

Parameter uncertainty, ceteris paribus, reduces the capacity of both parties. The insurer may benefit from an XL contract, since it relieves him of most of the risk stemming from the imperfect information. However, since the reinsurer has to risk more capacity in order to meet the ruin constraint under the increased uncertainty, he must charge a higher premium or, rather, stick to the quota-share agreement.

Section 5 examines the case of asymmetric information, and shows that proportional contracts protect the reinsurer better against the potential adverse effects of his possession of inferior information.

Concluding remarks are presented in section 6 of the paper.

2. The general framework

The main task of this paper is to analyze whether sharing a given risk (among many insurers) will generate any overall economic benefit. The analysis is carried out by focusing on a simplified environment, with only two parties, predestined to be an insurer and a reinsurer (though the analysis may be extended to the more complicated case of multiple insurers). Both parties are subject to regulation: minimum capital requirements, and a probability of ruin that should not be exceeded. Further, a single-period framework, in which premiums are collected at the beginning of the period and claims are paid at its end is assumed.

The insurer's policy. The insurer invests the capital and the interim cash flows in the capital market. For the sake of simplicity assume that all investments are made in the riskfree asset. The insurer pays claims $X$ at the end of the period. $X$ is a random variable with distribution $F(X)$. The insurer collects premiums $P$ ($P \geq E(X)$). $K$ and $Y$ denote the end-of-the-period values of the insurer's equity and premiums income, respectively.

The insurer's objective is to maximize the expected value of the end-of-year

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3Solvency regulation is needed in the model since risk neutrality of (reinsurers may, otherwise, lead to the corner solution with extreme insurance leverage (little equity).

4This assumption is somewhat restrictive, since it imposes a complete separation between the assets and the insurance portfolio. It represents, however, the regulatory situation in many countries.
profits, subject to an external regulatory solvency constraint. This constraint is characterized by a parameter, \( z \) \((0 < z < 1)\), which is a permissible probability of ruin.\(^5\) That is, the objective function is

\[
\max E[\max(Y - X, -K)], \quad \text{subject to}
\]

\[
\text{prob}[(K + Y - X) < 0] \leq z.
\]  

Denote by \( K^* \) the minimal amount of equity, which is needed to underwrite a given portfolio \((Y, X)\) while maintaining the ruin constraint \((2).\)\(^6\)

2.1. The insurer's underwriting criterion

From the objective function \((1)\) and the ruin constraint \((2)\) we can derive the following underwriting criterion: for two alternative insurance portfolios \(a, b\) the insurer should prefer to underwrite portfolio \(a\) if

\[
Y^a - E(X^a) \geq Y^b - E(X^b) \quad \text{and}
\]

\[
K^a \geq K^b.
\]

where only one of the inequalities is strong.\(^8\)

If the insurer is interested in underwriting the entire proposed business he may take one or more of the following steps: raise more capital \((K)\), increase the premiums \((P)\), or buy appropriate reinsurance. The decision depends, of course, on the marginal costs and benefits of each option.

Reinsurance. Buying reinsurance is normally the most convenient and practical option for an insurer with a capacity problem; raising capital is often a quite complicated and lengthy procedure (besides the difficulties of raising more capital while in financial distress). The ability to collect higher

\(^5\)There could be an optimal \( z \) for which the 'social cost' of insurer's bankruptcy is minimized [see Kahane, Tapiero and Jacque (1986)].

\(^6\)If \( K > K^* \) (the outstanding equity is higher than the required equity) then the insurer has 'free capacity' of \( K - K^* \). If \( K < K^* \) then the insurer lacks capacity (he is not allowed to underwrite the entire proposed business).

\(^8\)This approach may be contrasted with that of Stone (1973) who defined capacity in terms of the 'exposure ratio' and that of Doherty (1980) who defined capacity in terms of a portfolio-performance measure.

\(^8\)It should be noted that though the insurer is an expected profit maximizer (and not an expected utility maximizer) he must set his underwriting policy according to portfolio considerations. As has been claimed by Borch (1985a) we do not need utility theory in order to analyse the insurer's policy.
insurance premiums (without changing the expenses) is limited due to the competition in the insurance market, and due to the insureds' demand function. Buying reinsurance is easily done and involves simple procedures.

After buying reinsurance, the insurer's ruin constraint becomes

\[ \text{prob}[K + Y_s - X_s < 0] \leq \alpha, \quad (4) \]

where the subscript \( s \) denotes the ceding company's share of the business (\( Y_s \) is the insurer's share in the premiums income, while \( X_s \) is its share in the claims).\(^9\) \( Y_s \) and \( X_s \) depend on the specific form of the reinsurance contracts. Two alternative forms are examined,

(i) A proportional (quota share) agreement.
(ii) A non-proportional (XL) contract.

In a quota-share agreement the insurer cedes the proportion \( (1-q) \) \((0 \leq q \leq 1)\) of the premiums, against the reinsurer's promise to share the same proportion \( (1-q) \) of the claims.\(^10\) In an XL contract the reinsurer agrees to pay all claims that exceed an agreed limit \( X_0 \).

The reinsurer expects to make a profit,\(^11\) i.e.,

\[ E(Y_r - X_r) > 0 \quad (5) \]

where the subscript \( r \) denotes the reinsurer's share in the premium and claims.

In addition, the loading factor charged by the reinsurer above the expected loss is equal to or higher than that required by the ceding company. In other words, the ceding company is not allowed to simply operate as a broker.

\[ [Y_r - E(X_r)]/Y_r \geq [Y_s - E(X_s)]/Y_s. \quad (6) \]

The reinsurer adopts the same underwriting criterion (3) as the insurer. For two alternative ceded portfolios (under two alternative reinsurance contracts) the reinsurer prefers the contract which requires less capacity and

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9 Reinsurer's bankruptcy may also cause the insurer's ruin. We assume implicitly that the reinsurer is solvent. The topic of risky insurance policies has been examined by Kahane et al. (1986).
10 We shall later refer to more complicated quota-share transactions where the reinsurer does not participate with the same proportion in premiums and claims.
11 It is assumed that the reinsurer is subject to the same solvency constraint \((\alpha)\) as the ceding company. Otherwise, if the reinsurer is subject to a less stringent solvency constraint \((\text{a higher } \alpha)\) the insurer may by-pass the local solvency requirements. (Moreover, it should be remembered that the insurer is fully responsible towards the insureds for the entire business and the bankruptcy of the reinsurer may cause the insurer's ruin.)
promises a higher expected profit. In addition, assume that the reinsurance activity does not incur any transaction and management costs. The effect of transaction cost on the reinsurance transaction has been studied elsewhere [see Blazenko (1986)].

The choice between the proposed reinsurance contracts (quota-share versus an $XL$ contract) is analyzed below under three alternative scenarios of the information possessed by the parties on the loss distribution:

Case 1 – Full information;
Case 2 – Imperfect but symmetric information;
Case 3 – Asymmetric information.

3. Full information

3.1. Reinsurance and market capacity

Let $(Y_0, X_{0t}, K^*_0)$ denote the premium income, claims, and required capital of the insurer and the reinsurer, respectively, at time 0 (before the reinsurance transaction).

Let $(Y_t, X_{1t}, K^*_1)$ denote these parameters after the reinsurance transaction.

Let $K^{**}$ denote the capacity which is required in order to underwrite the combined portfolio $(Y_{0t} + Y_{0t}, X_{0t} + X_{0t})$ by a 'super global insurer'. Then

(a) $K^*_0 + K^*_1 - K^{**}$ represents the risk-pooling effect.
(b) $K^{**} - (K^*_1 + K^*_1)$ is the pure redistribution effect.

Proposition 1.

(i) The risk-pooling effect is positive, i.e. $(K^*_0 + K^*_1 - K^{**}) \geq 0$.
(ii) The redistribution effect is not positive, i.e. $(K^{**} - (K^*_1 + K^*_1) \leq 0$.

Proof. Part (i) can be proved by simple statistical arguments. By pooling stochastically independent portfolios, additional capacity is usually gained. The pooling effect is maximized (and the required capacity is minimized to $K^{**}$) when the insurance companies pool their portfolios and agree that each shall pay a proportional part of the claims (reciprocal contract). Alternatively, the optimum is reached when all the independent portfolios are ceded to one 'super' international reinsurer, while the other insurer acts as a broker (without sharing the risk). This case has been examined by Eden and Kahane (1986).

If, however, the parties agree upon a different reinsurance contract the market capacity is reduced. In this case the separate portfolios are still subject to further pooling and diversification.
We do often observe reinsurance transactions in which the aggregate effect is to increase the capacity, i.e., $K_0^* > (K_r^* + K_r^* - K_0^*)$ or $K_0^* + K_r^* > K_r^* + K_r^*$. But this synergistic effect is a direct consequence of the risk-pooling effect and not of the reinsurance transaction.

If the reinsurer's portfolio cannot be further diversified then the capacity of the market will always be equal to or higher than the sum of the capacities of the insurer and the reinsurer.

To illustrate this point using a more realistic case, assume that an insurance company 'A' has founded a subsidiary company 'B'. 'A' transfers to 'B' some of its business and then reinsurance it. In such a case no synergistic effect results from the allocation of the risk between the two insurance companies. The capacity of companies 'A' and 'B' must be equal to or lower than the capacity of 'A' before the foundation of 'B'.

3.2. Quota-share and excess-of-loss reinsurance must be priced differently

The size of equity capital needed to insure a risk is affected by the form of the reinsurance contract; a proportional contract, ceteris paribus, reduces the insurance leverage (the ratio of retained premiums to the equity), i.e., $Y/K \leq Y/K$, without changing the nature of the loss distribution. An $XL$ contract, on the other hand improves, ceteris paribus, the insurer's capacity by cutting the extreme claims (truncated loss distribution). The insurer's choice between the two alternative contracts depends on the reinsurer's pricing policy.

A quota-share contract reduces proportionally all risks, including those that the insurer can retain. Hence, from the insurer's point of view there is a 'wasted' element in this contract. On the other hand, in an excess-of-loss coverage only risks that cannot be retained by the insurer are reinsured. When the premiums for quota-share and $XL$ reinsurance contracts are naïvely set as a fixed proportion, $\delta$, of the expected claims of the ceded business (i.e., $Y = \delta E(X)$, $\delta \geq 1$) the insurer must prefer an $XL$ contract.

A naïve reinsurance pricing (same $\delta$ for both quota share and $XL$ contracts) means that the reinsurer freely transfers capacity to the insurer. This is unacceptable to the reinsurer, since under an $XL$ contract he has to maintain higher equity in order to meet the ruin constraint. (See Example 2 in the appendix.) In other words, the reinsurer must charge a higher loading.

12A reallocation of a given portfolio between the insurer and the reinsurer may sometimes be unfavorable to the insureds. Example 4 in the appendix demonstrates the case in which the direct insurer faces a severe claim (loss) which makes him insolvent, whereas the reinsurer remains solvent, even though the sum of the capital of the two companies would have been sufficient to fully cover all the claim. The insureds have no direct access to the reinsurer, and have no right to transfer the claims against the reinsurer. Hence, from the insured's point of view the effective capacity may be less than the sum of the capacities of the insurer and the reinsurer [see Borch (1985b)].
factor for XL coverages, to reflect the economic value of capacity [see Stone (1973)].

One possible costing technique is to charge reinsurance premiums, \( Y_t \), so that the ratio of the expected profit \( (Y_t - X_t) \) to the additional required capital, \( AK^*_t \), is greater than or equal to a target rate of return \( R^* \). That is, the reinsurance contract will maintain

\[
\frac{Y_t - E(X_t)}{AK^*_t} \geq R^*, \quad 13
\]

(7)

where \( AK^*_t \) is the additional capital which is required to underwrite the ceded portfolio.

We can modify our underwriting criterion (3). For two alternative reinsurance contracts, \( a, b \), the reinsurer prefers the contract that promises a higher rate of return on the additional required capital \( (AK^*_t) \), \( AK^*_t = K^*_t - K^*_0 \), that is, the reinsurer prefers contract \( A \) to \( B \), if

(i) \[ \frac{Y_t - E(X_t)a}{(AK^*_t)a} > \frac{Y_t - E(Y)b}{(AK^*_t)b} \geq R^*, \]

(ii) \[ K^*_0 + (AK^*_t)a \leq K. \]

Condition (8ii) stems from the ruin constraint (2).

Let us now examine the case when the reinsurer’s portfolio is not subject to further diversification (i.e., there is no risk-pooling effect).

With quota-share reinsurance the rate of return on the required capacity is always the same, irrespective of the size of the retention, \( q \). If the reinsurer is unsatisfied with the ‘original terms’ (i.e., if \( (Y - E(X))/(K^*) < R^* \)) there is no acceptable quota-share treaty (under the original terms). This may lead to one of the following possibilities.

(a) No reinsurance. The insurer may prefer to supply partial coverage. In such a case the risk will be shared only between the insurer and the insured.

(b) Partial coverage. Here, the risk is shared by the insured, the insurer and the reinsurer. A risk-averse insured prefers coinsurance and deductibles arrangements. On the other hand, the reinsurer may prefer a policy which imposes an upper limit on the maximal claim. The design of an optimal insurance policy has to satisfy the needs and preferences of all three parties, an issue that has been overlooked in the traditional discussion in the

\[ 13 \] \( R^* \) may be affected by the market price of risk.
literature [e.g., Raviv (1979)], and more recently handled by Blazenko and by Eden and Kahane (1986).

(c) A positive reinsurance premium. If a transaction is carried out, the reinsurer's share in the premiums increases, enabling him to improve his performance. In such a case the reinsurer achieves a higher rate of return than the insurer. Apparently, a positive reinsurance premium violates the underwriting criterion (3), since the insurer would be better off giving up on the reinsurance transaction and supplying only partial coverage to the insureds.

However, in many cases partial coverage does not fit the insured's needs and preferences. If the insurer faces the dilemma of whether to pay a positive reinsurance premium or to reject the entire business, he may prefer to accept the proposed business and to reinsure some part of it, paying a positive reinsurance premium.\(^{14}\)

The insurer's expected profit (after paying a positive reinsurance premium) is

\[
Y -, E(X) = \beta q Y - q X; \quad \beta < 1,^{15}
\]

Proposition 2. If there is no risk pooling effect, then for every acceptable XL contract there is a quota-share contract that leads to the same allocation of expected profit, and the same returns on required capital.\(^{16}\)

Proof. This proposition is self-evident since the equivalent quota-share contract has two arbitrary parameters: the quota-share retention \(q\), \((q < 0 < 1)\), and a loading \(\beta \leq 1\), which generates a difference between the premium actually retained and the pure quota share \(q Y\).

4. Imperfect but symmetric information

In this section the analysis is extended to the more realistic case in which

\(^{14}\)This approach is different from the traditional explanation that the ceding company gives up part of its expected profits in order to reduce the probability of ruin [see Borch (1961)]. The analogy to the insured-reinsurer relationship is misleading, since the ceding, unlike the insured, is not endowed with the risk.

\(^{15}\)Hence we depart from the original definition of quota-share treaties and allow the premium to be apportioned differently to the claims. Note also that according to condition (6) which does not allow the ceding company to simply earn on its brokerage activity, there is no 'negative reinsurance premium' in our model (i.e., in the general case \(\beta \leq 1\)). Relaxing this assumption, one has to cope with moral hazard and the ceding's underwriting quality. For an attempt to explain the structure of the insurance market on the basis of differences in the effectiveness of monitoring moral hazard, see Eden and Kahane (1986).

\(^{16}\)This statement is incorrect when there is a positive risk-pooling effect. In such a case, the rate of return may be increased as the retention, \(q\), decreases, reflecting the decrease in the required capital due to the risk-pooling effect.
the parameters of the loss distribution are unknown. Our main interest is to
examine the effects of such uncertainty on the structure of the reinsurance
contracts.

It has been shown by Houston (1964) and Venezian (1983) that insurers of
reasonable size have little to gain in financial efficiency by pooling such
'fundamental' risks, or by pooling information.

For the sake of simplicity, assume that the relevant loss distributions are
known, up to a scale parameter. That is, loss distributions are linear
transformations of each other: if \( F_i(X) \) and \( F_j(X) \) are two of the \( N \)
possible distributions, then \( F_i(X) = F_j(\lambda X) \), where \( \lambda \) is a random variable, the
distribution of which is fully known.

It is further assumed that the scale parameter is determined by an
independent economic process. This may be the case if the parameter
uncertainty results from unexpected changes in the political or legal
environment.

Under this special case, the expected loss distribution is \( \mathbb{E}(\lambda) F(X) \) while the
expected value of the loss is \( \mathbb{E}(\lambda) \mathbb{E}(X) \), and the solvency margin, \( K^*(\lambda, x) \),
which is required in order to meet the ruin constraint (2) with a given
reliability level, \( 1 - x \), may be calculated accordingly. For the sake of
simplicity the following discussion relates to the special case where \( \mathbb{E}(\lambda) = 1 \).

The case of full information is a private case of this general problem where
\( \lambda \) is constant rather than a random variable.

**Proposition 3.** Uncertainty in the parameters of the loss distribution reduces
the capacity of both insurer and reinsurer.

Let \( K^*_1 \) be the required capital for which the loss distribution \( F_i(X) \) is fully
known, and let \( K^*_2 \) be the required solvency margin for the case of parameter
uncertainty, then for a given solvency standard, \( x \), and for a given premium and
expected profit it is argued that \( K^*_2 > K^*_1 \). (The subscript 1 denotes the case of
full information and the subscript 2 denotes the case of imperfect, but symmetric
information.

**Proof.** For the case of quota-share agreement. Under a quota-share contract,
the ceding company is unhappy with a reinsurance contract having the same
retention parameter, \( q \).

This claim can easily be proved by substituting the maximal rate of self-
retention, \( q^* \) in the ruin constraint (2):

- (a) \( \text{prob}[(K + q^*_1 Y - q^*X) < 0] = x \) (the case of full information),
- (b) \( \text{prob}[(K + q^*_2 Y - q^*_2\lambda X) < 0] = x \) (the case of parameter uncertainty).

Hence, \( q^*_1 > q^*_2 \) for all \( \lambda > 1 \).
Since $\lambda$ is a random variable with an expected value of 1, the insurer has to meet all cases of $\lambda$ exceeding 1. Therefore his capacity is being decreased.

For the case of XL coverage. Examine the most probable loss $X_m$ that still maintains the reinsurer’s ruin constraint (2) $\text{prob} [(K_r^* + Y - X_m) < 0] = \alpha$. Similarly to the case of a quota share, $\lambda X_m$ violates the equality for all $\lambda > 1$. Therefore $K_r^*$ must be larger than $K_{2r}^*$. The insurer may try to use an XL contract in order to overcome most of the parameter uncertainty. If an XL contract is acceptable for the insurer in the case of full information, then $\text{prob} [(K + Y - X_e) < 0] \leq \alpha$. This inequality is still valid also for the case of imperfect information and the contract is acceptable for the insurer. However, this contract is unacceptable to the reinsurer, since he is not compensated for the increase in his required assets, $K_{2r}$. [The offered contract violates conditions (6) and (7).]

5. Asymmetric information

Under this case the insurer and reinsurer do not have the same information about the loss distribution. Assume that the ceding company has better information than the reinsurer. The reinsurer’s disadvantage could be expressed in the following ways:

(a) The reinsurer estimates correctly the expected value of the loss distribution, but underestimates its variance. For example, the reinsurer may be unaware of the parameter uncertainty and may act as if he is facing a fully known loss distribution. This error may lead to underestimation of the required capital and, as a consequence, to overestimation of the expected return on equity.

(b) The reinsurer has incorrect estimates for both the mean and the variance of the loss distribution. Therefore, he has biased ideas concerning the expected profitability of the ceded business and concerning the required capital.

The reinsurer may be unaware of his inferior information and ask for too low premiums. This may be observed by the ceding company and it may attempt to take advantage of the situation. In this case the ceding company will have no motivation to correct the situation. In the reverse case, when the reinsurer asks for excessive premiums - the ceding company will have a strong incentive to disclose its superior information to the reinsurer.

This is a special form of the moral hazard problem. Our case, however, differs from the well-known principle-agent problem [see Shavell (1979), Barnea et al. (1985)]. In our case, the ceding company is absolutely passive; the actual losses are in no way the outcome of its activity. Yet we are dealing
with a case of asymmetry in information which leads to an incentive problem.

Our main interest is to examine the implications of the two alternative reinsurance contracts on this moral-hazard problem.

Let us examine first the quota-share agreements. The reinsurer may have inferior information and its actual rate of return may be less than its target rate. Yet, the reinsurer’s rate of return cannot be lower than the ceder’s. The ceder has no motivation to reveal the right information, but cannot use his superiority of information to achieve a higher rate of return.

Thus quota-share contracts by themselves reveal some information to the reinsurer. Conditions (4), (5), (6) are maintained despite the reinsurer’s inferior information.\(^{17}\)

In contrast with quota share contracts, XL contracts by themselves do not reveal information. Under XL contracts the reinsurer is less protected from the moral hazard risks. In particular, the ceder may use its superior information to obtain a higher rate of return than the reinsurer.

If the reinsurer has inferior information about both the expected value and the variance of the loss distribution, the ceder may use his advantage, by negotiating an ‘unfair’ reinsurance contract. (In an ‘unfair contract’ the expected value of the ceded business is negative.)

There could be other situations, where the reinsurer has better information than the insurer. This may be the case in certain lines where the reinsurer sees the global picture (earthquake, natural risks in agriculture, certain liability lines, aviation, etcetera).

6. Concluding remarks

(1) The capacity which is actually ‘tied up’ in the reinsurance transaction should be considered in the pricing of reinsurance contracts.

(2) The reinsurance transaction has a dual effect on the capacities of the two parties.
(a) A positive risk-pooling effect;
(b) A non-positive allocation (reinsurance) effect.

(3) In the case where there is no risk-pooling effect, then if quota-share and XL contracts are priced properly, they will lead to the same allocation of expected profits and returns.

(4) Solvency regulation should not focus only on the insurance leverage, but should consider also the reinsurance coverage and its form. Limiting the

\(^{17}\) This statement is valid even if we depart from the narrow definition of the quota share and allow the premium to be apportioned differently to the claims (i.e., \(\beta \leq 1\)). In practice the reinsurer pays a commission to reimburse the direct insurer for his costs. If the reinsurance commission deviates from the insurer’s expense ratio the well-known leverage problem occurs. For a further discussion, see Eden and Kahane (1986).
leverage (in order to regulate the probability of insolvency) is inadequate when XL reinsurance contracts are used.

(5) Reinsurance contracts have to protect the reinsurer in cases where he has inferior information and charges insufficient reinsurance premiums, while the cedee has, naturally, no motivation to reveal the 'true' information. Quota-share contracts, by their very nature, better protect the reinsurer against such adverse effects in comparison to XL contracts.

(6) The design of an optimal insurance policy at the insurer-reinsurer interface is interrelated with the form of the reinsurance contract.

Appendix

The purpose of this appendix is to present a few numerical illustrations of the issues discussed in the article. All examples are based on similar initial parameters which are presented in the first two examples.

Example 1. The insurer's underwriting criterion

The purpose of this example is to demonstrate – under somewhat artificial assumptions – the impact of solvency constraints on the underwriting policy of an expected profit maximizer insurer. This insurer is able to discriminate among insureds, and is thus able to absorb the entire 'consumer surplus'.

Assume an insurer with equity capital of $4.1M, and a portfolio consisting of six identical risks (Bernoulli trials). Each unit may suffer $1M loss with a probability $p$, or remain undamaged with a probability $(1 - p)$ and all risks are stochastically independent. Each unit may suffer, at most, one loss during the period. The aggregated loss distribution $F(X)$ is binomial with a parameter $p = 0.1$. Thus $E(X) = 0.6M$, and $\sigma(X) = 0.73M$.

Due to his specific tastes (utility) the insured is willing to pay a premium of $150,000 to fully cover each risk (aggregated premium $0.9M$). However, he insists on having a full coverage to all six risks.

Assume that the insurer is subject to solvency regulation with a permissible probability of ruin of $\alpha = 0.0001$. Given the binomial loss distribution, the insurer must be prepared to meet a loss of up to $5M$ (a loss of 5 independent units). Such a loss can be handled by holding equity of $K^* = -0.9 + 5 = 4.1M$ (the maximum permissible loss minus the premium), and by assumption, the insurer is endowed with such equity.

Assume now that a second customer is interested in buying insurance to cover four additional exposure units which are stochastically independent and identical to the previous ones. Given his specific utility function this customer is willing to pay an aggregated premium of $0.7M$ for this coverage.

With these policies the insurer has a portfolio of 10 units, and is exposed
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assumed value under</th>
<th>6 units portfolio</th>
<th>4 units portfolio</th>
<th>Combined portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Notation</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Premiums</td>
<td>$y$</td>
<td>0.9</td>
<td>0.7</td>
<td>1.6</td>
</tr>
<tr>
<td>Expected claims</td>
<td>$E(x)$</td>
<td>0.6</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td>Expected profit</td>
<td>$y - E(x)$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Actual loading factor</td>
<td>$y / E(x)$</td>
<td>1.5</td>
<td>1.75</td>
<td>1.60</td>
</tr>
<tr>
<td>Maximum permissible loss</td>
<td>$M$</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Required capital</td>
<td>$K^*$</td>
<td>4.1</td>
<td>3.3</td>
<td>4.4</td>
</tr>
</tbody>
</table>

to a new binomial loss distribution ($p = 0.1$, $N = 10$). The regulatory constraint means that the insurer must be capable of handling a loss of $6M$ (the loss of 6 independent units) without going bankrupt. Given the premium income of $6.6M$ the required capital is $K^* = 6 - 1.6 = 4.4M$. Since the equity at this point is only $4.1M$ the insurer is not allowed to retain the entire business unless more capital is raised, or unless some parts of the risks are ceded to reinsurers.

Assuming that each insured insists on obtaining full coverage to all units he owns, the insurer will have to choose between two alternative portfolios:

(a) The initial 6 units portfolio.
(b) The proposed 4 units portfolio.

Portfolios A and B generate the same expected profit ($0.3M$), but portfolio B ties less capacity, and is, therefore, preferred from the insurer’s point of view. (See table 1.)

**Example 2. Naive reinsurance pricing**

The purpose of this example is to demonstrate that naive reinsurance pricing – where both the quota-share and non-proportional contracts are sold with the same loading factor – cannot be the equilibrium case.

Assume that the insurer in the previous example has access to the reinsurance market. Assume that the reinsurer already holds a portfolio of 10 other stochastically independent exposures which are similar to those discussed above, but are covered for a total premium of $1.6M$.

The reinsurer is subject to the same solvency requirement with a permissible probability of ruin of $\alpha = 0.0001$, which means that he must be prepared to face a loss of up to $6M$ with his current portfolio. Assuming that the reinsurer’s equity is $5.4M$ and the premium income is $1.6M$ the reinsurer has free capacity of $1M$ ($5.4 + 1.6 - 6$).

Being approached by the ceding company as discussed above, the reinsurer considers whether to extend his portfolio.
Table 2  
Quota-shares vs. non-proportional coverage (10 additional units with $1.6M premium).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Quota-share (with q = 0.93)</th>
<th>Non-proportional coverage (with Xₚ = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Insurer</td>
<td>Reinsurer</td>
</tr>
<tr>
<td>Premiums</td>
<td>Y</td>
<td>1.49</td>
<td>0.11</td>
</tr>
<tr>
<td>Expected claims</td>
<td>E(X)</td>
<td>0.93</td>
<td>0.07</td>
</tr>
<tr>
<td>Expected profit</td>
<td>Y - E(X)</td>
<td>0.56</td>
<td>0.04</td>
</tr>
<tr>
<td>Loading factor</td>
<td>Y/E(X)</td>
<td>1.60</td>
<td>1.60</td>
</tr>
<tr>
<td>Maximum permissible loss</td>
<td>M</td>
<td>5.58</td>
<td>-</td>
</tr>
<tr>
<td>Required capital</td>
<td>K⁺</td>
<td>4.10</td>
<td>-</td>
</tr>
</tbody>
</table>

*The maximum permissible loss for the 10-unit portfolio (under permissible α = 0.0001) is $6M. (A loss of 6 independent units.) Hence, the insurer’s MPL under the quota-share contract is 6.093 – 5.58.

Let us examine the optional reinsurance contracts:

**Quota share** (see a summary of this example in tables 2, 3)

If the reinsurer is satisfied with the same loading as the ‘original’ business (i.e., an average loading factor of 1.60 – see table 1), the maximum quota q, which can be retained by the ceding company can easily be found by solving the boundary condition:

\[ K + q(Y - X) = 0, \]

for the maximum permissible loss \(X = $6M\) in our case.

In other words:

\[ 4.1 + 1.6q - 6q = 0 \Rightarrow q = 0.93. \]

Under such a quota-share arrangement, the ceding company retains 93% of the business, its net retained premium income is 1.49 \((Y = 0.93 \cdot 1.6)\), its expected claims 0.93 \((qE(X) = 0.93 \cdot 1)\) and its expected profit is 0.56. The reinsurer takes the rest of the premiums, claims and profits, i.e., his additional premium is \((1 - 0.93) \cdot 1.6 = 0.11\) while his expected claims are \((1 - 0.93) \cdot 1 = 0.07\), leaving a marginal gross profit of 0.04.

**Non-proportional coverage**

The insurer’s ruin constraint is fulfilled surely with non-proportional coverage with an excess point \(Xₚ = $4M\) (it may even be fulfilled with a lower point). The reinsurer’s expected claims from such an additional contract are \(E(Xₚ) = 0.0082\). (This figure is obtained by calculating the probabilities of having more than 4 damaged units.)
Table 3
Examination of the combined reinsurers’ portfolio.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Assumed value under</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Before transaction</td>
</tr>
<tr>
<td>Equity</td>
<td>$K$</td>
<td>5.40</td>
</tr>
<tr>
<td>Premiums</td>
<td>$Y$</td>
<td>1.6</td>
</tr>
<tr>
<td>Expected claims</td>
<td>$E(X)$</td>
<td>1.0</td>
</tr>
<tr>
<td>Expected profit</td>
<td>$Y - E(X)$</td>
<td>0.6</td>
</tr>
<tr>
<td>Loading factor</td>
<td>$Y/E(X)$</td>
<td>1.6</td>
</tr>
<tr>
<td>Max. permissible loss*</td>
<td>$M$</td>
<td>6.42</td>
</tr>
<tr>
<td>Required capital</td>
<td>$K^*$</td>
<td>4.4</td>
</tr>
<tr>
<td>Free capacity</td>
<td>$K - K^*$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*Before the additional transaction the maximum permissible loss (under $z = 0.0001$) was the loss of 6 out of 10 exposures. With the new quota-share contract the reinsurer has two independent portfolios: the initial portfolio of 10 units of $1M each which may bring about a loss of $6M and the ceded portfolio of 10 units where 6 losses at $0.07M each may bring about a loss of $0.42M. Therefore, the most probable loss is less than 6.42.

If the reinsurer accepts the proposed non-proportional contract he copes with a 20-exposures portfolio. The maximum permissible loss is $9M. The ceder retains losses of $2M, i.e., the reinsurer faces a maximum permissible loss of $9 - 2 = 7M.

Assuming that the reinsurer still charges the same loading factor, he requires a premium of $0.014 = (1.60 - 0.008)$, leaving the ceding company with retained premium of $1.586M. Given the possibility of a $4M loss, the ceding company must hold a capital of at least $2.41M.

The insurer prefers the non-proportional coverage contract, which improves his expected profits and creates excess capacity. However, this contract is inferior from the reinsurer’s point of view: the new business has to be considered with the existing portfolio. Given the new binomial distribution, the maximum permissible loss is at least $7M$, i.e., the reinsurer must tie up almost all his $1M free capacity. The alternative quota-share contract ‘ties up’ less capacity (only an additional $0.31M) and carries a higher expected profit. These results are presented in table 3.

This example demonstrates that the reinsurer must charge a higher loading factor for non-proportional coverage contracts. Otherwise, he transfers capacity to the insurer without receiving the economic compensation for it [violating the underwriting criterion (3)].

Example 3. Reinsurance and the market capacity

By uniting the portfolios of the insurer and the reinsurer we get a combined portfolio of 20 independent exposures. The maximum loss which the insurer is still expected to handle (under the permissible $z = 0.0001$) is $9M (the loss of 9 units). The initial premium incomes of the insurer and the reinsurer are $Y_{o} = 1.6$ and $Y_{o} = 1.6$. Hence the capacity which is required in
Table 4
The dual effect of the reinsurance transaction.\textsuperscript{*}

<table>
<thead>
<tr>
<th>Insurer</th>
<th>Reinsurer</th>
<th>Total required capacity (rounded)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y$</td>
<td>$MPL$</td>
</tr>
<tr>
<td>(a) No reinsurance</td>
<td>1.6</td>
<td>6</td>
</tr>
<tr>
<td>(b) Ceding all business to the reinsurer</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Risk-pooling effect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) QS reinsurance with $q=0.93$</td>
<td>1.49</td>
<td>5.58</td>
</tr>
<tr>
<td>Reinsurance effect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d) Non-proportional coverage reinsurance with $X_1=4$</td>
<td>1.586</td>
<td>4</td>
</tr>
<tr>
<td>Reinsurance effect</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{*}$Y$-Premium income, $M$-Maximum permissible loss (under $x=0.0001$), $K^*$-Required capacity.

In order to underwrite the combined portfolio is $K^{**}=9-1.6-1.6=5.8$. The capacity which was required before the reinsurance transaction was $K^*_s+K^*_r=4.4+4.4=8.8$. In other words, the 'risk-pooling effect', i.e., the reduction of the required equity obtained by handling all risks by a 'super global insurer' is $8.8-5.8=53$M. The large saving which is demonstrated in this example stems from the specific binomial nature of the problem, and the limited diversification reached in the assumed initial portfolios.

Under the proposed quota-share contract (with $q=0.93$) the required capacities ($K^*_s + K^*_r$) are $4.1+4.71=8.8$. Hence the 'negative' pure reinsurance effect is $5.8-8.8=-3.0$M. Under the alternative non-proportional coverage contract the required capacities are $2.41+5.39=7.80$ and the pure reinsurance effect is $5.8-7.8=-2.0$M. In other words, most of the potential saving due to the risk-pooling effect has not been reached. Moreover, the quota-share agreement proves to be the least efficient, since a very large proportion of the risk is retained, so that for practical purposes we have not come closer to the desired pooling.

Table 4 summarizes these results.

Example 4. Insurer's limited liability

This example demonstrates that the limited liability of the insurance company may decrease the effective market capacity.
Let us re-examine cases $b$ and $d$ of table 4. In case $b$ (where the reinsurer underwrites the whole combined portfolio) the maximum permissible loss is $9$M, and the system will handle all losses up to this level.

In case $d$ (XL contract) the coverage is split between a ceding company and a reinsurer. There could be cases where less than 9 units are lost, without being properly covered, due to bankruptcy of the ceding company. For example, if more than 6 exposures of the ceder’s portfolio are lost, the insureds will not be fully indemnified, due to the insurer’s insolvency (despite the fact that the reinsurer still has free capacity to handle all claims).

Example 5. Proper reinsurance pricing

Let us examine the implications of the reinsurer’s conditions (7), (8) with the above numerical example. From table 3 we learn that under the non-proportional coverage contract the rate of return on the ceded portfolio is only $0.006/0.986 = 0.0065$; while under the QS contract the rate of return is $0.04/0.31 = 0.129$.

Therefore, the proposed non-proportional coverage contract will not be accepted by the reinsurer. The reinsurer may agree to a non-proportional coverage contract with excess point being $X_c = $4M only if his share in the premium is $0.12$M, promising him a rate of return of $0.129$.

The results of the revised non-proportional coverage contract are shown in table 5.

If the ceder accepts this revised non-proportional coverage contract he has to increase the ceded premiums from $0.014$M (under the first proposed contract) to $0.1213$M. However, his rate of return under the revised non-proportional coverage contract (0.193) is still higher than the expected rate of return under the QS contract $R_c = 0.56/4.10 = 0.137$.

Hence, the revised non-proportional coverage solution may be accepted by both parties.
Example 6. Asymmetric information leading to inferior non-proportional coverage contracts

Let us modify Example 2 by assuming that the actual loss distribution on the insurer's portfolio is binomial with \( n = 10 \); \( p = 0.19 \), i.e., \( E(X) = 1.9M \), and assume that the premium income is \( Y = 2M \).

The correct distribution is known solely to the ceding company. The reinsurer wrongly believes that the loss distribution is binomial with \( n = 10 \); \( p = 0.10 \). That is \( E(X) = 1.0M \).

The ceding company is looking for a non-proportional coverage with an excess point \( X_c = 0.4M \). The proposed ceded premium is \( Y_c = 0.1212 \). The reinsurer may accept this offer, since according to his information his expected loss is \( E(X_c) = 0.008 \) (see table 5, Example 5). The reinsurer has no way of knowing that the expected loss in the ceded business is \( E(X_c) = 0.1398 \), and that the amount of the ceded premiums \( Y_c = 0.1213 \) is lower than the expected loss.

It is noteworthy that although this proposed non-proportional coverage is actuarially unfair for the reinsurer, the reinsurer has good chances of making a profit: \( \text{prob}(X < (X_c + Y_c)) = \text{prob}(X < (4 + 0.1398)) = 0.9734 \).

Retroactive arrangements (such as profit commission) cannot provide a full remedy to this problem. This example demonstrates that under a non-proportional contract, the fact that the reinsurer enjoys a profit does not necessarily mean that the reinsurance contract was fair.

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