

Sponsored by "THE GENEVA ASSOCIATION"

RISK, INFORMATION AND INSURANCE

Essays in the Memory of Karl H. Borch
Edited by Henri LOUBERGÉ, University of Geneva

CONTENTS

Foreword, by Orio Giarini

Editorial introduction
Henri Loubergé

Borch's Theorem: A historical survey of applications
Jean Lemaire

Part I: Economics of Uncertainty

The role of securities and labour contracts in the optimal allocation of risk-bearing
Jacques H. Drèze

Proportional risk aversion and saving decisions under uncertainty
Georges Dionne, Louis Eeckhoudt and Eric Briys

Part II: Economics of Information

The welfare economics of moral hazard
Richard Arnott and Joseph Stiglitz

Problems of equilibria in insurance markets with asymmetric information
Roland Eisen

Moral hazard and insurance market structure
Yoram Eden and Yehuda Kahane

The effects of organizational form on capital structure: The case of stock and mutual property-liability insurance firms
Prabir Datta and Neil A. Doherty

Part III: Economics of Insurance

Strategic growth and profit policy of insurance companies
Dieter Farny

(De)regulation of insurance markets
Lutgart van den Berghe

Price regulation and segmented insurance markets
J. François Outreville

The microstructure of the London insurance market
Robert L. Carter and Stephen R. Diacon

Do shares in other insurance companies reduce the solvency margin of an insurer?
Walter Karten

KLUWER ACADEMIC PUBLISHERS
MARTINUS NIJHOFF PUBLISHING
KLUWER-NIJHOFF PUBLISHING
Norwell, Massachusetts, USA, 1989

D-9

7 MORAL HAZARD AND INSURANCE MARKET STRUCTURE

Yoram Eden
Yehuda Kahane*

1 Introduction

The objective of this article is to suggest an explanation for the observed market structure of the insurance industry, which is based on three parties—insureds, insurers, and reinsurers. This structure should not be viewed simply as a special version of the common marketing system in which the reinsurer is analogous to a wholesaler and the insurers are analogous to the retailers. Careful examination reveals a much more complicated framework involving agency and information problems as well as trading of financial capacity. Despite its practical importance, little has been written about this issue and most of the articles that do appear in the literature are not really in tune with reality, since the authors restrict themselves to dealing with

* We would like to thank the participants of the ASTIN Colloquium in Tel Aviv and the members of the Risk Theory Seminar in Austin, Texas, for very helpful comments and suggestions. All common caveats apply.

only two parties at a time (insured and insurer or insurer and reinsurer). Moreover, they deal with market structure and retention problems as two distinct issues.

The market structure of the insurance industry has been analyzed from various points of view: Leland and Pyle [1977], Campbell and Kracaw [1980], and others view *information-production* as the main motive for financial intermediation. Others explain the risk exchange as resulting from different risk-aversion parameters among participants (see Borch [1962], Buhlmann [1980], Gerber [1984a]) or from a special attitude toward insolvency (Crouhy and Galai [1986]).¹ The use of risk aversion in the explanation of the insurer-reinsurer interface is, however, not satisfactory, since the insurer, unlike the insured, is not endowed with the risk. If the insurer cannot cope with it, or if the price is inappropriate it does not have to underwrite the risk (see Doherty and Tinic [1981], Blazenko [1986]). Moreover, when the risk is correctly priced in the capital market (Biger and Kahane [1978]), reinsurance is simply redundant. Therefore, the common use of reinsurance in practice should be explained in other ways: Borch [1985] explains the willingness of (risk neutral) insurers to reinsure by the effects of limited liability and capacity problems. Blazenko [1986] applied micro-economic analysis of monopolistic and monopsonistic situations to explain the reinsurance mechanism. The present article suggests another approach, based on differences in the effectiveness of monitoring moral hazards, to explain the insurance market structure, and especially the coexistence of the reinsurer alongside the insured and insurer.

The optimal insurance and retention problem has been discussed by quite a number of papers (to mention just a few: Arrow [1963]; Borch [1962]; Doherty and Schlesinger [1983]; Gerber [1984a, b]; Kahane and Kroll [1985]; Mayers and Smith [1981]; Raviv [1979]; Rothschild and Stiglitz [1976]). These studies are confined to two-party situations. A by-product of the analysis in this paper is a simultaneous solution leading toward optimal allocation of a given risk among all parties. It will be shown that at the optimum both the insurer and reinsurer may retain a nonzero portion of the risk. The solution, therefore, is instrumental in explaining both the insurance market structure and the optimal retention problem.

The model presented has at most three types of players, which are predestined as insureds, (local) insurance companies, and (international) reinsurers. These players differ with regard to the information they hold, their capital, and their portfolios. The insured faces a single nondiversifiable risk and has full information concerning its distribution. Being a risk-avertter the insured is interested, at the right price, in buying insurance (expected utility maximization approach).² The insurer lacks information on the in-

sured's risks and therefore employs monitoring in order to control the problems of moral hazard. The premium the insurer charges covers the expected loss, the cost of monitoring, and a loading covering its residual risk (due to its inability to fully diversify the risks). The reinsurer operates in another (international) arena, and therefore is less efficient in monitoring information about local risks. The reinsurer, therefore, may wish to rely on the local insurer's monitoring and on signaling from the local insurer concerning its monitoring policy. The key signals are the insurer's capital and retention level. Both the insurer and reinsurer are subject to the market's risk-return trade-offs, and to ruin constraints. But the reinsurer because it holds a large and well-diversified portfolio, can use the risk-pooling effect to completely avoid the nonsystematic risk. The difference between the ability of the insurer and reinsurer to diversify the risk away may stem from the incompleteness of the local market (Doherty and Schlesinger [1983]).

This simplistic, and somewhat stylized, background "story" generates a solution with a trade-off between moral hazard, monitoring costs, self-retention, capital, and risk loading at each of two interfaces: insured-insurer and insurer-reinsurer. A too-low premium prompts the insured to buy full coverage, which the insurer and reinsurer are unwilling and unable to sell. For a higher premium, the insurer's interest in selling full coverage is stronger; but the insured's willingness to buy diminishes, and may even disappear. Interesting trade-offs exist at the insurer-reinsurer interface as well (moral hazard vs. monitoring costs).

The analysis of such a framework leads to specific results concerning the overall risk-sharing among the three parties. Important properties of the solution are:

- a. A direct transaction between the insured and the reinsurer may be less efficient than risk allocation among all three players—insured, insurer, and reinsurer.
- b. At each level of retention, the insurer reaches the optimal solution by changing both its loading policy and the monitoring activities.
- c. There is an optimal sharing of the risks between the insurer and reinsurer. There is, however, room for negotiating the exact sharing of the premium between them.

The present analysis enables us to show that a three-party solution may often be preferable to partial solutions, and to offer some insight into the coexistence of small local insurers. Our model, however, makes no pretensions to providing a full representation of the detailed bargaining mechanisms that take place in reality, and determine the equilibrium.

2 The Participants in a Three-Player Model

Assume an economy with three types of participants (insureds, insurer, and reinsurer) operating within a single-period framework in which the premiums are collected at the beginning of the period, and claims paid at its end.³

The Insureds

There are N identical individual insureds, each having an initial wealth, W , and facing a pure risk x . \tilde{x} is a random variable, with a distribution $F(x)$. X is the cumulative function of the N risks where $F(X) = F(\Sigma \tilde{x})$. Each insured has a convex utility function, $u(\tilde{W}_1)$, where \tilde{W}_1 is his terminal wealth. The insured is an expected utility maximizer.

An Insurer

The insurer (denoted, herein, by subscript a) operates in a capital market, obeying a general CAPM framework. Therefore, its goal is to try to utilize various market imperfections in order to optimize the value of the firm, i.e., its market performance. Specifically we shall assume that the insurer is an expected profit maximizer subject to a regulatory ruin constraint, and to the market risk-return trade-off (expressed as a second constraint).⁴ Each insurer maintains a ruin constraint, which may be imposed by a regulator or may be adopted voluntarily:

$$\text{Prob}\{[(K_0 + P_a)(1 + R) - X] < 0\} < \alpha$$

where

K_0 is the amount of equity at the beginning of the period.

P_a is the net premiums retained by the local insurer, i.e., premiums collected from the insureds minus the amounts ceded to reinsurers.

R is the riskless rate of interest.

α is the permissible probability of ruin.⁵

In order to maintain the ruin constraint, the insurer has to raise capital in a competitive market, which, in turn, determines the "normal return" the insurer must earn.⁶ Hence the premiums charged by the insurer must cover at least the expected loss plus a loading factor to cover its monitoring costs, C_a , and another loading factor $L_a(\alpha, \sigma, s)$ to cover the risk in its

portfolio (where σ is the standard deviation of the rate of return on the equity capital and s is the price of risk in the capital market).⁷

A Reinsurer

The reinsurer denoted, herein, by subscript b maintains a large and well-diversified portfolio, and is subject to a similar ruin constraint. However, due to the well-diversified portfolio the reinsurer may sell reinsurance at a fair actuarial rate without any loading factor to cover the risk.⁸ Therefore, the reinsurance premiums cover the expected loss plus only one loading factor. This loading has to cover the monitoring costs of the original risk (in case of a direct insured-reinsurer transaction), or the reinsurance commissions paid to the local insurer, which serves as the monitoring agent for the reinsurer (in an insurer-reinsurer transaction).

Additional constraints are needed in order to justify the independent existence of the local insurer, and especially its need for equity capital. Otherwise, what is to prevent the reinsurer from purchasing the insurer's shares—thus exercising its superiority in the capital market, while benefiting from the preferred monitoring instruments of the local insurer. Such direct involvement of the reinsurer in the local market is typically discouraged by regulators in practice: (a) through the common requirement that local insurers must be owned by citizens of the country in question, and (b) through the constraint preventing a local insurer owned by another (re)insurer from relying on the equity of its parent company. Such companies must retain certain local funds, intended to limit the possibility of abusing the limited liability of corporations.

3 The Rules of the Model

3.1 The Insured-Insurer Interface (Moral Hazard and Monitoring)

There is a maximum premium that the insured is willing to pay. Pratt [1964] has shown that the maximum risk premium, Π , that a (risk averter) insured with a utility function $u(W)$ is willing to pay for a small risk x is approximately

$$\Pi = -\frac{1}{2} \frac{u''(W)}{u'(W)} \sigma^2(x) \quad \Pi > 0 \quad \text{since } u''(\cdot) < 0 \quad \text{and} \quad u'(\cdot) > 0 \quad (7.1)$$

This approximation may be used for defining a demand constraint: the present value of the premium, p , paid by the insured ($N_p = P$ being the total premium income) should be lower than the present value of the expected loss plus the risk premium.

$$p \leq \left(\frac{1}{1+R} \right) [E(x) + \Pi] \equiv \left(\frac{1}{1+R} \right) \left[E(x) - \frac{1}{2} \frac{u''(W)}{u'(W)} \sigma^2(x) \right] \quad (7.1a)$$

After acquiring insurance, the insured will change his behavior and become more careless, and the new risk will become X_1 . A coefficient, λ , ($\lambda \geq 1$) describes this moral-hazard effect. We assume a multiplicative effect: $\tilde{X}_1 = \lambda \tilde{X}$, where λ could be constant—following Dionne [1981]—or a function of the size of the loss.

The insurer is aware of the moral-hazard problem and tries to control it by monitoring. By spending C , at the beginning of the period, the insurer is able to prevent some of the moral hazard. The effectiveness of monitoring is described by a *monitoring function* $\delta = V(C)$, ($0 \leq \delta \leq 1$) (and assume $V(0) = 0$, $V'(C) > 0$, $V''(C) < 0$).⁹ The expected value of the cumulative damage function, after monitoring, is $E[\tilde{X}|\delta]$ where

$$E[\tilde{X}|\delta] = E(X_1) - \delta[E(\tilde{X}_1) - E(\tilde{X})]$$

The insurer's behavior is affected by the reinsurance coverage. Therefore, it is essential to examine the insurer-reinsurer interface before analyzing the insured-insurer relationship within a three-player framework.

3.2 The Insurer-Reinsurer Relationship

The insurer buys a quota share (coinsurance) policy, retaining a proportion, q , of the risk, and ceding $1 - q$ to the reinsurer. In return for transferring premiums it accepts a reinsurance commission, at a rate Z , which is a function of ceded premiums, $Z = Z(P, q)$, $0 < Z < 1$.

Reinsurance may be desirable due to the following trade-off: on the one hand, the reinsurer's portfolio is more diversified than that of the local insurer. On the other hand, due to the distance from the local market, the reinsurer's ability to monitor and react to the behavior of the insureds is limited, and more expensive. ($V_a(C)$ denotes the monitoring function of the local insurer. $V_b(C)$ is the monitoring function of the reinsurer. By assumption, $V_a > V_b$ for all values of C and δ).

3.3 The Reinsurer's Constraint

Let us examine first the possibility of a direct transaction between the insured and the reinsurer. It would be worthwhile for the reinsurer to spend C_b on monitoring only if

$$V'_b(C_b)[E(X_1) - E(X)] \geq (1 + R) \quad (7.2)$$

From inequality (7.2), we can find the optimal monitoring cost, C_b^* . In the event of a direct transaction, the reinsurer receives the full original premium. The total premium income should cover the expected losses plus the monitoring costs, C_b .

$$N_p \geq \frac{1}{1+R} E[X|V_b(C_b)] + C_b \quad (7.3)$$

A necessary but not sufficient condition for a direct insured-reinsurer transaction (excluding the local insurer) is that p fulfill both (7.1a) and (7.3).

A combined three-party transaction may be beneficial due to the trade-off between the reinsurer's ability to reach better diversification of its portfolio and the local insurer's superior monitoring mechanism. In other words, the reinsurer replaces its inefficient monitoring mechanism with the more efficient mechanism of the local insurer, and in return pays reinsurance commissions, at a rate Z .

The reinsurer must make a profit (the reinsurance premium income must exceed the reinsurer's share in the losses), that is,

$$(1 - q)\{P(1 + R)(1 - Z) - E[X|V_a(C_a)]\} \geq 0 \quad (7.4)$$

The underwriting profits of the reinsurer depend on C_a —the monitoring costs of the local insurer.

The coinsurance rate, q , may serve as a signal for the quality of the monitoring done by the local insurer; if the reinsurer knows the monitoring function of the insurer, $V_a(C_a)$, it can correctly identify C_a (the monitoring costs that were actually expended by the ceder) by examining q , the rate of the insurer's retention (this is proved later).

3.4 The Insurer's Constraint

The insurer's expected profits per policy, A , equal its share in the premiums, minus its share in the losses, minus its monitoring costs, plus the reinsurance commission:

$$A = q\{(1 + R)P - E[X|V_a(C_a^*)]\} - C_a^*(1 + R) + (1 - q)P(1 + R)Z \quad (7.5)$$

Note that the reinsurance commissions, Z , are temporarily treated as constant (this assumption is relaxed later when Z is allowed to depend on q ($dz/dq \geq 0$)). In other words, the reinsurer is assumed to be a price fixer, and once it has declared the commission rate, it has to accept any amount of business ceded by the local insurer.

The local insurer must meet the capital market risk-return relationships, in order to be able to raise the required capital. Therefore,¹⁰

$$\frac{K(\alpha)R + A}{K(\alpha)} - \phi(s) \geq R \quad \text{or} \quad \frac{A}{K(\alpha)} \geq \phi(s) \quad (7.6)$$

where

$K(\alpha)$ is the equity required (in the beginning of the period) in order to maintain the solvency level α .

$\phi(s)$ is the risk premium required in the capital market.¹¹

In CAPM equilibrium, we have

$$\mu(\alpha) = R + \phi(s) \quad (7.6a)$$

$\mu(\alpha)$ is the required rate of return on the insurer's equity. The insurer will outperform the market by maximizing $\mu(\alpha)$, if market imperfections enable it to do so.

At this stage it is possible to return to the reinsurer's problem. The

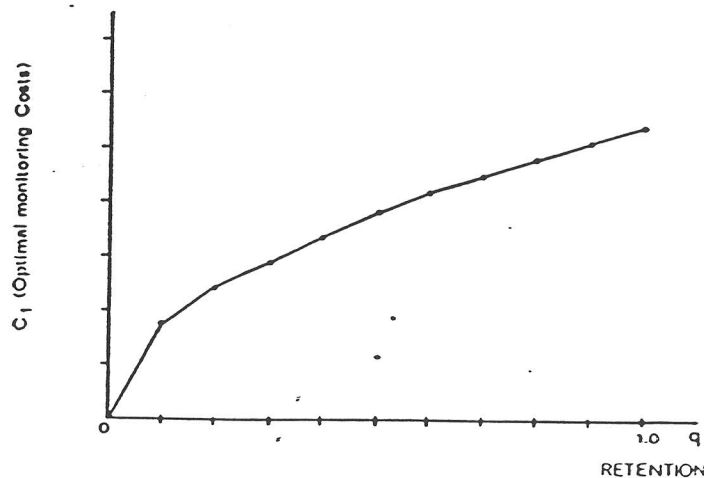


Figure 7-1. Optimal Monitoring Costs Schedule.

reinsurer's performance depends on the quality of the local insurer's monitoring C_a^* , which is determined by (7.5) and (7.6a). The first-order condition for C_a^* is¹²

$$\frac{d\mu(\alpha)}{dC_a} = qV'(C_a)[E(X_1) - E(X)] - (1 + R) = 0 \quad (7.7)$$

Equation (7.7) shows that C_a^* (the monitoring cost in equilibrium) is a monotonically increasing function of q , the rate of the self-retention, but does not depend on the reinsurance commission, Z (see figure 7-1).¹³

By using the monitoring function $V_a(C_a)$, and knowing the relationship between the expected loss $E(X_1|C_a^*)$ and the retention q , (see figure 7-2), the reinsurer can determine the expected value of damage function $E(X_1|C_a^*)$.

The expected loss $E(X_1)$ is monotonically decreasing with the rate of the ceder's retention (q) and asymptotically approaches, $E(X)$, the expected loss where no moral hazard exists. The loading factor to be included in the premium is shown in figure 7-3 and is described by¹⁴

$$L(\alpha) = q^2\alpha S \quad (7.8)$$

4 Results: The Insurance Market Structure

Figure 7-4 demonstrates the solution graphically. The line \overline{CF} represents the demand constraint, that is, the maximum premium the insured is willing

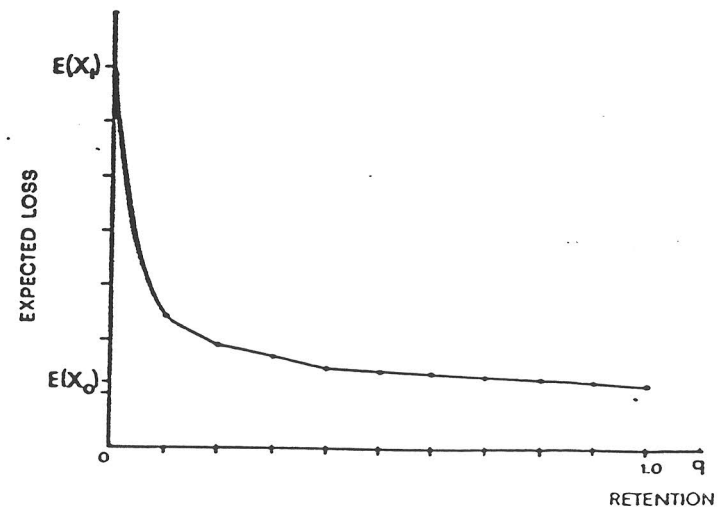


Figure 7-2. Expected Loss Schedule.

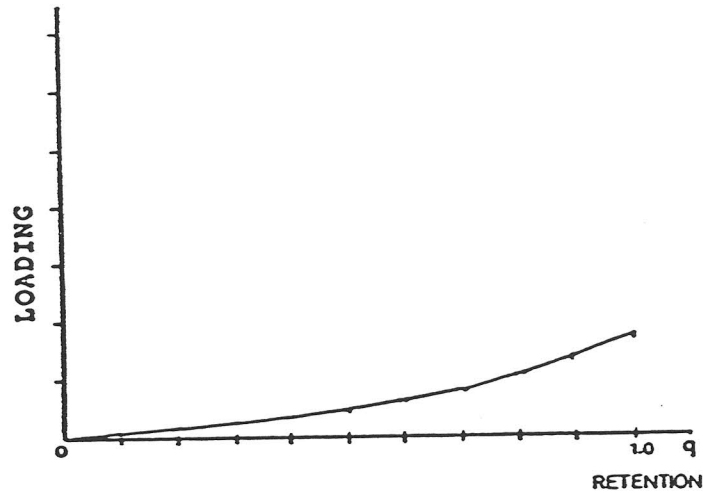


Figure 7-3. Loading Factor Schedule.

to pay (equation (7.1)). The curve \overline{ADG} describes the premium requirements of both insurer and reinsurer, for each level of q . This curve is obtained by summing (for each level of q) all the cost elements: the expected loss, the monitoring cost, and the loading factor. Graphically, this curve is obtained as a sum of figures 7-1, 7-2, and 7-3.¹⁵ For high levels of reinsurance (small q) the reduction of the expected loss $E(X)$, due to monitoring, is the dominant factor. For higher levels of retention the loading factor is dominant. These effects operate in opposite directions, and thereby create the U shape in figure 7-4. The line \overline{BE} represents the supply constraint relevant for the case where monitoring is done directly by the reinsurer (equation (7.3)).

4.1 Corner Solutions

A few corner solutions may be obtained, where no insurance is transacted, or where it is transacted by two parties only.

- a. **No Insurance Transaction:** This happens when the premium requested by the insurer/reinsurer exceeds the premium the insured is willing to pay. In zone I most of the risk would anyhow be covered by the reinsurer

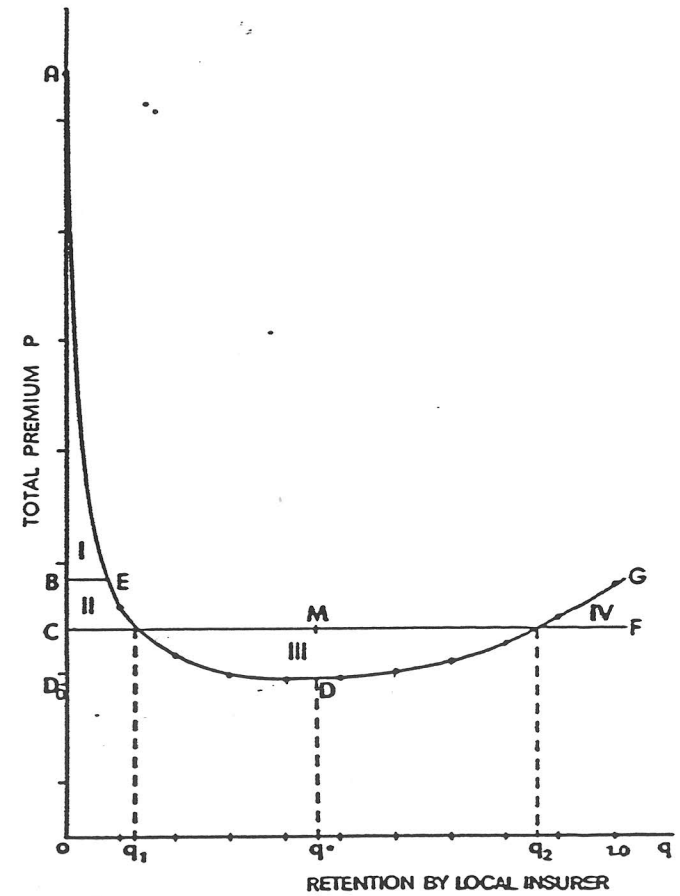


Figure 7-4. Insurance Market Structure: The Solutions.

and, therefore, the motivation of the ceding company to monitor is so low that the reinsurer prefers to monitor by himself. The minimum premium needed to maintain such a direct sale is \overline{OB} (equation (7.4)). The insured is unwilling to pay more than \overline{OC} . Hence this is a zone of disagreement among all three players; the risk is fully retained by the insured, who buys no insurance.

In zones II and IV the insurer and the reinsurer agree that the monitoring should be done by the insurer but the premium they demand exceeds the amount the insured is willing to pay, hence, again, the entire risk is assumed by the insured, and no insurance is sold.

b. **No Reinsurance:** Another corner solution is obtained when an insurance transaction takes place between the insured and the local insurer alone, without the reinsurer. This happens when the local insurer is willing to accept the business with a low loading (due to weak capital requirements and/or his ability to diversify away almost the entire risk by holding a large insurance portfolio). Such corner solutions are often found in practice in lines where full diversification may easily be reached locally while the reinsurer is inefficient in monitoring (e.g., automobile insurance).

c. **Direct Insured-Reinsurer Interface:** The entire risk is covered by the reinsurer alone. This may happen when the capital requirements are tight or in those lines of business in which the local insurer has no advantage in monitoring, for example, earthquake risks. Note that a private case of this situation is when monitoring is inefficient, thereby making the absorption of the effects of moral hazard by the (re-)insurer less expensive than monitoring. Such solutions are common in practice, for example, in cases of large enterprises making only limited use of local insurers and transferring their risks to reinsurers via captive insurance companies.

4.2 A Combined Three-Party Solution

We seek the reinsurance arrangements (in terms of p , q , Z) for which a three-party solution is obtained. In zone III all three players mutually benefit from insurance, and coexistence of the insurer and reinsurer in the marketplace is justified. The proportion of the risk retained by the local insurer is in the range $[q_1, q_2]$ —in which $q = q^*$ (point D)—are optimal. From the insured's point of view, the contract $[p = D_0, q = q^*]$ is optimal. Note, that there is a bargaining range where the insurer/reinsurer may set the premiums above the minimal level D (but not higher than M), and thereby they may enjoy, and share, some profit.

If the reinsurer has a monopolistic power in the reinsurance market, it can set the premiums at the maximum level M , thus forcing the local insurer to demand the maximum premium \overline{OC} the insured are willing to pay. In this case, there will be no room for any price competition in the local insurance market.

On the other hand, in a competitive reinsurance market, the reinsurers may increase the commissions Z , thus enabling the local insurer to enjoy an abnormal profit. This may trigger price competition in the local insurance market, to the benefit of the insured.

The maximum commission the reinsurer can afford to pay is its total

expected profit, that is, its share in the premiums minus its share in the expected losses:

$$(1 - q)ZP \leq (1 - q)P - \frac{(1 - q)E[X_1|C_a^*]}{1 + R}$$

Our model assumes that the reinsurer declares the commission rate, Z , to which the local insurer reacts by setting its policy concerning the premium p , the retention q , and the monitoring cost C . This may trigger a dynamic process where the reinsurer is motivated to change the commissions. In other words, our implicit assumption concerning the sequencing may be of great importance. At present we assume that once the commission rate is set, the local insurer is not permitted to change its decision. An alternative, but more complicated, analysis may be based on the assumption that the commission rate Z depends on q . In this alternative analysis, the reinsurer fixes the premiums p , ($\overline{OD}_0 < p/N < \overline{OC}$), and offers the local insurer a commission schedule that is a function of q . Z is negative in the range $[0, q_1]$, positive only in the range $[q_1, q_2]$, and is maximized at q^* . Such a commission schedule may change the main results of our model.

5 Conclusion

In this article, we present a simple model of an insurance market with three players: insureds, a local insurer, and an international reinsurer. We have tried to explain why local intermediaries coexist and engage in trade with larger, and better diversified, international reinsurers. The model analyzes the motivation of all participants and helps to explain their coexistence in the market. We assumed that local insurers possess an advantage not available to the reinsurers: They can monitor the behavior of the insureds at a lower cost and, for standard moral-hazard reasons, monitoring reduces the insureds' losses. The large, international, reinsurers possess an advantage not available to local insurers: They are better diversified. Hence, there is an economic rationale for three-party risk sharing, resulting from a trade-off between the costs of monitoring moral hazards and the ability to diversify risks internationally. Similar results would probably be reached by a slightly different model where the inferiority of the local insurer stems from his being subject to more stringent capital requirement-imposed by the local regulatory authorities.

A direct practical result which follows from the model is that the rate of reinsurance commission should be a function of the part of the risk retained by the insurer (q).

An interesting by-product of this model of insurance market structure may be the call for additional research in the (endogenous) determination of the monitoring policy, δ . Even when the insured has only little interest in controlling moral hazard, an optimal solution could be obtained as a result of the cost structure of the insurer and reinsurer. In other words, the market structure in itself may lead to self-control of moral hazards!

Further research is clearly needed in order to analyze sequencing problems in the determination of premiums and sharing arrangements and more complicated trade-offs among the participants. Another useful line of inquiry could be in the direction of a two-tiered principal-agent relationship.

Notes

1. Crouhy and Galai [1986] argue that the customers of financial intermediaries can correctly identify the probability of insolvency, and consider this probability in their demand function. The argument could, perhaps, be used to explain reinsurance purchasing by the insureds—as stemming from the insureds' demand. Such an explanation is, however, deficient, in view of the fact that insureds have no information concerning the reinsurance coverage of the insurers, and that in practice, the insureds have no direct claims against the reinsurers.

2. The assumption of risk aversion can be replaced, without loss of generality, by the assumption that the insured is willing to pay a positive risk premium due to income-tax considerations or the fear of financial distress (Main [1982, 1983]).

3. The single-period framework is somewhat restrictive since it does not allow for the treatment of moral-hazard problems through multiperiod and repeated agreements among the participants (e.g., the possible effects of the local insurer building up a reputation).

4. This policy may be effected by increasing the firm's leverage (such that it operates with as little capital as possible, while still maintaining the solvency constraint). Note that the Modigliani and Miller theorem concerning the irrelevance of the firm's leverage is inapplicable in our case. In practice the insureds cannot determine the insurer's risk and rely, instead, on the regulatory ruin constraint. Therefore, the insurance leverage hardly affects the premium income of the insurer, which will try to maximize its leverage.

5. There could be an optimal α for which the societal cost of insurers' bankruptcy is minimized. In the present analysis α is assumed to be exogenous to the system.

6. Note that the regulatory technique commonly used in many countries imposes a capital requirement that is typically a function of the premium volume (and is often independent of the composition of the insurer's portfolio—see Kahane [1977]).

7. Biger and Kahane [1978] have shown that insurance pricing should regard only the systematic risk ("beta"). However, in most cases full diversification cannot be attained and the insurer cannot ignore the standard deviation in its pricing considerations.

8. For the sake of simplicity our analysis assumes that the reinsurer's portfolio is risk free; thus, no loading is needed to cover either the systematic or nonsystematic risks. This assumption can easily be relaxed.

9. This is similar to the description of the effectiveness of the agents' effort in the principal-agent problem (Shavell [1979]). Note that a different model may be obtained if, instead of monitoring, the insurer uses a selection procedure that will enable him to charge insureds with

various levels of λ differential premiums. This topic is discussed in another study by the authors.

10. Assuming investments in the riskless assets.

11. In the context of the CAPM, $\phi(s) = s^* \text{cov}(A/K(\alpha), M)$; s^* is the market price of risk; and M is the rate of return on the market portfolio.

12. Note that (7.6a) is an equilibrium condition in the capital market. As such, of course, this equation is not used as the objective function for the insurer. The derivative (7.7) is used only in order to find the monitoring level in equilibrium. For the sake of simplicity, the possible effects of monitoring costs on the standard deviation of the loss are ignored.

13. Figures 7-1 to 7-4 are based on the numerical values obtained in the example in the appendix.

14. This equation is obtained from Sharpe's performance measure:

$$L(\alpha) = \mu(\alpha) - R = S\sigma$$

Once parts of the risk are being ceded, the standard deviation becomes $q\sigma$.

15. The figures are based on the numerical values obtained in the appendix. They are not, however, drawn to the same scale.

References

- Arrow, K.J. 1963. "Uncertainty and the welfare economics of medical care." *American Economic Review* 53(5):941-970.
- Biger, N., and Kahane, Y. 1978. "Risk considerations in insurance ratemaking." *The Journal of Risk and Insurance* 45(1):121-132.
- Blazenko, G. 1986. "The economics of reinsurance." *The Journal of Risk and Insurance* 53(2):258-277.
- Borch, K. 1962. "Equilibrium in a reinsurance market." *Econometrica* 30(3):424-444.
- Borch, K. 1985. "Do we need utility theory in insurance?" 12th Seminar of the European Group of Risk and Insurance Economists, Bruxelles (September).
- Bühlmann, H. 1980. "An economic premium principle." *Astin Bulletin* 11(1):52-60.
- Campbell T., and Kracaw, M. 1980. "Information production, market signalling and the theory of financial intermediation." *Journal of Finance* 35(4):863-882.
- Crouhy, M., and Galai, D. 1986. "A new look at the theory of the banking firm." *Journal of Banking and Finance* 10(2):231-342.
- Dionne, G. 1981. "Moral hazard and the search activity." *Journal of Risk and Insurance* 48(3):422-434.
- Doherty, N.A., 1980. "Portfolio theory of insurance capacity." *Journal of Risk and Insurance* 47(3):405-420.
- Doherty, N.A., and Schlesinger, H. 1983. "Optimal insurance in incomplete markets." *Journal of Political Economy* 91(6):1045-1054.
- Doherty, N.A., and Tinic, S.M. 1981. "A note on reinsurance under conditions of capital market equilibrium" *Journal of Finance* 36(4):949-954.
- Gerber, H.U. 1984a. "Chains of reinsurance." *Insurance: Mathematics and Eco-*

- nomics* 3(2):43–48.
- Gerber, H. U. 1984b. "Equilibria in a proportional reinsurance market." *Insurance: Mathematics and Economics* 3(2):97–100.
- Johnson, R. E. 1977. "Reinsurance theory: The new applications and future." *The Journal of Risk and Insurance* 44(1):55–66.
- Kahane, Y. 1977. "Capital adequacy and the regulation of financial intermediaries." *The Journal of Banking and Finance* 1(2):207–218.
- Kahane, Y., and Kroll, Y. 1985. "Optimal insurance coverage and speculative risk and the risk free asset." *Insurance: Mathematics and Economics* 4(2):191–199.
- Leland, H. E. and Pyle, D. H. 1977. "Information asymmetries, financial structure and financial intermediation." *Journal of Finance* 32(2):371–387.
- Main, G. M. 1982. "The firm's insurance decisions. Some questions raised by the capital asset pricing model." *Managerial and Decisions Economics* 3(1): 7–15.
- Main G. M. 1983. "Corporate insurance purchases and taxes." *Journal of Risk and Insurance* 50(2):197–223.
- Mayers, D., Smith, C. W. 1981. "Contractual provisions, organizational structure and conflict control in insurance markets." *Journal of Business* 54(3):407–434.
- Pratt, J. W. 1964. "Risk aversion in the small and in the large." *Econometrica* 32(1–2):122–136.
- Raviv, A. 1979. "The design of optimal insurance policy." *American Economic Review* 69(1):84–96.
- Rothschild, M., and Stiglitz, J. 1976. "Equilibrium in competitive insurance markets: An essay in the economics of imperfect information." *Quarterly Journal of Economics* 90:629–650.
- Shavell, S. 1979. "Risk sharing and incentives in the principal and agent relationship." *Bell Journal of Economics* 10(1):55–73.

APPENDIX: A NUMERICAL EXAMPLE

A simple example may assist in clarifying the forces operating in this model. Assume that there are $N = 10,000$ identical insureds. Each insured has a utility function $u(W) = \ln(W)$ and an initial wealth of $W = \$1000$. (Note that once the insured's utility is fully specified, the maximum premium she/he is willing to pay may be calculated directly. In order to be consistent with the text, we later use the Arrow–Pratt approximation.) Among the assets there is one that is exposed to a pure risk. The asset may suffer a \$600 loss with a probability pr , or remain undamaged with a probability $(1 - pr)$. $F(\bar{x})$ is a binomial loss distribution; $pr = 0.01$ —that is, $E(\bar{x}) = \$6$, $\sigma(\bar{x}) = \$59.7$.

After acquiring insurance, the insured may change his behavior and become more careless. Assume a moral-hazard coefficient of $\lambda = 2$, which means a doubling of the losses—that is, $E(x_1) = \$12$, $\sigma(x_1) = \$119.4$.

The monitoring function of the local insurer, $V_a(C_a)$, is assumed to be

$$\delta = \begin{cases} 1 - \frac{500}{C_a} & C_a \geq \$500 \\ 0 & \text{otherwise} \end{cases} \quad (7A.1)$$

while the monitoring function of the reinsurer, $V_b(C_b)$, is assumed to be

$$\delta = \begin{cases} 1 - \frac{2000}{C_b} & C_b \geq \$2000 \\ 0 & \text{otherwise} \end{cases} \quad (7A.2)$$

In order to simplify the numerical example, it is further assumed that the regulatory authorities impose a constraint on the leverage of the insurer. The constraint is $P_a/K \leq 10$ (the maximum permissible leverage is 10). The capital market constraints are:

1. The price of risk in the capital market is $S = (E_n - R)/\sigma_n = 2$ (demonstrating Doherty's [1980] use of Sharpe's "reward to variability").
2. The riskless interest rate is $R = 0.05$.

Let us examine several possible solutions:

Case a: (A corner solution) $q = 0$

In this case all premiums are ceded to the reinsurer, which performs the monitoring alone. From equation (7.2) we can figure its monitoring costs, C_b^* :

$$\frac{\$2,000}{(C_b^*)^2} (\$120,000 - \$60,000) = 1.05$$

$$C_b^* = \$10,690$$

From (7.A2) we get

$$\delta = 1 - \frac{2,000}{10,690} = \underline{0.813}$$

The reinsurer is only willing to sell insurance for a premium, p , which maintains (equation (7.3)):

$$p \geq \frac{E[X|V_b(C_b)] + C_b^*(1 + R)}{10,000 \times 1.05}$$

$$= \frac{120,000 - 0.813 \times (120,000 - 60,000) + 10,690 \times 1.05}{10,000 \times 1.05}$$

$$= \frac{82,455}{10,500} = \$7.852$$

Note that the figure of 82,455 in the numerator is the expected cost to the reinsurer. The maximum risk premium that each insured is prepared to pay is obtained from equation (7.1):

$$\Pi_0 = \left(-\frac{1}{2} \right) \frac{u''(W_0)}{u'(W_0)} \sigma^2(x)$$

$$= \frac{1}{2} \frac{1/W^2}{1/W} \sigma^2(x)$$

$$= \frac{1}{2,000} \times 3,564$$

$$= 1.782$$

And the maximum premium he/she is willing to pay is obtained from equation (7.1a):

$$\frac{6 + 1.782}{1.05} = \underline{\$7.411}$$

Since this premium is more than the insured is willing to pay ($7.852 > 7.411$) there will be no transaction between the insured and the reinsurer.

Case b: (A corner solution) $q = 1$

In this case the entire premium is retained by the local insurer. From (7.7) we find that the insurer will spend $C_a = \$5,345$ on monitoring. The monitoring level δ is as follows (see equation (7A.1)):

$$\delta = 1 - \frac{500}{5,345} = 0.9065$$

By substituting $S = 2$ and $K(\alpha) = 0.1P$ into inequality (7.6) we can find P , the total premium required by the local insurer:

$$P = 120,000 - 0.9065 \times 60,000 + 5,345 \times 1.05 + 0.1P[2 \times 0.6528]$$

$$= \$81,917$$

This means that each insured must pay at least

$$p = \frac{81,917}{10,000} \times 1.05 = \underline{7.802}$$

In this case there will be no transaction since the insured is prepared to pay at most 7.411, while the minimum premium required by the insurer is higher (7.802).

Case c: (A combined three-player solution)

Let us examine a solution with $0 < q < 1$, that is, a three-player solution. For example, assume that the self-retention by the local insurer is 0.50 ($q = 0.50$). From (7.7) we find that the local insurer spends $C_a^* = \$3,780$ on monitoring and this corresponds to an accuracy level of (see equation (7A.1))

$$\delta = 1 - \frac{500}{3,780} = 0.8677$$

The reinsurer will require a minimum income of P_b

$$\begin{aligned} P_b &= \frac{1}{1 + 0.05} (1 - 0.5) \times [120,000 - 0.8677(120,000 - 60,000)] \\ &= \underline{\$32,351} \end{aligned}$$

Actually the reinsurer needs a higher income—to cover commission. The local insurer's share in the premium is obtained by substituting $S = 2$ and $K = 0.1P$ into (7.6):

$$A = q(0.1P)q\sigma_n^2$$

Substituting (7.5) for A we get (assuming that the reinsurance commission $Z = 0$)

$$\begin{aligned} 1.05 \times 0.5P - q[120,000 - 0.8677(120,000 - 60,000)] + 3780 \times 1.05 \\ = 0.5P \times 0.10 \times 0.5 \times 0.676 \times 2 \end{aligned}$$

Isolating the insurer's net income qP we obtain

$$0.5P = 38,750$$

The minimum premium (per policy) required by both the insurer and reinsurer is

$$\frac{32,350 + 38,750}{10,000} = \underline{7.110}$$

which is lower than the premium the insured is willing to pay. In this case

the reinsurer is ready to pay to the insurer a commission that does not exceed

$$(1 - 0.5)(71,110) - 32,350 = \underline{3,200}$$

This solution is acceptable to all three players. Note that it is not necessarily the optimal solution, since it has been assumed arbitrarily that $q = 0.5$. (The optimum can be easily calculated by numerical methods and will lead to $q^* \cong 0.40$.)