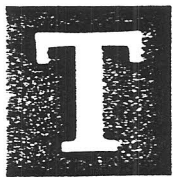


On the Application of New Loss Reserving Techniques in Automobile Insurance

by Yehuda Kahane, Ph.D.

ABSTRACT: *This article demonstrates the application of a modern and sophisticated loss reserving method in "long-tail" insurance lines. Under this method, a statistically testable pattern is being fitted to the past loss experience and used for the forecasting of future payments for the outstanding claims (and IBNR). Although the analysis is based on the same data used for common actuarial techniques, the new method is superior on both theoretical and practical grounds: it requires estimates of a smaller number of parameters, its significance may be tested, and it avoids the bias that results from the implicit use of the medians rather than means under most actuarial techniques. In addition, the forecast is based on the paid losses figures and thereby avoids the bias caused by the use of statistically dependent observations (cumulative, or incurred loss figures) in traditional analysis.*



he adequacy of the loss reserving technique is of major concern to insurance executives, as well as to the regulatory and supervisory agencies: the loss reserves determine the solvency and profitability of insurers, and are the key to the ratemaking process.

This article demonstrates the application of a modern loss reserving technique. It uses the same data which are prepared for most traditional actuarial techniques. In essence, the method is used to identify the pattern of the

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past loss experience (basically using a "least squares" approach). This pattern is then used for the forecasting of future payments for the outstanding claims and incurred but not reported losses (IBNR).

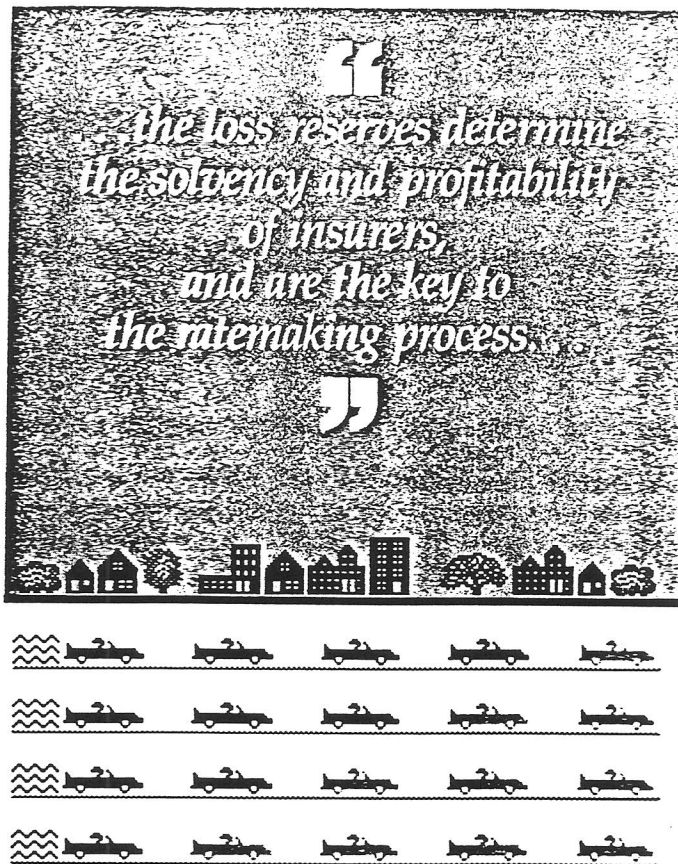
The method is superior to common actuarial techniques: first, it is statistically testable, unlike most traditional methods. Second, the fitted pattern is based on estimates of a smaller number of parameters and, therefore, reaches better parsimony and gives a more reliable forecast. Third, the suggested method is based on the losses actually paid—rather than on cumulative figures, or incurred loss figures, which are used by common forecasting techniques. It thereby avoids other sources of bias that are related to the use of statistically dependent observations. Fourth, this method avoids the bias of most actuarial techniques, that result from the implicit use of the medians rather than means in making the forecasts.

The new technique may apply a varying parameter model, rather than fixed parameter regression models, thus enabling the loss reserver to assign weights to certain observations and to introduce more professional input into the fitting and forecasting process. This enables the loss reserver to reach a good fit faster and to understand the underlying processes better.

Loss reserving could also be of much relevance to insurance ratemaking. Insurance rates are based on past performance, and the measurement of the latter is determined by the reserves, hence the strong tie—which often remains unnoticed—between reserving and ratemaking.

This study is based on an example taken from a no-fault automobile insurance portfolio. It analyzes the experience of the compulsory no-fault automobile insurance program in Israel, which, despite its geographical distance, could be of relevance to the American environment.

The second section of this article discusses some of the theoretical and practical problems related to reserving. The third section deals with the application of loss reserv-



ing techniques, and the fourth section deals with some implications for ratemaking.

Loss Reserving: Theory and Practice

The Case Reserving Method. Many companies set the loss reserves on a case estimate basis, i.e., estimates for the value of the outstanding claim amount in each file. Such estimates are based on professional skill, practical experience, and subjective beliefs of the lawyers and loss adjusters. Insurers often apply additional subjective judgment before publishing the figures in their financial statements. There is no way of verifying the validity of the estimates until all claims are finally settled. Case reserving is stated in terms of the ultimate expected payments and offers neither an accurate prediction for the timing of the payments (i.e., a cash flow projection) nor adequate recognition of the present value calculations.

Chain Ladder Techniques. More objective loss reserving methods are based on the assumption that the future can be predicted by the analysis of the pattern that characterizes the actual payments made by the insurers in the past. The key to a successful forecast is in the ability to accurately identify the pattern and its trends as well as the ability to use it for statistical projections.

The traditional actuarial literature suggests a few methods to identify the pattern (see a summary in Salzmann

[1984], Taylor [1986,1988], and Van Eeghen [1981]). Most popular are several variations of the so-called "chain ladder" technique. Given the claims in the first development year, and the assumed ratio between the first and second years, one can estimate the claims for the second development year. The figures for the next years can then be estimated by the use of a recursive chain that employs the ratios between the second and third and the third and fourth years, etc. (hence the name "chain ladder"). The role of the forecaster is to use the past records in order to come up with estimates of these ratios (referred to as "loss development factors").

The differences between various chain ladder techniques stem from the use of different inputs (claims amount, claims count, cumulative or noncumulative figures, incurred or actually paid figures, opened claims or finally settled claims, etc.), and from the use of various assumptions concerning the basic model that governs the behavior of the loss development factors and between accident and payment years (additive or multiplicative model, for example).

The most common forecasting techniques are based on the cumulative loss figures, since such data appear to have a smoother pattern, which, mistakenly, appears to be superior for forecasting purposes. It should not be forgotten that cumulative figures are strongly dependent on each other and, therefore, do not give independent inputs for the forecasting process. Similarly, the use of incurred loss figures, which is common in practice, could also be misleading, since the incurred losses are affected by estimates of future claims, and may, therefore, be subject to bias with unassessable direction and magnitude. The use of average claim figures may be problematic as well: the averages are affected by the claims amount (the numerator) and the number of claims (the denominator), and are, thus, subject to the combined forecasting errors of the two parameters.

More Sophisticated Reserving Techniques. Recently, actuaries have proposed to improve the chain ladder techniques by the use of statistical regression analysis (see Taylor [1986, 1988], Zehnwrith [1985], and others). Zehnwrith had suggested replacing the chain ladder techniques with a sophisticated nonlinear regression analysis (Zehnwrith 1985). This method is interpreted as an attempt to fit a function (or a series of functions) to the traditional loss triangle. The parameters of the function are estimated by basically using what is known in statistics as a "least squares" approach. In other words, there is an attempt to find a curve with such parameters, so that the sum of the squared deviations between the ob-

served and fitted figures will be minimized.

The function which has been suggested by Zehnwirth, is the one known as Hoerl's curve. This function describes a very rich family of curves, depending on the specific values given to its parameters. The function has only three parameters—denoted alpha, beta, and gamma—which have no immediate economic or insurance interpretation. Generally speaking, alpha determines the height (scale) of the curve, beta determines the general shape (a curve with a negative beta is always decreasing, and a curve with a positive beta has a maximum). Gamma and beta determine jointly the shape of the tail. The same method may be applied to other functions as well. The use of the Hoerl curve has been suggested due to its simplicity and generality.

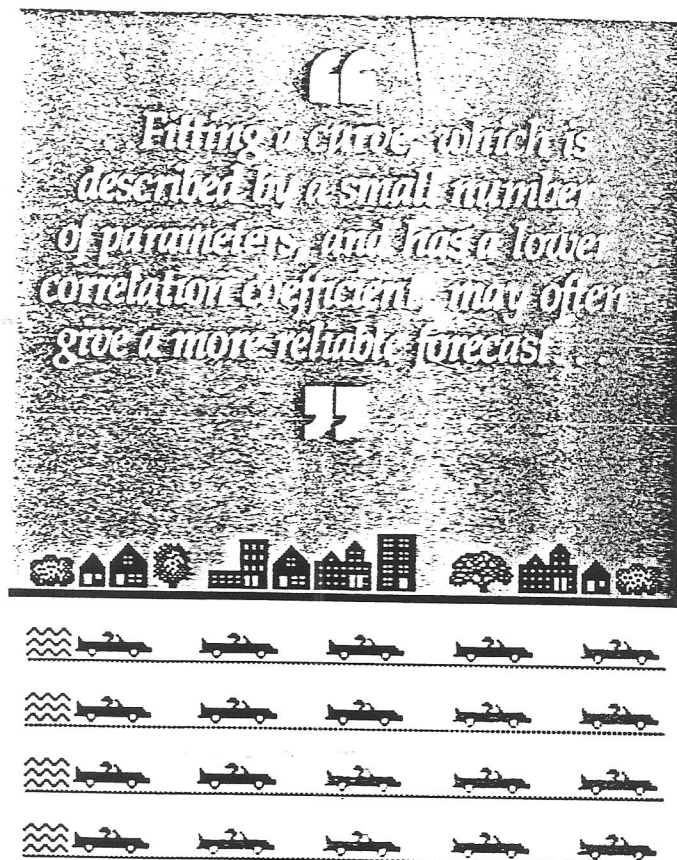
The function describes the development of claims settlement for a specific year of occurrence (i.e., how much is paid during the accident year, how much during the next year, etc.). For each accident or underwriting year the following curve is fitted by estimating the appropriate parameters, alpha, beta, and gamma:

$$\log(h(d)) = a + b \log(1 + d) + cd + e$$

where $h(d)$ denotes the claims for the said accident (or underwriting) year, paid during development year d ; a , b , and c are the estimated values for the parameters alpha, beta, and gamma; and e is the error term. A forecast could be made with the use of the same estimates of alpha, beta, and gamma for all years or, alternatively, with the use of specific parameters for each year. Such functions can, of course, be fitted to other types of data, where each cell of the loss triangle represents accrued losses, cumulative losses, average claim, or number of claims, instead of the paid claims figures. The interpretation of h has to be changed accordingly.

The Advantages of the New Method. The quality of a model is recognized by its parsimony, i.e., the use of a small number of parameters, while still reaching a good fit and having a predictive power. The suggested method has such properties: it uses a remarkably smaller number of parameters than the chain ladder techniques and avoids overparameterization, thus reaching more accurate and significant forecasts. (The chain-ladder process requires the estimation of many parameters for each triangle; there may be a special parameter for each policy—or occurrence—year, a parameter for each development year, and sometimes a parameter for each payment year.) Using too many parameters may result in a perfect fit, but no predictive power whatsoever.

Overparameterization of a model is typically undesired,



since it leads to excessive prediction error. The fitted pattern may give a fairly close estimate of the already observed claims, but its predictive power may be weak.¹ One can always find an equation that gives a 100 percent fit to any set of observed figures. Such a formula, however, does not necessarily give an appropriate forecasting tool. In fact, an additional observation is most likely to greatly deviate from the forecasted figure. New information, e.g., data for an additional payment year, may result in a completely new set of estimated loss development factors and may cause a strong deviation from the previous forecast. In other words, overparameterization is often undesirable. Parsimony is a most important property of a model and is commonly misunderstood in practice. Fitting a curve, which is described by a small number of parameters and has a lower correlation coefficient, may often give a more reliable forecast.

The new model, based on the concept of statistical regression analysis (simple nonlinear regression analysis or a varying parameter model) is statistically testable. It is, therefore, superior to traditional chain ladder techniques. The suggested method has an additional advantage, as it enables one to consider the credibility of certain data points and to affect the statistical significance of the forecast.²

A varying parameter model enables increased economy in the estimation of parameters, i.e., improves the parsimony of the model. For example, instead of using a differ-

ent function to describe the payments for each year of occurrence, it is possible to use the same function for all years, but to allow some controlled changes of one (or more) of the parameters. This means that the loss reserver does not sacrifice too many degrees of freedom while still having a distinct equation for each year. If, say, there are ten years, all having the same beta and gamma parameters but each having a different alpha, one needs 12 parameters. However, if the alpha parameters are determined using a varying parameter model, it is possible that the number of degrees of freedom sacrificed is equivalent to, say, only 3.5 (one beta and one gamma, and, say, only 1.5 for alpha).

The traditional loss reserving techniques may introduce a bias that is often ignored in practice. Such a bias occurs in multiplicative chain ladder models because the forecast is typically obtained by using a logarithmic transformation of the data and the fitted equation. In such cases, much care should be taken in the interpretation of the results, since the (log) residuals are no longer symmetrically distributed around the means. The resulting fitted formula reflects the medians rather than the means, and a direct use of the fitted formula results in a biased forecast. Assuming log-normal distribution around the fitted curve, the mean forecast for $h(d)$ would typically be above the curve.³

The suggested technique, thus, is based on a better model, which is more economical and which may reach a more meaningful forecast. The number of degrees of freedom that are sacrificed is much smaller, a statistical significance test may be performed, credibility considerations may be included, and the statistical bias produced by the use of the medians—rather than the means—is avoided.

Recently, computer software, which does the analysis in an elegant and efficient way, has been developed and marketed under the name Interactive Claims Reserving Forecasting System (ICRFS). This system goes far beyond the simple regression model that may be used for the fitting of the above equation. The system allows the user to estimate, test, and validate a wide variety of regression models, as well as a wide variety of varying regression models. This package was used in the analysis below.

Demonstration: No-Fault Auto Insurance

The case of the compulsory automobile insurance program in Israel is used here in order to demonstrate the application of loss reserving techniques for practical purposes.

A few introductory words about the background of the Israeli system are needed at this point. All car owners in

Israel have to carry unlimited compulsory insurance coverage. This covers bodily injury caused to pedestrians, drivers, and passengers in traffic accidents. Compensation is paid on a pure no-fault basis by the insurers of the car that hit the pedestrians or in which the victims were traveling. The coverage is marketed by private insurers under a uniform policy wording and subject to a uniform tariff. The entire line is coinsured and reinsured by a special national pool founded for this purpose with the establishment of the no-fault scheme in 1976.

Since the rates for this compulsory plan are set by law, a standing committee and the Commissioner of Insurance follow the financial statistics of the pool and all insurers and advise the Finance Committee of the Knesset (the country's legislative body) on required changes. The analysis in this article uses a database that has been prepared and analyzed by the author for that purpose.

Data. The database covers a portfolio of approximately 900,000 cars (only half of that figure in 1976), and about 300,000 claims, two-thirds of which have been settled. All claims were classified according to the date of occurrence. Within each accident year, all payments were classified by the delay (payment made within the accident year was referred to as delay 0; payments made during the next year were referred to as delay 1, etc.). The data include all payments, such as partial payments for still unsettled claims. The statistics relating to claims count (number of closed claims, for example) is less reliable, and not reported in this study.

The fitting technique discussed above can also be used to detect superimposed claims inflation. However, because of the high and variable monthly inflation rates which prevailed throughout the period (monthly inflation rates of 0-27%), all the figures had to be deflated. The consumer price index (CPI) was used for that purpose.⁴ The deflated loss statistics for the years 1977-1988 (January-December) are presented in Table 1. Each row in Table 1 represents the payments originated in a specific year of origin. Each column represents the delay between the year of origin and the year of payment. For more recent years of origin, there are fewer observed development years, and this generates the triangular table. The diagonals represent the payments made during each payment year.^{5,6}

The outstanding claims are represented by the still unknown data cells lying at the southeast part, below the paid claims triangle, i.e., the loss-reserver has to estimate the still missing numbers for each accident year (each row). Having done that, the loss reserver has supplied the missing triangle, or actually, "squared the triangle."

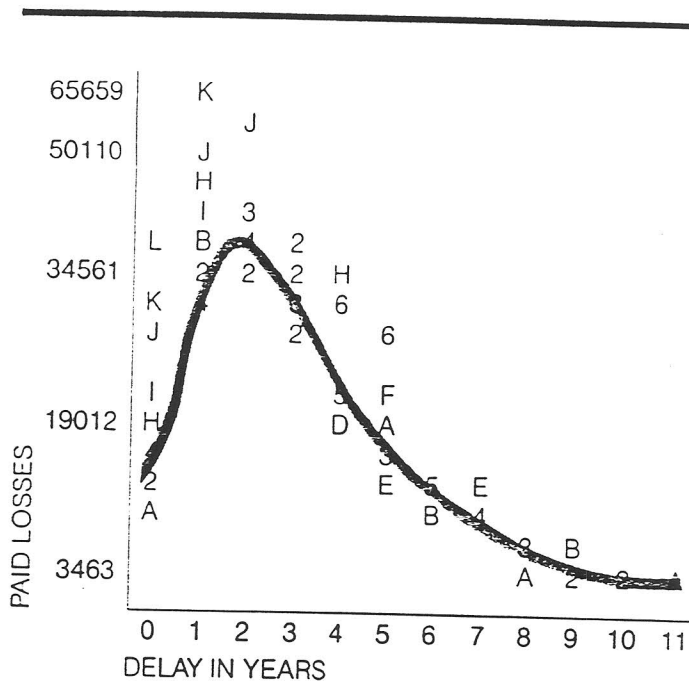


FIGURE 1. Adjusted Data vs. Delay for Accident Years 1977-1988

NOTE: Letters represent occurrence years. A = 1977, B = 1978, etc.; where values overlap, a number represents the number of overlapping observations.

Analysis and Findings. The plot of the raw data, versus delay, for each accident year, helps to make some initial statements about the loss settlement pattern. The data are presented in Figure 1, in which each year is denoted by a letter. The first year (1977) is denoted by A; the second

accident year, by B; and 1988, by L. Overlapping observations are denoted by their number, instead of having all letters printed one on top of the other. The plot, reproduced in Figure 1, reveals an obvious pattern, with a "hump," which corresponds with a Hoerl curve with a positive beta parameter.

Fitting one curve, describing this general "hump" pattern, to all accident years resulted with a good fit: a correlation coefficient of 89.5 percent, and highly significant estimated parameters. It shows that, in this specific case, three parameter forecasts can reach a very good fit without the need for a complicated chain ladder model, which estimates a much larger number of parameters. The use of a more sophisticated analysis enabled reaching even a better fit (correlation coefficient of 94 percent).⁷

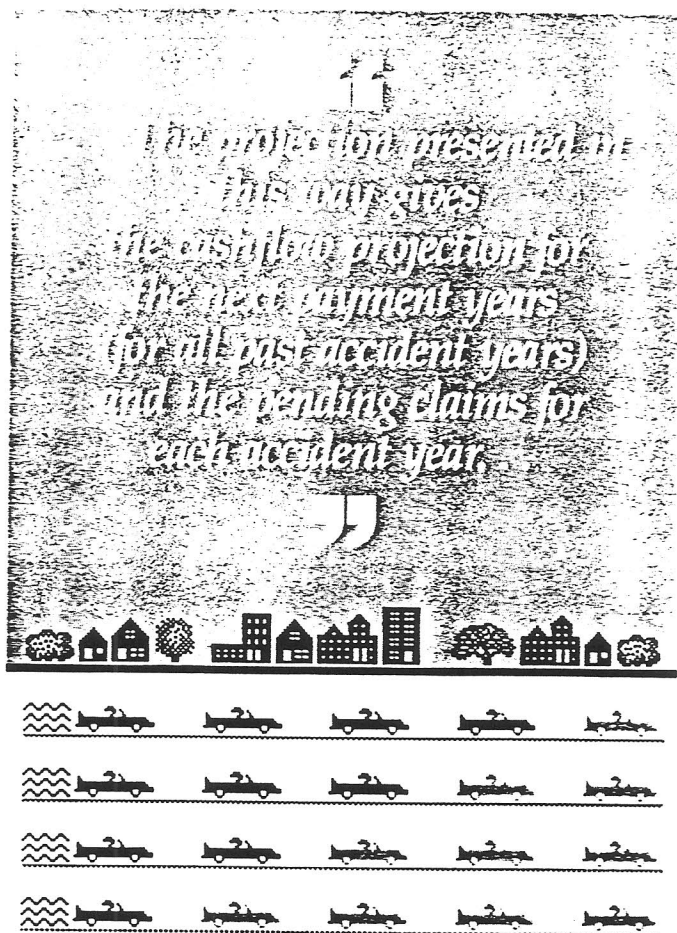
It is interesting to note that improved results could have been obtained even with the use of fewer data points; for example, when the first six payment years are omitted (46 data points out of 78). The stability of the model, and its forecasting ability, may be tested by examining the goodness of fit of the model against observations that did not participate in the derivation of the pattern. For that purpose, it is also possible to assign zero weight for the last payment year, and then check the forecast made by the fitted model against the recently observed data.

Additional information concerning possible bias of the model may be obtained from the analysis of the residuals (the deviations of the observed values from the fitted values).

TABLE 1. Paid Losses by Accident Year and Delay

Accident Year	Delay in Years											
	0	1	2	3	4	5	6	7	8	9	10	11
1977	10395*	31291	35931	30317	21874	18382	12072	10308	3775	3911	3463	3982
1978	13359	37335	37264	35041	23496	16650	10725	9189	6414	5502	4105	
1979	14497	32022	38735	29609	23534	15487	12128	8761	6100	3657		
1980	12003	32787	33577	28386	20447	16139	11643	11206	6952			
1981	15744	34892	38206	35064	21545	13206	12575	11676				
1982	15680	30128	39807	30357	22562	21639	13832					
1983	16568	35817	39371	32406	29984	28301						
1984	18142	42490	40244	38179	54968							
1985	23198	41048	38881	37311								
1986	27284	50526	53635									
1987	32905	65659										
1988	37123											

*Values are in thousands of New Israeli Shekels (N.I.S.), at December 1986 prices. 1 N.I.S. = US\$0.67



The Use of the Model in Forecasting. The fitted pattern can be easily used to forecast the next payment year and also the entire missing (unknown) part of the claims triangle. This is also shown in Table 2. For each occurrence and development year (i.e., each cell in the table), two figures are shown. The upper figures always represent the values forecasted by the model. The lower figures give the actually observed payments in the past. For the future years, the lower figures give the standard errors of the estimates.

The projection presented in this way is useful in practice. The bottom of the table gives the sums of each diagonal, i.e., the cash flow projection for the next payment years (for all past accident years). This is an important instrument for decision making. The projected values are followed by the estimated standard error: a small number means that the real figures are not expected to deviate much from the projected value.

The right column represents the pending claims for each accident year. This information is useful for the analysis of profitability of each accident year. (Similar figures could be calculated for policy years, if the basic data were sorted by policy years.) The pending claims, plus the already paid claims, should be compared to the net premium income for the said year, in order to get the underwriting profit in that year.

Table 2 reports the forecast already in terms of discounted expected payments (a real annual interest rate of 4 percent has been selected for this demonstration). The present value of the next payment year (1989) is obtained by the first forecasted diagonal (N.I.S. 275.2 million in total). The present value of the total outstanding claims is given at the bottom right of the table, with a reasonably low standard error (83.5 million, in comparison to the aggregate value of 1,075.6 million—i.e., some 7.7 percent!).

This forecast already includes estimates of claims payments for the years beyond the twelfth development year (such estimates cannot be obtained by traditional chain ladder techniques). This extension has easily been reached by using the same fitted curve for the forecasting of the still unobserved tail. Such forecasting, beyond the horizon, cannot be handled by traditional loss reserving methods.

Loss Reserving as a Ratemaking Tool

The loss reserver may affect the ratemaking at two points: first, in determining the expected losses from the next policy year, and secondly, in determining possible gaps between the premiums and claims for the past policy years (such gaps may sometimes be covered by a charge levied on future policies).

Much effort is expended on the analysis of loss experience in order to determine the equitable premium for the individual risk.⁸ However, sometimes—and especially in “long-tail” insurance lines—it is advisable to first estimate the aggregate losses and to give priority to the setting of the overall amount of premiums to be collected from the entire portfolio. In such cases, only as a second step, other arbitrary methods are used to split this amount among the various subgroups in order to obtain individual rates.⁹

The forecasting technique described earlier can be used to detect the implicit claims settlement pattern. The claims of the following underwriting year may be forecasted by applying the formula for a next, yet unknown, underwriting year. This estimate, plus the information about the investment income is then used to derive the net premium. The expense loading may then be added for estimating the gross premium.

The Israeli experience reveals a continuous growth in the expected claims settlement. The pending claims estimates, represented in constant prices, grew quickly, beyond the original expectations (based on the old techniques). Due to the growth of the premium income—which resulted from the growth of the portfolio—no sig-

nificant cash flow drain has been observed. However, this deficit puts a heavy strain on the insurers, since they have to maintain outstanding claims reserves without generating sufficient surplus on the current business. Since the outstanding claims are roughly three times larger than the annual premium, we have a "multiplier" effect: An

unexpected growth of the annual claims, of the order of, say, 5 percent, which leads to a revised estimate of the loss reserves, may generate a deficit of roughly 15 percent of one annual aggregate premium.

The new loss reserving method enables one to identify such gaps which have to be covered by a special charge

TABLE 2. Present Values of Forecasts

Accident Year	Delay in Years															Forecast Totals Standard Errors
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
1977	17117*	30064	33231	30374	25017	19292	14218	10137	7047	4803	3222	2133	1370	855	529	2754
	10395	31291	35931	30317	21874	18382	12072	10308	3775	3911	3463	3982	271	175	112	401
1978	17443	30617	33838	30934	25485	19659	14494	10338	7190	4903	3291	2138	1347	841	521	4847
	13359	37335	37264	35041	23496	16650	10725	9189	6414	5502	4105	420	274	179	116	687
1979	15969	28029	30980	28324	23337	18004	13275	9470	6587	4493	2957	1884	1187	741	459	7229
	14497	32022	38735	29609	23534	15487	11213	8761	6100	3657	573	377	247	161	105	972
1980	16022	28121	31082	28418	23415	18066	13322	9504	6612	4422	2854	1819	1146	716	443	11400
	12003	32787	33577	28386	20447	16139	11643	11206	6952	845	560	369	242	158	103	1457
1981	17023	29876	33021	30192	24878	19196	14156	10099	6890	4519	2917	1859	1172	732	453	18542
	15744	34892	38206	35064	21545	13206	12575	11676	1304	874	381	383	251	164	107	2261
1982	17857	31340	34640	31673	26100	20140	14852	10391	6952	4560	2944	1876	1182	738	458	29101
	15680	30128	39807	30357	22562	21639	13832	1957	1331	893	594	392	257	168	109	3411
1983	20184	35424	39157	35805	29506	22768	16466	11297	7558	4958	3201	2040	1286	803	498	48105
	16568	35817	39371	32406	29984	28301	3101	2151	1464	983	654	432	283	185	120	5472
1984	21858	38368	42414	38784	31962	24186	17152	11768	7873	5165	3334	2125	1339	836	518	74296
	18142	42490	40244	38179	34968	4580	3271	2270	1545	1037	690	455	299	195	127	8308
1985	23229	40782	45087	41231	33321	24724	17534	12030	8049	5279	3408	2172	1369	855	530	109271
	23198	41048	38881	37311	6384	4758	3398	2357	1603	1075	714	471	308	201	131	12233
1986	28109	49367	54586	48952	38793	28785	20413	14005	9369	6146	3967	2528	1593	995	617	176163
	27284	50526	53635	9564	7612	5672	4047	2804	1904	1274	844	555	363	236	153	20274
1987	33238	38403	63336	55700	44141	32753	23225	15933	10659	6991	4512	2875	1812	1131	701	263769
	32905	65659	12797	11344	9029	6722	4789	3310	2240	1494	986	646	421	273	176	32718
1988	35278	60807	64673	56879	45076	33445	23715	16268	10882	7136	4606	2934	1849	1154	715	330142
	37123	13308	14355	12726	10123	7527	5352	3689	2438	1652	1085	707	457	295	189	49439
Total for Payment Years	275236	230750	178518	130313	91311	62029	41083	26612	16880	10448	6271	3597	1855	715	1075618	
Standard Errors	24973	23249	19301	14880	10936	7789	5431	3726	2519	1674	1087	682	397	189	83466	

*Values are in thousands of New Israeli Shekels (N.I.S.), at December 1986 prices. 1 N.I.S. = US\$0.67

against future premiums. Note that in a politically sensitive line, like automobile insurance, the authorities are often reluctant and slow in approving required rate increases. Given the above mentioned multiplier effect, a slow reaction may cause a disaster, because the required cumulative rate increases may quickly reach a very high level.

Summary and Concluding Remarks

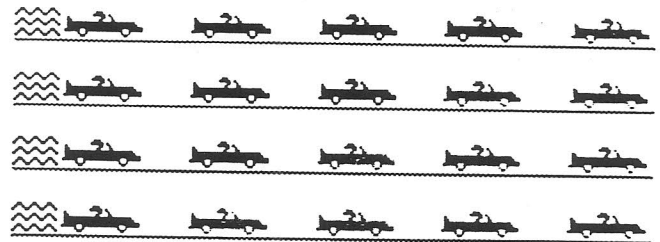
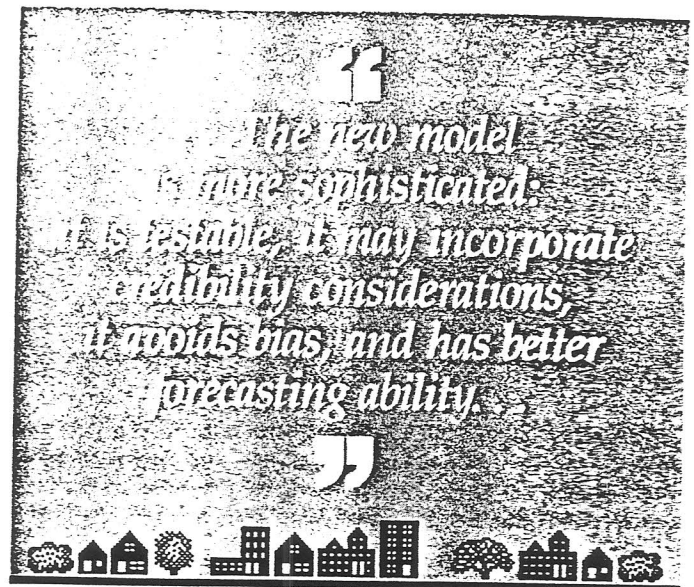
In this article we have presented and demonstrated the use of the modern statistical method for more sophisticated and meaningful loss reserving. This method is superior to commonly accepted techniques, due to certain theoretical advantages. In fact, some of the existing techniques are only private cases of the more sophisticated tool.

The appropriateness of the loss reserves is the key to decision making in insurance: the loss reserves determine the solvency and profitability of the insurer and affect the ratemaking in "long tail" insurance lines. Therefore, this study is of great relevance to insurance executives, in financial, underwriting, and claims departments.

The models discussed in this article also can be used easily by regulatory authorities wishing to get a quick test for the validity of the loss reserves held by insurance companies. The discussion of this important aspect, which is related to the control of insurers' solvency, goes beyond the scope of this paper.

The new model requires the same data that are already used in the application of common reserving techniques (i.e., various "chain ladder" techniques). Therefore, the new method can be immediately applied by firms. The basic inputs to the model are the traditional loss triangles, which are used to identify a pattern, which is in turn used for forecasting the future development years. The system is more sophisticated than the traditional loss reserving techniques, as explained in the article: it is testable, it may incorporate credibility considerations, it avoids bias, and it has better forecasting ability. It is also more sophisticated than a simple statistical regression analysis, since it is based on variable parameters regression, which leaves more room for additional professional input in the analysis.

The no-fault automobile insurance scheme in Israel has been used here for demonstration purposes. It has been shown that the new method enables the loss reserver to get estimates of the outstanding claims and their spread in time. In other words, the forecast gives clear estimates of the future cash flows, of the expected claims for a specific year, estimates for the loss reserves and its credi-



bility level. The same example was used to show how the loss reserving tool may be applied for insurance ratemaking. Due to information problems in "long-tail" insurance lines, such as the liability part of automobile insurance, priority is often given to the setting of the overall amount of premiums rather than to the pricing of individual risks. Since the ultimate loss figures are known only after a long delay, the ratemaking process in these lines depends on the appropriateness of the loss reserving technique.

The analysis of the specific case reveals a recent trend of rapid growth of the required loss reserves. Even if the current premium income is sufficient to cover the current loss payments and creates no cash problems, it may still generate a financial strain due to the need to maintain growing loss reserves, which are unpaid for. The inability to immediately correct such an effect, and a slow reaction to these trends, may be disastrous due to the "multiplier" effect. The author believes that this lesson is valid not only for Israel but for other environments as well. ■

Endnotes

1. This is similar to a regression analysis in which there are many parameters and a small number of observations: it is possible to fit a curve that passes through all points—a correlation coefficient of 1—but with no predictive power.

2. Whereas regression analysis is typically a fixed parameter model, the suggested technique is a varying parameter model. The fitted curve is obtained by minimization of a more complicated objective function, which depends on the squared deviations, and certain weights that are assigned to the observations and parameters and reflect *credibility* considerations.
3. Using the fitted logarithmic formula directly for forecasting would produce estimates of the medians, rather than means, and would typically generate a downward biased forecast. The forecast should, therefore, be carried out with the estimated mean values, which could be obtained with additional statistical manipulations (see Zehnwirth 1985).
4. The inflation affected mainly the monetary values. No clear effects were detected when the number of claims was analyzed, though such effects may theoretically be possible (especially in lines where higher, and unindexed, deductibles are common).
5. The technique employed in this study enables the analysis of truncated loss triangles.
6. Often it is helpful to normalize the data to reflect the changes in the size of the portfolio. The number of insured cars, or the number of traffic accidents, could serve as an appropriate normalization basis. For the specific data at hand, these factors did not affect the goodness of fit and the forecast.
7. More detailed analysis and information may be obtained directly from the author. Figure 1 suggests that we have a series of curves where later accident years are at higher levels. This implies that an even better fit may be reached by using the varying parameter regression. Such a regression, combined with credibility analysis, has been used on the data, with the ICRFS system. Most other statistical packages (such as LOTUS 1-2-3) cannot handle such an analysis.
8. See a survey of the common ratemaking problems in Van Eeghen (1983). One of the first attempts to apply a large-scale multiple regression analysis to the ratemaking problem was made in Israel in 1969 (a summary in English was published only later—see Kahane and Levy (1975)). More sophisticated techniques and studies were later used, thanks to the developments of more powerful computers. A special report concerning the ratemaking problems in the Israeli no-fault program was prepared by the author for the Knesset and the Israeli Ministry of Transportation (1975). Only a short summary in English is available (Kahane and Lang (1978)).
9. Some methods to determine the premium relativities of various groups are discussed by Kahane and Lang (1978), and Bennett (1987).

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