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THE UNEARNED PREMIUM RESERVE AND THE FINANCIAL POLICY OF INSURANCE COMPANIES UNDER INFLATION

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INTRODUCTION

When financial analysis is carried in real rather than nominal terms, one often discovers that monetary inflation, per se, has little effect on insurers. Most problems alleged to result from inflation are artificial and disappear after real figures are examined. However, inflation may still affect certain financial results expressed in real terms. Therefore, financial analysis should be carried out only after all values are brought to a common monetary base.

The passive side of the insurer's balance sheet consists primarily of liabilities in the form of insurance reserves: the loss reserve reflecting the pending (outstanding) claims, and the unearned premium reserve. These reserves are based on estimates, which may be severely affected by inflationary effects. Since the unearned premium reserve plus the loss reserve often account for 90 percent, or even more, of the liabilities, any estimation error may significantly affect the balance sheet, and thereby the reported profitability. This in turn may bias the analysis of an insurer's financial standing.

This paper focuses on the inflationary effects on the unearned premium reserve. The effects on the loss reserves are discussed elsewhere (see, for example, Lundberg 1977).

Section I explains the meaning and methods of calculation of the unearned premium reserve. Section II discusses the effects of inflationary growth of the written premiums during the year of account on the estimated reserves and examines whether their real value adequately reflects the real unexpired risks. Section III analyzes situations where the common methods of calculating reserves may yield biased results and therefore should be replaced by formulas based on real values. The possibility of a continuous series of

implicit surplus, or losses, in the unearned premium reserve resulting from an inflationary premium growth which extends over a period of a few years, is examined in Secion IV. The last section summarizes the paper and presents some concluding remarks.

I. THE UNEARNED PREMIUM RESERVE AND ITS CALCULATION

Premiums are normally written and collected by insurance companies in advance, except in lines where retroactive premiums are used (certain assessable policies, reporting forms, etc.). Even in these lines, a large portion of the premiums is prepaid. This prepaid element generates a problem since the end of the policy period does not always coincide with the accounting reporting date. As of the reporting date, the insurer has collected premiums which have not yet been "earned," i.e. they will have to cover future unexpired risks, that may become real before the end of the policy period. Accountants distinguish between the premiums "written" throughout the year of account, and the premiums "earned" during that period. At the end of the accounting period, part of the premiums is set aside in the form of an "unearned premium reserve," often referred to as "reserve for unexpired risks."

In order to understand the effects of inflation on the unearned premium reserve, it is necessary to examine first the bases used in its calculation. A common method is based on the calculation of the reserve on each individual policy. For example, an annual policy issued January 1st would have no unearned premium reserve on December 31st, when the firm prepares its annual report. An annual policy issued at the middle of the accounting year (June 30), would still have half of the premium unearned at the end of the year, since it has to cover risks until the middle of the next year. A three-year policy issued and paid at the middle of the accounting year would still have

to cover risks during the next two and a half years beyond the end of the current year. The unearned premium reserve for such a policy would thus be 83.3 percent (2.5/3.0) of the total premium. On very short-term policies, e.g. marine insurance for cargo in transit, the unearned premium reserve should take into consideration only the risks on shipments which were not yet terminated at balance sheet date. The unearned premium reserve in such cases would be only a small percentage of the annual written premiums, since the risks on the shipments made during the first months of the year, had expired already.

Computations for the unearned premium reserve are often made by formula, rather than by computing the reserve on each policy separately. In some states, however, regulators insist on the calculation of reserves on each individual policy, on a pro-rata basis. The use of computers today, facilitates the computation of the unearned premium reserve on a daily pro-rata basis. However, in most countries, formulas are still the most popular basis of computation.

There are several formulas available. The simplest one is based on the assumption that policies are issued at an even rate throughout the calendar year. It can, thus, be assumed that at the valuation date, half of the net premiums written on annual policies are still unearned.

Premiums are not usually written evenly throughout the year. In such cases calculations are based on more detailed statistics of written premiums: semi-annual, quarterly or monthly. The corresponding methods are referred to as the "quarters," "eighths," or the "monthly pro-rata" methods, respectively.

When semi-annual statistics are used, it is assumed that the premiums are evenly distributed throughout each semi-annual period. At the end of the

year, a quarter of the annual premiums written during the first half-year and three quarters of the premiums written during the second half are still unearned. The reserve is, thus, equivalent to the sum of 1/4 and 3/4 of the first and second semi-annual premiums, respectively: hence the name "quarters method." Similarly, if quarterly data are used, the unearned premium reserve at the end of the year would be the sum of 1/8 of the first quarter's written premium, and 3/8, 5/8 and 7/8 of the second, third and fourth quarter's written premium, respectively. When monthly data are used ("monthly pro-rata method" or "semi-monthly method"), the unearned premium reserve will be the sum of 1/24, 3/24, 5/24, . . . 23/24 of the first, second, third, etc., and last month, respectively.

II. INFLATIONARY EFFECTS ON THE UNEARNED PREMIUM RESERVE

Inflation affects the premium series throughout the year and this may drastically distort the calculation of the unearned premium reserve. This may have serious effects on the financial reports and on the stability of the firm.

The distortions will be examined through a simplified example; assume that annual policies are written at a constant rate throughout the year, (say \$1 premium per month). The unearned premium reserve should equal 6 times the monthly premium at either the annual, semi-annual, quarterly or monthly calculation.*

Now assume that the above premium stream represents the <u>real</u> values. The nominal premiums are affected by the inflation assumed to be at a constant monthly rate of i percent. Assume further, for the sake of simplicity, that

^{*}The reserve should be a constant proportion of this figure, when allowance is made for the "loading" element in the gross premium which is intended to cover administrative costs.

prices remain constant throughout the month, and are adjusted only on the last day of the month.

At a hypothetical monthly inflation rate of i percent, the nominal monthly premium series would be \$1, $$1 \times (1 + i)$, $$1 \times (1 + i)^2$, $$1 \times (1 + i)^3$, etc., for the first, second, third and fourth month, respectively.* This series is used for calculating the unearned premium reserve at the end of the year, as presented in Table 1.

The first column of Table 1 presents the monthly written premiums at constant prices as at the beginning of the year. The second column presents the current, nominal, values of the premium stream, as actually written during the year. The unearned premium reserve according to the monthly pro-rata method is obtained by multiplying the monthly written premiums, Pi, by the appropriate weights and summing all products.**

$$U = \sum_{k=1}^{12} P_k (2k - 1) / {24}$$

where P_k represents written net premiums for month k. The real value of the reserve $\mathbf{U_r}$, at beginning of year prices, would be

$$U_r = U/_{(1 + i)}^{12}$$
.

If all monthly premiums are equal in real values, say \$1 each month, the nominal premium written in the kth month would be:

$$P_{K} = \$1 (1 + i)^{k}.$$

The unearned premium reserve would be:

$$U = \sum_{k=1}^{12} (1 + i)^{k-1} (2k - 1)/_{24}.$$

(continued)

^{*}Inflation is a compounded, rather than an additive process.

^{**}For simplicity, we ignore the reserves related to the business written in the previous years, and deal only with the premiums written during the year of account. The unearned premium reserve may be presented mathematically as:

It is often believed that the common methods of calculating the unearned premium reserve correct the inflationary effects since the calculation is based on the nominal, i.e. inflated, premium stream. It is our contention, however, that this is not always a correct assumption, since the end of the year reserve does not preserve its real value; in order to obtain the real value (beginning of year prices), the end of year reserve should be deflated by the full annual price increase:* $(1+i)^{12}$. However, since each periodical premium stream is affected only by the price changes since the beginning of the year until the relevant month, each component of the reserve does not reflect the full annual pure increase. It is thus clear that the real value of the end-of-year reserve is always smaller than 6.

This means that the unearned premium reserves do not preserve their real value. Similar effects are obtained when the calculation is carried on a semi-annual or quarterly basis and also when the reserve is calculated "accurately" (for each individual policy). For example, assume a monthly inflation rate of 2 percent (compounded). The annual written premium is \$12 (\$1 per month) at beginning of year prices. The written premiums at current prices amount to \$13.411 (see Table 2). The calculation of the reserves is demonstrated in the table: the nominal unearned premium at the end of the year will be \$6.975 (per each \$1 of premium written during the first month). This is equivalent to only \$5.50 (i.e. $6.975/(1.02)^{12}$) when deflated to

$$U_r = U/(1+1)^{12} = \sum_{k=1}^{12} (1+1)^{-(11-k)} \cdot (2k-1)/_{24}.$$

Its real value $U_{\mathbf{r}}$ at beginning of year prices, would be:

^{*}On the last day of the year, prices are assumed to make their final "jump"; therefore the real value is calculated by dividing the end-of-year figure by $(1+i)^{12}$, rather than by only $(1+i)^{11}$.

reflect the prices at the beginning of the year. The unearned premium reserve is highly biased downward in real terms. The firm actually holds only \$5.50 in terms of prices at the beginning of the year, instead of \$6 that should have been kept in order to preserve the same real value of the reserve. This means that the reserve is some 8 percent short of the appropriate reserve in real value.

The semi-annual or quarterly calculations which are also presented in Table 2 lead to similar results,* with even a slightly stronger bias: the quarterly calculation leads to a nominal reserve of \$6.954, which at the beginning of the year prices is equivalent to only \$5.48 (6.954/(1.02)¹²). The semi-annual calculation leads to nominal end-of-year reserve of \$6.904, which is equivalent to only \$5.44 in real terms—compared to \$6 in real terms which should have been held.

Similar calculations were carried for monthly inflation rates of 1, 5 and 10 percent and the findings are summarized in Table 3. It can be seen that the bias is significant for low rates of inflation and becomes extremely large in times of rapid inflation. At a 10 percent monthly inflation, for example, the end-of-year unearned premium reserve would be only 67%-57% of its required real value according to the formula in use. The bias is weaker but still noticeable at lower inflation rates.

The percentage bias is roughly equivalent to some 3-4 times the monthly inflation rate: when the inflation rate is 1 percent per month there is an under-reserving of some 4.4 percent (with a monthly calculation) and when the

^{*}The calculation is sensitive to the assumption about the date of price changes. It has been assumed that prices "jump" at the end of each month, but are constant throughout the month. Calculations of the average price level for the month, under the assumption that premiums grow at a continuous rate during the month (rather than a step function) will change the results only slightly.

monthly inflation rate is 2 percent, the bias reaches 8.3 percent of the reserve. Such biases are extremely important since they are of the same order of magnitude of the annual profits of a typical insurance firm. From Table 3 it is evident that the bias is much stronger when the reserve is calculated according to the annual premium method.

The above analysis was based on a uniform premium distribution throughout the year. In practice, premiums are spread unevenly throughout the year. The bias would have been much stronger if the (real) written premiums were concentrated at the beginning of the year rather than at the end. It is well known that seasonality in the premiums can drastically affect the appropriateness of the unearned premium reserve. These effects are magnified by the incidence of inflation since the premiums written in current prices are not uniformly spread throughout the year even when the premiums in real terms are evenly spread.

In the above examples, it has been assumed that nominal premiums increase according to the general inflation rate. This assumption may be incorrect in practice, firstly because insureds are often unaware of increased insurance needs and therefore do not purchase additional insurance; secondly, because the demand for insurance could be affected by possible changes of variables such as the disposable income of individuals, liquidity of firms, inventory holding policies by firms, etc., which do not necessarily react linearily to the inflation rate. Consequently, the nominal unearned premium reserves may be further changed during inflationary periods and this, in turn, would affect the real reserve.

III. SHOULD THE FIRM HOLD THE REAL UNEARNED PREMIUM RESERVE?

The above examples have shown substantial gaps between the nominal unearned premium reserve and the reserve which should be held if the reserve

were to reflect the <u>real</u> value of the unearned premiums. What is the correct size of the unearned premium reserve? Should it reflect the real value? The answer depends on the terms of the insurance contracts.

When the coverages are fully indexed, or inflation-proof in other ways, ("inflation guard endorsement," "waiver of coinsurance penalty," etc.), the liability of the insurer toward risks which have not yet been realized, will grow wih inflation. In such a case the premium is consumed throughout the year at a constant or almost constant rate, in real terms. Hence, the common technique for calculating the end-of-year unearned premium reserve, based on a constant rate of consumption of nominal premiums, will be biased. This bias is of the utmost importance to the insured, insurer and regulator.

If the insurance coverages are not indexed, the liabilities of the insurers retain their nominal values, but are devaluated in real terms due to inflation. For example, a policy with a coinsurance clause provides adequate coverage to the insured only during the first few days, but becomes inadequate as time passes. The coverage is not uniform throughout the year, thus it may be argued that the premium does not expire at a uniform rate.

If coverages are not indexed, risk expires at a non-uniform rate throughout the year and the unearned premium reserve reflects this feature. It will be shown that the conventional technique for the calculation of the unearned premium reserve implicitly reflects the expiration of real insurance protection which occurs at a non-linear rate throughout the year. In other words, the common formulas for the computation of the unearned premium reserve, which assume pro-rata consumption of the nominal premium, could prove, after all, to be correct for non-indexed policies.

Holding the unearned premium reserve which is based on the nominal premium series actually reflects non-linear use of the real premium. The

expiration of premiums in real terms can be calculated in the following way: assume that a \$1200 net premium is Written at the beginning of the policy year to cover the annual risk. Under the monthly po-rata method, it is assumed that the premium expires according to a straight line method; that is, \$100 are consumed at the end of each month throughout the policy year. The unearned premium is obtained by deducting the accumulated monthly premiums which have already expired from the annual written premium. The premium reserve could be transformed into its real-term equivalent (by deflating by the price changes which occurred from the beginning of the year up to the date of the reserve calculation). The difference between two consecutive figures of the reserves in real values would generate the series of premium expiration in real terms. For example, at the date of policy issue, the unearned premium is \$1200 in nominal terms. At the end of the first month, the remaining unearned premium is \$1100. Under 2 percent monthly inflation, the real value of the reserve at the end of the first month is only \$1078.43 (\$1100/(1 + .02)). This means that at this inflation rate \$121.56 of the premiums have actually expired in real terms (\$1200 - \$1078.43), not just \$100.

Table 4 shows that a linear expiration of the nominal premium leads to a non-linear expiration of the real premium (the detailed calculation is tabulated in the Appendix). It can be seen that at very high inflation rates (10 percent per month, for example), most of the premium has actually been considered expired after only a few months. For example, during the first month of the policy year 16.67 percent of the premiums are consumed, while during the last month only 2.92 percent of the real premiums are implicitly consumed.

IV. INFLATION EFFECTS ON IMPLICIT SURPLUSES OR LOSSES IN THE UNEARNED PREMIUM RESERVE

Inflation may also affect the unearned premium reserve by creating possible gaps between the actual and assumed loss ratios. In the calculation of the reserves an assumption is made concerning the loss ratio (or its complement—the expense and profit ratio). The assumption may bias the calculation even under non-inflationary circumstances; however, the effect may be different under inflation. It is therefore, essential to examine, first, the effects under zero inflation, and to analyze the inflationary effects later.

Over-reserving and under-reserving at zero inflation: In the computation of the unearned premium reserve only a certain part of the gross premium is taken into consideration. Part of the gross premium is intended to cover costs which are incurred near the date of policy issue, and no reserve should be retained for these amounts. If 20 percent of the premiums represent expenses which are paid at the date of policy issue, the prepaid element of the premiums should be calculated on the basis of the remaining 80 percent of the gross premium. Thus, if premiums are written at an even rate throughout the year, it is sufficient to hold an unearned premium reserve equivalent to 40 percent of the annual gross premium written (half of the 80 percent of the gross annual premium). Differences between the actual and assumed expense ratios may create situations of over- or under-reserving.

Over-reserving occurs when the actual expense ratio is higher than that assumed in the unearned premium reserve formula. For example, over-reserving occurs when the reserves are based on the full gross premiums. Since the bulk of the operating expenses is typically incurred at the inception of the policy, whereas the nominal premium is assumed earned on a straight-line basis, a reserve based on the full gross premiums is biased upward, even in

non-inflationary circumstances. This creates a hidden profit in the firm's income statement, (The deduction of an excessive end-of-year reserve from the annual income leads to a lower reported profit figure or even a loss.*).

Financial analysts often believe that at zero inflation, the unearned premium reserves would typically be excessive, in comparison with the actual future liability of the insurer. The unearned premium reserve, with its element of excess valuation often creates a considerable implicit equity. It is often believed that there may be an implicit surplus of up to 30-40 percent hidden in this reserve.

Under-reserving occurs when the deduction for assumed immediate expenses exceeds the actual expenses. This leads to reporting of exaggerated profits during the first months, but the loss becomes explicit at a later reporting date. For example, assume that \$1200 in annual premiums are written, and the expected losses are \$1080 (\$90 per month). If the expenses are \$120 (10 percent of premiums), no profit is expected for the year. Assume, however, that the calculation of the reserve is based on 75 percent of written premium (reflecting an assumed expense ratio of 25 percent). The unearned premium reserve at the end of the first month would be \$825 (i.e., 75% x \$1200 x 23/24). The income statement will show:

Written premium		\$1200
Deduct: increase of unearned premium reserve		825
Earned premium	s 120	\$ 375
Actual expenses (immediate) losses	90	210
Underwriting profit		\$ 165

^{*}Example: Assume a \$1200 premium is written during the last month of the year. This creates an \$1150 unearned premium reserve (1200 x 23/24) and the unearned premium is \$50. However, expenses and commissions might be \$200. This would generate a reported loss of \$150 at the end-of-the-year report, since the actual expenses (\$200) were higher than assumed (\$0). The profit is shifted to the following year.

Under-reserving means that the current underwriting profits are exaggerated; i.e., the unearned premium reserve hides losses.*

Over- and Under-Reserving in Times of Inflation: A continuous inflationary period may lead to a technical growth of the nominal premium volume and may, therefore, affect the size of hidden profits or losses in the firm's reporting. In cases of over-reserving, the lower reported current profits are normally offset by the higher profits in the next reporting period, when the unearned premium reserves have turned into earned premium. However, if a rapid and accelerating expansion of premium volume occurs during consecutive periods (due to inflation), there would be a continuous series of reported underwriting losses, since the increase in the unearned premium reserve for the newly written premium overcomes the decline in the reserves due to the old maturing policies. That is, if the reserving formula generates over-reserving, profits will be continuously understated (or even take the form of a continuous series of technical losses) when the unearned premiums are increased due to inflation. Although these losses "hide" some profits, they may not be revealed for quite some time.

The continuous technical losses may give the impression that the capital is eroded, and that the firm approaches technical insolvency. Such a trend in itself creates capacity problems, since the insurers' capital requirements are related to the volume of premiums. This triggers the demand for increased capacity which can be achieved by raising additional capital or by slowing down the expansion of the firm. In other words, an artificial technical accounting problem may create real effects.

^{*}Such a situation could be quite common in Israel, for example. The insureds pay premiums plus additional "fees." These fees often cover all the immediate expenses. The unearned premium reserve, however, is still based on only a certain percentage of the premium, excluding fees. In other words, the actual remaining costs are often lower than those accounted for in the calculation of reserves.

Similarly, if the reserving formula creates under-reserving, a continuous increase of the nominal premium, due to inflation, creates a series of exaggerated reported profits, while hiding an ever accumulated loss. This may lead to superfluous taxation, erosion of the firm's capital (see Kahane 1981a) and finally even to the failure of the firm.*

Slow-down of the inflation rate could also have disastrous effects on the insurers, since the real under-reserving would be discovered in the reports of the following periods: the unearned premium reserve held at the beginning of the year will turn into earned premiums, but would be only partially replaced by the new unearned premium reserve generated by the slowly growing, newly written premiums.

V. CONCLUDING REMARKS

Much thought should be given to inflation accounting in the insurance industry. This paper discussed only some of the problems and did not analyze additional important effects, such as inflationary taxation. However, even the partial analysis demonstrates the possible bias of the entire reporting system, and the possibly crucial effects on the financial policy of insurers.

Inflation may drastically affect the calculation of the unearned premium reserve, and thus may distort the financial reports of insurance companies. The main bias in the reserve arises from the well-known effects of over-reserving or under-reserving resulting from the use of incorrect assumptions concerning the expense ratio. This could lead to either over-reserving or under-reserving, and the effects could be maintained over a long period due to the growing premium series. Inflation may magnify the effects of over-reserving or under-reserving as a result of use of inaccurate assumptions made

^{*}The interesting simulation by Johnson (1977) which bears the title, "How Insolvent Are We?" discusses these effects.

in the calculation concerning the loss and expense rates. Such effects may become quite dramatic when inflation accelerates or decelerates.

Another bias of the unearned premium reserve is related to the implicit assumption concerning the expiration of the risk (and premium) throughout the year. The common methods of calculating the unearned premium reserve assume expiration of the nominal premium at a constant rate. In times of inflation this means a decreasing real expiration rate. Such assumption could be considered sound only when the insurance protection deteriorates with inflation. This is often the case in practice, since insurance contracts often shift the entire inflationary burden on the shoulders of the insured due to co-insurance (under-insurance) arrangements and similar clauses. However, when the insurance policies preserve, partially or fully, the real value of protection ("inflation guard" policies, indexed policies, etc.), the real premium expires at a constant rate, and therefore the assumption made in the calculation of unearned premium reserve leads to under-reserving. The growth of nominal premiums throughout the year does not fully compensate for this under-reserving.

Such bias could have either a supporting effect or a contradictory effect to the bias resulting from the assumption concerning the cost element. If the assumptions concerning the cost element generate under-reserving as well, and this effect is augmented due to the artificial inflationary growth in premium volume, the two effects bias the reserves in the same direction and lead to drastic under-reserving. If the allowance for administrative expenses creates over-reserving, (resulting in a continuous series of reported losses), the two effects are contradictory and the end-result depends on the relative intensity of the two effects.

The possible bias of the unearned premium reserve should be of much concern to managements and insurance regulators. This is especially true

since the possibility of under-reserving stands in contradiction to the common belief that there is a sizeable "hidden equity" in this reserve. Some of the major implications of under-reserving are: first, the stability of the firm is threatened because of under-reserving in real terms, the effect remaining unobserved for a long period due to the increased premium flows resulting from inflation. Second, due to under-reserving the current profit is biased upward and this may bring about extra inflationary taxation, which will contribute to the erosion of the firm's capital. Third, under-reserving may have a bearing on the liquidity of the firm and its cash flow problems.

Financial analysts and insurance regulators should be very careful when analyzing the reports of non-life insurance companies in times of inflation. An understanding of the actuarial assumptions and their meaning becomes essential, and no rules of thumb should be applied without a re-examination. The biases discussed in this paper could easily amount to a few percents of the reserves (Table 3, for example, shows biases between 4-44 percent, depending on the inflation rate and the specific actuarial formula in use). Such biases could drastically distort the reported profits since the profit of non-life insurers seldom exceeds some 10 percent of the unearned premium reserve. Even a possible bias of only 4 percent, which is likely at relatively low inflation rates, may bias the reported profit by 40 percent!

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TABLE 1:

THE UNEARNED PREMIUM RESERVE AT MONTHLY INFLATION OF I PERCENT

Month	Monthly Written Premium (in begin- ning of year prices)	Monthly Written Premium (at current prices)	Weight Assigned for Determining Reserve (Monthly Calculation)
1	» \$ 1	$1 \times (1 + i)^{0}$	1/24
2	\$ 1	$1 \times (1 + i)^{1}$	3/24
3	\$ 1	$1 \times (1 + i)^2$	5/24
4	\$ 1	$1 \times (1 + i)^3$	7/24
•	•	•	•
•	•	•	
•	•	•	• •
k	\$ 1	$1 \times (1 + i)^{k-1}$	(2k - 1)/24
•	•	•	•
•	•	. •	•
•,	• *,	•	•
11	\$ 1	$1 \times (1 + i)^{10}$	21/24
12	\$ 1	$1 \times (1 + i)^{11}$	23/24

TABLE 2:
UNEARNED PREMIUM RESERVE CALCULATED AT 2 PERCENT MONTHLY INFLATION RATE

Period	Real Written Premiums (Real terms) (\$)	Nominal Written Premium (current prices) (\$)	Nominal Unearned Premium Reserve (\$)	
(1)	(2)	(3)	(4)	$(5) = (3) \times (4)$
		Monthly Calculati		
1st Month 2nd " 3rd " 4th " 5th " 6th " 7th " 8th " 9th " 10th " 11th " 12th "	1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00	1.000 1.020 1.040 1.061 1.082 1.104 1.126 1.149 1.172 1.172 1.195 1.219 1.243	0.048 0.125 0.208 0.292 0.375 0.458 0.542 0.625 0.708 0.792 0.875 0.958	0.048 0.128 0.216 0.310 0.406 0.506 0.610 0.718 0.830 0.946 1.067 1.190 6.975 => \$5.50 at
Sum	12.00			8.0.Y. Prices*
		Quarterly Calculat	ion	
1st Quarter 2nd " 3rd " 4th " Sum	3.00 3.00 3.00 3.00	3.060 3.247 3.447 3.657	0.125 0.375 0.625 0.875	0.383 1.218 2.154 3.200 6.954 => \$5.48 at B.O.Y.
		Comi Dunual Coloula		Prices*
		Semi-Annual Calcula	Andrew Control of Cont	
lst Half	6.00	6.307	0.25 0.75	1.577 5.328
Sum	12.00	13.411		6.904 => \$5.44 at B.O.Y. Prices*

^{*\$1} at the beginning of the year is equivalent to \$1.268 at the last day of the year.

TABLE 3:

THE END OF YEAR UNEARNED PREMIUM RESERVE AT BEGINNING OF YEAR PRICES,

AT VARIOUS INFLATION RATES (REAL WRITTEN PREMIUMS OF \$1 PER MONTH)

Monthly Inflation	Real Reserve	Real Value of Reserve Calculated					
Rate	should be (\$)	Monthly (\$)	Quarterly (\$)	Semi-Annually (\$)	Annually (\$)		
1%	6.00	5.74	5.73	5.71	5.63		
2%	6.00	5.50	5.48	5.44	5.28		
5%	6.00	4.85	4.83	4.75	4.43		
10%	6.00	4.04	4.00	3.88	3.40		

TABLE 4:

THE EXPIRATION OF THE PREMIUM THROUGHOUT THE YEAR AT

VARIOUS INFLATION RATES*

End of	Expiration of Nominal	Implicit Expiration of Real Premium When Monthly Inflation Rate:						
Month	Premiums	l percent	2 percent	5 percent	10 percent			
1	8.33	9.24	10.13	12.70	16.67			
2	8.33	9.06	9.77	11.72	14.46			
3	8.33	8.88	9.42	10.78	12.52			
4	8.33	8.77	9.09	9.94	10.81			
5	8.33	8.53	8.76	9.14	. 9.31			
6	8.33	8.42	8.43	8.40	7.99			
7	8.33	8.21	8.13	7.70	6.84			
8	8.33	8.09	7.82	7.05	5.83			
9	8.33	7.90	7.53	6.44	7.94			
10	8.33	7.74	7.20	5.88	4.17			
11	8.33	7.61	6.97	5.36	3.15			
12	8.33	7.47	6.70	4.87	2.92			
Total Annual Real					100 000			
Premium	100.00%	100.00%	100.00%	100.00%	100.00%			

^{*}Nominal premium expires at linear rate throughout the year. The unearned nominal premium reserve is translated into real terms (beginning of year prices), and its expiration is calculated by chained differences of consecutive values. See Appendix.

The expiration of \$1200 premium net throughout the year at various inflation rates.

Time	Expiration of		2% monthly inflation Real Value Expiration Percent of		5% monthly inflation						
month) amount	the nominal amount during the month	Premium reserve at the end of month	Real Value of unearned premium reserve*	expiration of real premiums	Percent of premium	Real value of reserve		of amount premium	Real value of reserve \$		of annual
0	0	1200	1200.00	0	0	1200.00	0	0	1200.00	0	0
1	100	1100	1078.43	121.56	10.13	1047.62	152.38	12.70	1000.00	200.00	16.67
2	100	1000	961.17	117,26	9.77	907.03	140.59	11.72	826.44	173.56	14.46
3	100	900	848.09	113.08	9.42	777.45	129.58	10.80	676.18	150.26	12.52
4	100	800	739.08	109.01	9.09	658.16	119.29	9.94	546.41	129.77	10,81
5	100	700	634.00	105.08	8.76	548.47	109.69	9.14	434.64	111.77	9.31
6	100	600	532.78	101.22	8.43	447.73	100.74	8.40	338.68	95.96	7.99
7	100	500	435.28	97.50	8.13	355.34	92.39	7.70	256.58	82.10	6.84
8	100	400	341.40	93.88	7.82	270.73	84.61	7.05	186.60	69.98	5.83
9	100	300	251.03	90.37	7.53	193.38	77.35	6.44	127.22	59.38	4.94
10	100	200	164.06	86.97	7.20	122.78	70.60	5.88	77.11	50.11	4.17
11	100	100	80.43	83.63	6.97	58.46	64.32	5.36	35.04	42.07	3.51
12	100	0	0	80.43	6.70	0	58.46	4.87	0	35.04	2.92
	1200			1200.00	100.00		1200.00	100.00		1200.00	100.00

^{*} The real value of the reserve in terms of the beginning of year prices,
equals nominal unearned premium reserve at the end of month divided by

(l+i)^t where i denotes the inflation rate and t denotes the time (months) since the inception of the policy.