DETERMINATION OF THE PRODUCT MIX AND THE BUSINESS POLICY OF AN INSURANCE COMPANY—A PORTFOLIO APPROACH*

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This paper sets out a model which simultaneously determines the optimal composition of the insurance and investment portfolios of an insurance company using Sharpe's Single-Index Technique. This technique can be explained for management as follows: different product lines that a multi-product firm offers have different rates of return and different risks associated with those rates of return. Taking into account both risks and rates of return, what is the best mix of product lines for a firm to offer in the marketplace? This approach is especially suitable for insurance because of data limitations. This type of analysis can make a useful contribution in shaping the firm's product mix and marketing policy.

1. Introduction

Recent publications have drawn attention to marketing problems of insurance companies, and especially to the problem of their marketing channels (see for example [3], [6]). However, only little attention has been given to the determination of the desired product mix of an insurance company—i.e., to the types and the number of policies which should be sold. This problem becomes quite complicated due to the existence of interrelationships between the sale of policies in various lines, and due to regulation which restricts the amount and the types of obligations that an insurer may take. The purpose of this paper is to suggest a model which may assist insurers in making their product mix decisions.

The product-mix dilemma may be formulated by using the terminology and tools of portfolio-selection theory [2], [13]. This approach is especially relevant in the insurance context since the activity of an insurance company may be viewed as the management of a portfolio of insurance policies, in addition to the handling of an investment portfolio. The profits from these insurance and investment activities are random variables, and the firm is assumed to have estimates concerning their distributions. The rates of profit of every two activities may be correlated, and this gives rise to interesting risk-reduction effects through diversification (e.g. through multiple-line operation). Therefore, the decision to extend the activity in a certain insurance line cannot be taken in isolation: Instead, the combined effect with other insurance lines should be examined [4]. Moreover, the insurance product mix should be determined simultaneously with the investment portfolio due to the possible correlation between the underwriting and investment incomes. The current literature suggests a model which applies quadratic programming techniques to find simultaneously the efficient compositions of the insurance and the investment portfolio of

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1 For example, the profits in workmen's compensation insurance may be negatively correlated with the economic cycles (less accidents occur in the time of depression and therefore profits tend to be higher). The profit in fire insurance, on the other hand, may be positively related with economic cycles (more arsons in times of recession). Thus the profit in these lines may be negatively correlated. Similar systematic relationships may also exist between insurance and investment activities (e.g. the investment income may be positively correlated with the economic trends—and thereby negatively related to the profit in workmen's compensation insurance).

2 This effect is similar to the one generated by diversification of portfolios of securities. Such an effect is the subject of extensive discussion in finance literature. (See, for example, a summary in [9].)
insurance companies [1], [4], [5], [7], [8], [11]. The purpose of the present paper is to suggest a simplified model to solve for the composition of the efficient portfolios. The suggested technique, which is based on the Sharpe-Single-Index Technique [12], uses a greatly reduced data set, which enables applying the model for practical purposes.

A second aim of the paper is to focus on some marketing considerations which should be added to the basic underlying framework. The portfolio approach implicitly assumes that the firm may increase or decrease the volume of its activity in each insurance line without changing the expected rate of profit, or the risk characteristics of the said line. This clearly ignores the possibility of market saturation on one hand, and the inability of insurers to completely abandon certain insurance lines (due to the nature of the marketing channels) on the other hand. In addition, some other factors and constraints should be considered in determining the product mix; for example, the customers may prefer to have all their insurance handled by one company, thus the insurer cannot offer only a certain type of insurance while refusing to sell policies in other complementary lines. Insurers are also subject to regulation concerning the amount of premium they may write with a given equity, which again may restrict their freedom in the choice of the optimal insurance mix. Such marketing relationships can often be expressed as linear constraints and be incorporated into the model.\(^3\)

In §2 the simplified portfolio model is presented; the data requirements of the model are set out in §3. The potential uses of the model are demonstrated with the above data in §4, and §5 concludes the paper with some general remarks and conclusions.

2. The Model

In this section a model for the simultaneous determination of the efficient composition of insurance and investment activities of an insurance company (industry) is presented. Earlier models (see for example [1], [5], [7], [8], [11]) use the Markowitz portfolio-selection approach [10], which requires a large data-set. The alternative model, suggested in this paper, applied the Sharpe-Single-Index approach [12], which cuts the data requirement substantially. The model is formulated as follows:

Assume that there are \( m \) insurance lines and \( n - m \) types of possible investments and that the return on these activities are random variables \( \tilde{r}_i \), with known distributions having finite expected values and finite (nonzero) variances.\(^4\) The return on equity, \( \tilde{y} \), is a linear combination of these random variables:

\[
\tilde{y} = \sum_{i=1}^{n} a_i \tilde{r}_i,
\]

where

- \( \tilde{r}_i \) = rate of profit on \( i \)th insurance activity, after reinsurance, (percent of premium)
  - for \( i = 1, 2, \ldots, m \);
- \( \tilde{r}_i \) = rate of return on investment \( i \) (percent of the assets) for \( i = m + 1, m + 2, \ldots, n \);
- \( a_i \) = ratio of premium of insurance line \( i \) to equity (\( i = 1, 2, \ldots, m \));
- \( a_i \) = ratio of investment in asset \( i \) to equity (\( i = m + 1, m + 2, \ldots, n \));
- \( \tilde{y} \) denotes a random variable and will be omitted hereafter.

\(^3\) In addition, the investment portfolio is subject to regulatory constraints on the types of assets and the amounts that may be invested. Such constraints are not discussed in the paper, but they can quite easily be incorporated into the model.

\(^4\) The expected returns may be negative. This may especially be true for the returns on the insurance activities—as the insurer may be willing to underwrite at a loss, since he expects to be compensated by the investment income generated from his insurance activity. The possibility of a risk-free asset (cash) has been excluded. This simplifying assumption does not severely affect the usefulness of the results, since the cash holdings of insurers are relatively small in practice.
In order to reduce the data requirements it is assumed that the various activities are related only through common relationships with an index of general market performance. This is a simplifying assumption, and its validity is discussed later in the paper. It is assumed that the return on activity \( i \) is related linearly to a certain index number, \( I \):

\[
\tilde{r}_i = A_i + B_i \tilde{I} + \tilde{C}_i \quad (i = 1, \ldots, n).
\]

(2)

\( A_i \) and \( B_i \) are parameters (which can be determined by regression analysis). \( \tilde{C}_i \) is a random variable (the tilde will be omitted hereafter) which is assumed to fulfill

\[
E(C_i) = 0 \quad (i = 1, \ldots, n),
\]

\[
\text{Var}(C_i) = 0 \quad (i = 1, \ldots, n),
\]

(3)

\[
\text{COV}(C_i, C_j) = 0 \quad (i \neq j, i, j = 1, \ldots, n),
\]

and the random variable \( I \) is assumed to have the distribution

\[
I = A_{n+1} + C_{n+1},
\]

(4)

where \( E(C_{n+1}) = 0 \), \( \text{Var}(C_{n+1}) = Q_{n+1} \), \( \text{COV}(C_{n+1}, C_i) = 0, i = 1, \ldots, n \), where the subscript \( n+1 \) is used for mathematical convenience. Using these assumptions the return on equity (1) becomes:

\[
y = \sum_{i=1}^{n} a_i r_i = \sum_{i=1}^{n} a_i [A_i + B_i \tilde{I} + C_i]
\]

\[
= \sum_{i=1}^{n} a_i (A_i + C_i) + \sum_{i=1}^{n} a_i B_i (A_{n+1} + C_{n+1}).
\]

(5)

This expression can be simplified by denoting

\[
\sum_{i=1}^{n} a_i B_i = a_{n+1}.
\]

(6)

(6) defines an imaginary activity \( n + 1 \), which may be called the “investment in the index.” This notation helps to describe the total return on equity as a weighted sum of the nonsystematic return on all activities and the systematic element resulting from the relationship to the index.

\[
y = \sum_{i=1}^{n+1} a_i (A_i + C_i).
\]

(7)

Assuming that the insurer wishes to minimize the variance of the rate of return on the equity, for any level of expected value,\(^6\) the objective becomes to find the set of \( a_i \) which minimizes \( L \):

\[
\min L = \text{Var}(y) - \lambda E(y),
\]

(8)

where (see (7))

\[
E(y) = \sum_{i=1}^{n+1} a_i A_i,
\]

(9)

\[
\text{Var}(y) = \sum_{i=1}^{n+1} a_i^2 Q_i.
\]

(10)

\(^5\) It is assumed that any additional dollar of premium, or investment, in activity \( i \) fulfills the relationship (2). This is clearly unrealistic as it implicitly assumes unsaturated insurance and investment markets. However, this assumption is acceptable when dealing with a small company and with limited changes in the composition of insurance and investment portfolios.

\(^6\) The limitations of the Mean-Variance approach are well known (and summarized for example in [9]). This criterion can, however, be safely used since \( r_i \) are approximately normally distributed and so is \( y \), and the Mean-Variance Criterion is workable in this case.
The unknowns $a_i$ are subject to the following constraints:

$$a_i \geq 0, \quad i = 1, 2, \ldots, n.$$  \hspace{1cm} (11)

$$a_{n+1} = \sum_{i=1}^{n} a_i B_i$$  \hspace{1cm} (12)

which reflect the common nonnegativity constraints, and the notation (6), respectively. In addition, a balance sheet constraint, which keeps the equality of the equity to the total assets minus liabilities, has to be introduced:

$$\sum_{i=m+1}^{n} a_i - \sum_{i=1}^{m} a_i g_i = 1.$$  \hspace{1cm} (13)

The term $\sum_{i=m+1}^{n} a_i$ represents the assets per one dollar of equity, and the term $\sum_{i=1}^{m} a_i g_i$ represents the total liabilities which are generated by the insurance business against each dollar of equity (the coefficients $g_i$ express the amount of reserves which are generated by one dollar of premiums in insurance line $i$). \(^7\) (13) is expressed in terms of assets and liabilities per dollar of equity, and therefore equals 1.

The efficient composition of the insurance and investment activities is derived by the simultaneous solution of (8) subject to (9)–(13). The portfolios obtained are efficient because no other balance sheet combination will have a lower degree of risk with the same level of expected return.

3. Data Requirements of the Model

Earlier portfolio models \([4], [5], [7], [8], [11]\) of insurance companies require the knowledge of quite a substantial number of parameters, which is much larger than what is needed for the suggested model. The use of the old models requires estimates for the expected return and the variance for each insurance line and for each asset, and estimates for the correlation matrix between the returns on all activities. The present model reduces the data requirements substantially by taking into account only the relationships of each activity with a general index, instead of studying the relationships between all activities. In this way only three parameters are needed for each activity ($A_i$, $B_i$ and $Q_i$), and another two parameters ($A_{n+1}$ and $Q_{n+1}$) for the index. The analysis of a problem with 20 activities requires, therefore, only 62 parameters as compared to 250 estimates which are needed for the previous model. The analysis of a problem with 100 activities requires only 302 estimates as compared to 5150 parameters in the earlier model \([12]\). The gain from such a saving is not only in the computational time, but mainly in the elimination of the need for data which are hardly attainable, and such a data reduction is extremely important when ex-ante parameters are used. Thus, the reduction of the number of parameters makes the present model much more applicable in practice.

Another major advantage of the new approach over the Markowitz portfolio model is revealed when parameters are estimated by analyzing historical records: The number of periods has to be larger than the number of activities studied, otherwise the variance-covariance matrix will be singular, and there will be no solution to the quadratic programming problem. \(^8\) This becomes a very significant problem when analyzing insurance companies’ data, since their data are generally compiled on an

\(^7\) The liabilities of an insurance company are generated through the selling of insurance policies and result from the delayed nature of payment of claims.

\(^8\) Assume $N$ variables each with $T$ observations, which are ordered in matrix $X$ ($TXN$). The variance-covariance matrix is $(X - JX/N)(X - JX/N)$. If $T < N$ the rank of the covariance matrix is $T$ (since $r(AB) < r(A), r(B)$).
annual basis (only in some rare cases are quarterly data available). Thus, in order to study a complex problem with many activities, data for an extremely long period are required. Even if such data are available there is always a danger that a long historical series no longer reflects current trends and relationships. The use of the single index approach eliminates such problems, and a relatively short period is sufficient for estimating the relevant parameters.

The potential use of the model for the determination of insurer's business policy and marketing decisions is demonstrated by the use of the industry's aggregated data. The parameters were estimated from historical records to the annual performance of U.S. stock insurance companies, and of capital market indices, for the period 1956–1973. For demonstration purposes 18 insurance lines and 2 assets (stock and bonds) were analyzed. The analysis can be extended to include more activities—depending on the availability of data. The use of aggregate historical data is subject, of course, to all reservations with regard to the use of ex-post data as estimators of ex-ante expectations.

The aggregated annual rate of underwriting profits was selected to serve as the general index. The relationships between the return on each line and the index were estimated by a simple linear regression (2) and the regression coefficients are reported in Table 1. The coefficients of determination ($R^2$) are significant at 5 percent confidence level for 14 out of the 20 activities. In the other cases $R^2$ is below 0.2 and insignificant. The low value of $R^2$ is not surprising, and has been reported also in

\[
T_i = A_i + B_i T + C_i
\]

<table>
<thead>
<tr>
<th>Activity</th>
<th>$A_i$</th>
<th>$B_i$</th>
<th>$R^2$</th>
<th>$T$ value for $B_i$</th>
<th>Unexplained Variance ($Q_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fire</td>
<td>2.6</td>
<td>1.9</td>
<td>0.53</td>
<td>4.23</td>
<td>13.8</td>
</tr>
<tr>
<td>Allied lines</td>
<td>4.0</td>
<td>3.6</td>
<td>0.52</td>
<td>4.20</td>
<td>50.4</td>
</tr>
<tr>
<td>Home owners</td>
<td>-8.2</td>
<td>3.4</td>
<td>0.30</td>
<td>2.63</td>
<td>111.4</td>
</tr>
<tr>
<td>Commercial</td>
<td>-4.4</td>
<td>4.1</td>
<td>0.39</td>
<td>3.21</td>
<td>111.5</td>
</tr>
<tr>
<td>Ocean marine</td>
<td>0.9</td>
<td>2.1</td>
<td>0.41</td>
<td>3.37</td>
<td>26.5</td>
</tr>
<tr>
<td>Inland marine</td>
<td>2.2</td>
<td>2.2</td>
<td>0.65</td>
<td>5.41</td>
<td>11.7</td>
</tr>
<tr>
<td>Group health</td>
<td>-0.5</td>
<td>-0.6</td>
<td>0.24</td>
<td>-2.22</td>
<td>4.4</td>
</tr>
<tr>
<td>Health</td>
<td>6.1</td>
<td>-0.5</td>
<td>0.12</td>
<td>-1.50</td>
<td>8.8</td>
</tr>
<tr>
<td>W. Compensation</td>
<td>2.8</td>
<td>-0.7</td>
<td>0.25</td>
<td>-2.32</td>
<td>5.6</td>
</tr>
<tr>
<td>Liability</td>
<td>-1.7</td>
<td>-1.8</td>
<td>0.24</td>
<td>-2.27</td>
<td>44.8</td>
</tr>
<tr>
<td>Auto-liability</td>
<td>-4.2</td>
<td>1.0</td>
<td>0.74</td>
<td>6.74</td>
<td>1.5</td>
</tr>
<tr>
<td>Auto-property</td>
<td>3.5</td>
<td>1.7</td>
<td>0.72</td>
<td>6.49</td>
<td>4.6</td>
</tr>
<tr>
<td>Fidelity</td>
<td>3.0</td>
<td>0.4</td>
<td>0.02</td>
<td>0.53</td>
<td>41.7</td>
</tr>
<tr>
<td>Surety</td>
<td>8.1</td>
<td>0.1</td>
<td>0.00</td>
<td>0.18</td>
<td>36.8</td>
</tr>
<tr>
<td>Glass</td>
<td>-2.7</td>
<td>1.4</td>
<td>0.39</td>
<td>3.22</td>
<td>12.1</td>
</tr>
<tr>
<td>Burglary</td>
<td>2.7</td>
<td>2.7</td>
<td>0.32</td>
<td>2.77</td>
<td>62.5</td>
</tr>
<tr>
<td>Boiler</td>
<td>2.1</td>
<td>0.7</td>
<td>0.04</td>
<td>0.86</td>
<td>41.8</td>
</tr>
<tr>
<td>Credit</td>
<td>5.1</td>
<td>-6.6</td>
<td>0.26</td>
<td>-2.40</td>
<td>513.6</td>
</tr>
<tr>
<td>Stocks</td>
<td>9.1</td>
<td>0.9</td>
<td>0.03</td>
<td>0.69</td>
<td>111.1</td>
</tr>
<tr>
<td>Bonds</td>
<td>1.7</td>
<td>1.1</td>
<td>0.18</td>
<td>1.87</td>
<td>25.4</td>
</tr>
<tr>
<td>Total Underwriting Profits</td>
<td>$-0.5$</td>
<td></td>
<td></td>
<td></td>
<td>4.0</td>
</tr>
<tr>
<td>(The Index)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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* Insurance data were gathered from Best's Property-Casualty Aggregates and Averages (1974), A.M. Best Co., New York. Rates of return on stocks were calculated from Moody's composite index, and realized returns on bonds were calculated from the Federal Reserve Bulletin.
studies of the Sharpe model in the context of capital markets. It should, however, be kept in mind that the cases with low $R^2$ do not invalidate the analysis, since the regression is used merely in order to isolate one source of variation (the reaction of the line to the general index). This element is captured by the term $a_{i+1} Q_{i+1}$ in (10), which is equal to $(\sum_{i=1}^{n} a_i B_i)^2 Q_{i+1}$. The remaining unexplained variances $Q_i (i = 1, \ldots, n)$ are taken into account, as well, in (10) (through the sum $\sum_{i=1}^{n} a_i^2 Q_i$).

The parameter $A_i$ is an estimate for the intercept of the regression line. This parameter represents the return obtained in activity $i$ when the index effect is neutralized. Some insurance lines have negative coefficients $A_i$, meaning that the insurers are expected to suffer underwriting losses in these lines when the general index is zero (i.e., no aggregate underwriting profits). (For example, Table 1 shows that in Auto Liability the expected loss is 4.2 percent of premium.) Insurers may be willing to underwrite in such lines despite the losses because they still may realize some investment income on the funds generated by these lines, and because of possible risk reduction effects that these lines may have when combined with the other activities.

The parameters $B_i$ represent the relationships between the return on the line and the general index, and reflect the systematic contribution of the particular activity to the portfolio risk. These parameters play, therefore, a major role in the determination of the portfolio composition. An insurer who may wish, for example, to generate a "defensive" product mix should operate mainly in lines with negative $B_i$. Thereby he would enjoy profits when other insurers suffer losses, but may lose when other insurers enjoy profits. However, this may affect also the expected return, through the specific values of $A_i$ in the chosen lines.

It is interesting to note that for certain activities a negative $B_i$ with a highly positive $A_i$ were found (for example Health and Credit insurance). This means that the insurer may reduce the risk and increase his expected return by specializing in such lines. Other lines (e.g., Homeowners and Commercial Multi peril lines) have highly positive $B_i$ and negative $A_i$, meaning that a priori they will tend to be excluded from the optimal portfolio. Such relationships may mean that the insurance markets are not in competitive equilibrium, or alternatively this may reflect the use of ex-post rather than expectational data.

Most of the $B_i$ coefficients are statistically significant (14 out of 20). At this stage it is impossible to offer an explanation for the specific values and signs of $B_i$, because there is no theory on this subject. Part of the observed relationships may result from the data limitations, and especially from the possibility that the distribution of the rates of profit in certain lines is nonstationary over time. Such a problem can be avoided when preparing data for an individual company as in such a case the raw data can be "massaged" to correct for changes which occurred over the period (e.g., changes in insurance rates). The problem does not exist when ex-ante parameters are used.

The estimated unexplained variance, $Q_i$, represents the random movements of the rate of profit in activity $i$. Thus, $Q_i$ generates the "unsystematic" portfolio risk, which can be minimized in a well-diversified portfolio. Some lines (including the index itself) have extremely low variability, where other activities have extremely large values of $Q_i$. Those lines, with the high unsystematic risk, may affect the total portfolio risk in a significant manner unless they are included in small proportions, and in a well diversified portfolio.

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10 It should be noted again that the parameters of (2) are estimated from historical data. It is assumed that the relationship found is stable and can be used for prediction purposes. Some partial tests on subperiods did not confirm this assumption of stability over time. Additional work is required to explain the behaviour of the parameters over time.
The model presented above requires a few statistical assumptions about the relationships between the random elements of the returns on all activities: (3) and (4) require that these random elements \( C_i \) be uncorrelated. Despite this requirement some of the correlation coefficients are substantially different from zero. However, a statistical test of the entire matrix could not reject the hypothesis that all the error terms \( C_i \) are uncorrelated. This means that the single-index model will give a reasonably good approximation to the results of the Markowitz solution.

The use of the above data in calculating the efficient product mix of the insurance industry is illustrated in the following section. Before turning to the results it should be noted that the ex-post data were used only for demonstration purposes. The same model is appropriate when ex-ante parameters, reflecting management's forecasts, are used.

4. The Efficient Product-Mix

The above parameters are used in the model to demonstrate the derivation of the efficiency frontier—i.e., the composition of insurance and investment activities which minimize the risk for any level of expected return on equity. However, using the model as presented in §2 may suggest that insurers should specialize in only the few most attractive lines. Such a solution may be obtained even though the said lines may represent very small fractions of the current insurance business.\(^{11}\)

In order to avoid such an unrealistic solution, some marketing constraints which have been ignored up to this point are introduced. A basic assumption common to the earlier models and the model presented in §2 is that there is no market saturation—i.e., that insurers can sell any amount of insurance and get the same expected return and standard deviation. However, this is an unrealistic assumption, because in reality an increase in underwriting is accomplished by approaching the marginal policyholders which means a lower profitability (and possibly even a higher risk).\(^{12}\)

The market saturation effect can be accounted for in a model in which the basic parameters set \( (A_i, B_j, Q_i) \) is allowed to be different for each vector of the unknowns \( q_i \). Unfortunately, such a mathematical technique is unavailable so that the problem has to be dealt with in another way. For this purpose let us assume that each activity is actually composed of sub-activities, each with its own parameters (i.e., the demand for the activity is approximated by linear segments). A certain activity can be described, for example, by an ordered set of \( k \) sub-activities (i.e., those activities with \( i = l, l + 1, \ldots, l + k \)). For mathematical convenience it is assumed that the sub-activities differ only in their expected return \( (A_i) \), but have the same relationships with the index and the same risk characteristics. It is assumed that the first policies will be most profitable and that the expected return from additional policies is diminishing. These assumptions can be summarized as:

\[
A_i > A_{i+1} > \cdots > A_{i+k},
\]

\[
B_{i+j} = B_i \quad (j = 1, \ldots, k),
\]

\[
Q_{i+j} = Q_i \quad (j = 1, \ldots, k).
\]  \(^{(14)}\)

\(^{11}\) This has been verified by comparing the efficiency frontier obtained with the above technique to the one derived when Markowitz approach is used. The expected returns and standard deviation of the return on equity were quite similar. However, the portfolio compositions were sometimes slightly different.

\(^{12}\) On the other hand, increasing the number of policies sold may decrease the risk in the said line, due to the law of large numbers. This secondary effect is ignored at this point, since it is assumed that the size of the insurance portfolio is already large enough and that reinsurance is used, so that the effect is practically neutralized.
Additional constraints have to be added to the model in order to preserve the sequence of the sub-activities in the solution. For example, activity \( i + 2 \) should not be taken before activity \( i + 1 \) reaches its upper limit. Therefore, the following constraints should be added in addition to the nonnegativity constraints on the weights \( a_j \).

\[
a_{i+1} \leq u_{i+j} \quad (j = 1, \ldots, k),
\]

where \( u_{i+j} \) is the upper limit of the weight that sub-activity \( j \) can get. The program will choose the sub-activities with the highest coefficient \( A \) first, and when it reaches the upper limit the sub-activity with the second highest \( A \) will be considered for inclusion in the portfolio.

The main advantage of the above technique is that it recognizes the risk reduction effect obtained by increasing the number of policies written in each individual line. There are still some technical problems involved in making this method practical, and the major one is in the measurement and estimation of the step function which describes \( A_j \).

An alternative approach to the market saturation is to assume that the parameters \( A, B \) and \( Q \) are valid only when the optimal product mix is constrained to be quite similar to the one which existed when the data has been collected. Under this method, which is used below, the efficient product-mix is calculated by replacing the nonnegativity constraints (12) by a set of market saturation constraints. These constraints bound the proportion of each insurance activity in the optimal mix between an upper and a lower limit \( u_i \) and \( 1_i \), respectively, i.e., the nonnegativity constraint (12) is replaced by

\[
1_i \leq a_i \leq u_i \quad (i = 1, \ldots, m).
\]

For demonstration purposes, it has been assumed that the weight of each insurance line \( i \) in the desired product mix has to be within the arbitrarily chosen range of 0.8 and 1.2 of its actual proportion in 1973 premiums. Additional marketing constraints may be added to the model, to reflect some structural relationships between insurance lines. For example, the possibility that a certain insurance policy \( i \) is considered to be a prerequisite for another one \( j \) may be presented by a constraint \( a_j \geq p_{ij}a_i \). Other constraints may also be used to reflect the regulation of both the insurance and investment portfolios. Such constraints are ignored in the following demonstration.

The use of the model for marketing and business policy decisions is demonstrated in Table 2. This table presents the composition of the portfolios obtained by using the above data in the model. Only three points on the efficient frontier are presented. The one with the lowest expected rate of return on equity (composition (A)), the highest return on equity (composition (C)) and one of the possible intermediate solutions (composition (B)). The composition of both the investment and insurance portfolios changes when a different risk-return combination is chosen by the insurer.

For demonstration purposes, Table 2 is based on the assumption that a premium-dollar in each insurance line generates exactly one dollar of insurance reserves. This assumption may clearly cause a bias against some lines: the lines which generate less reserves tend, for example, to be less profitable than what could be seen from the

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13 Since \( a_i \) and \( a_j \) are defined in terms of premium dollars, a constant \( p_{ij} \), reflecting the ratio between the insurance rates in these lines is introduced, so that the constraint relates to the face amount of insurance.

14 Table 2 is calculated under the assumption that the total volume of insurance business remains constant. The specific assumption made in Table 2 is that each dollar of equity can be used to operate only one dollar of insurance business (premiums) (i.e., the model is solved under an additional constraint that \( \sum c_i = 1 \)). The same calculation can be repeated for other insurance/equity ratios, and the envelope of all the individual efficiency frontiers can be derived [7].
model since they generate less investment profits. This unrealistic assumption can be replaced by some other estimates for the parameters g. This, however, involves some arbitrary assumptions which were avoided since the demonstration, rather than the actual numerical results, are considered to be the main goal of this article.

The model, when used with the a priori ex-ante data, can play an important role in determining an insurance company’s policy. For example, by comparing the desired product mix to the existing one, activities which should be extended can be identified, thereby providing guidelines for advertising and marketing decisions. The potential of this method can be observed from Table 2, which summarizes the composition of the insurance portfolio over the entire possible range. Some insurance lines are consistently getting the proportions which are equal to their lower constraint—i.e., these lines tend to be “undesired” (denoted by the letter “u”). Other lines are consistently getting the upper proportion—meaning that they are “desired” (denoted by “d”). The desirability of some other lines (e.g., Fire, Marine, Liability lines) depends on the insurer’s choice of the specific risk-return combination. It should be noted that desirability is determined according to the risk-return combination and the entire set of constraints, i.e., according to the performance of each activity with all the other ones, and not only on the basis of the expected return of this activity in isolation (e.g., Group-Health insurance is considered as “desired” in spite of its negative A coefficient).

5. Summary and Concluding Remarks

The model presented in this paper is potentially a powerful tool for aiding in the determination of an insurance company’s policy. The insurance company is viewed as

\[
\text{TABLE 2}
\]

The Composition of Insurance and Investment Portfolios for Selected Points on the Efficient Frontier

<table>
<thead>
<tr>
<th>Composition of Insurance Portfolio</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected return on equity</td>
<td>3.7</td>
<td>8.2</td>
<td>16.4</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>11.2</td>
<td>12.7</td>
<td>21.9</td>
</tr>
<tr>
<td>Fire and Allied lines</td>
<td>7.9a</td>
<td>11.8</td>
<td>11.8d</td>
</tr>
<tr>
<td>Homeowners</td>
<td>7.8a</td>
<td>7.8</td>
<td>7.8a</td>
</tr>
<tr>
<td>Commercial Multi peril</td>
<td>5.8a</td>
<td>5.8</td>
<td>5.8a</td>
</tr>
<tr>
<td>Marine (Ocean and Inland)</td>
<td>4.1a</td>
<td>5.8</td>
<td>6.2d</td>
</tr>
<tr>
<td>Health (Group and Nongroup)</td>
<td>4.8a</td>
<td>4.8</td>
<td>4.8a</td>
</tr>
<tr>
<td>Workmen’s Compensation</td>
<td>14.2a</td>
<td>14.2a</td>
<td>14.2a</td>
</tr>
<tr>
<td>Liability</td>
<td>9.1d</td>
<td>9.1</td>
<td>8.7a</td>
</tr>
<tr>
<td>Auto Liability</td>
<td>29.0d</td>
<td>19.3</td>
<td>19.3a</td>
</tr>
<tr>
<td>Auto P.D.</td>
<td>13.6*</td>
<td>17.5</td>
<td>17.5d</td>
</tr>
<tr>
<td>Other lines</td>
<td>3.8</td>
<td>3.8</td>
<td>3.8</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Composition of Investment Portfolio</th>
<th>Stock</th>
<th>Bonds</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>19.8</td>
<td>45.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Bonds</td>
<td>80.2</td>
<td>55.0</td>
<td>—</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

- Undesired line—proportion equals lower limit.
- Desired line—proportion equals upper limit.
- Proportion between upper and lower limits.
holding a portfolio of insurance policies and, in addition, an investment portfolio. We
have suggested a method for solving simultaneously for the composition of both these
portfolios, under the assumption that the firm is trying to minimize the variance of the
return on the equity (risk), for any given level of profitability (expected return).

The suggested method requires a reduced data set, which can be generated from
historical records of insurance companies, or else by management ex-ante predictions.
The model incorporates some constraints which are intended to reflect various
marketing relationships, and account for the possibility of market saturation. Additional
linear constraints reflecting legal requirements on both the insurance and
investment composition can be readily introduced.

The workings of the model are illustrated by the use of aggregate data for U.S.
stock insurance companies. The specific numerical results, therefore, have only
limited value for practical purposes. However, they do show the ability of the
approach to differentiate between desired and undesired activities. Such information
can provide valuable guidelines for the allocation of advertising budgets, for directing
other marketing efforts and for setting up an incentive system for agents. The method
may also prove useful for analyzing ratemaking and other financial decisions
confronting the company.13

13 The author would like to thank Professor George Haines and two anonymous referees for helpful
comments on an earlier draft of this paper.

References

Constrained Programming to Portfolio Selection in a Casualty Insurance Firm," Management
Vertical Marketing System: The Case of the Property and Casualty Insurance Industry," Unpub-
lished Ph.D. thesis, Graduate School of Business Administration, University of California, Berkeley 
(1974).
4. FERRARI, J. ROBERT, "A Theoretical Portfolio Selection Approach for Insuring Property and Liability
5. HAUGEN, ROBERT A. AND KRONKE, CHARLES O., "Optimizing the Structure of Capital Claims and
6. JOSKOW, PAUL L., "Cartels, Competition and Regulation and the Cost of Capital in the Insurance
8. KROUSE, CLEMENT G., "Portfolio Balancing Corporate Assets and Liabilities with Special Application 
77–105.
9. LEVI, H. AND SARNAT, M., Investment and Portfolio Analysis, John Wiley & Sons Inc., New York, 
1972.
77–91.
11. QURIN, G. D. ET AL., Competition, Economic Efficiency and Profitability in the Canadian Property and
(January 1963), pp. 277–293.