

# Optimal insurance coverage in situations of pure and speculative risk and the risk-free asset \*

Yehuda KAHANE

*Faculty of Management, Tel Aviv University, Tel Aviv, Israel*

Yoram KROLL

*School of Business, The Hebrew University, Jerusalem, Israel*

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The insured's portfolio consists of an insurable (pure) risk, an uninsurable (speculative) risk, a (proportional) insurance policy and a risk-free asset. The optimal insurance policy (i.e., the proportion to be insured) is examined from the insured's point of view, using the reward to variability concept. The importance of the risk-free asset in reaching an exact and explicit solution is analyzed, while emphasizing the possibility of substitution of the risk-free investment and insurance mechanisms. The paper demonstrates possibilities of improving the insured's welfare by the use of the risk-free rate - which is sometimes less expensive than other risk reduction instruments. The analysis leads to a two-step solution, similar to the well-known Hirschleifer investment model and to the famous Capital Assets Pricing Model.

*Keywords:* Correlation, Risk-free rate, Portfolio, Optimal insurance, Risk loading, Reward to variability, Proportional insurance, Capital assets pricing model.

## 1. Introduction

The problem of optimal insurance policies has been discussed in quite a number of recent articles. Most have viewed the insurance policy as a contract between two parties, and used expected utility functional analysis to select the optimal form of the contract [Arrow (1965), Adar and Neumann (1978), Doherty and Schlesinger (1983), Raviv (1979), Smith (1968)]. An alternative approach has been to use efficiency criteria [Doherty (1980,1985)

and Kroll (1985)] which only implicitly assume a certain admissible group of utility functions. In this paper the second approach is employed. It is assumed that the insured can incorporate in his portfolio a risk-free asset and thus can use the reward to variability criterion in evaluating risky portfolios. Although this measure suffers from the well-known limitations of the mean variance criterion, it is used in the present analysis since it provides a good approximation for decision makers even when the utility functions and the distribution functions do not fulfill all the necessary assumptions [see Levy and Markowitz (1979), Kroll, Levy and Markowitz (1985)].

The insured's decision is examined in the context of a portfolio of risks, which includes an insurable (pure) risk, an uninsurable (speculative) risk, insurance policy, and a risk-free asset. Previous studies of the optimal insurance problem have not emphasized the impact of the risk-free asset on the insured's optimal strategy. It will be shown that by incorporating the risk-free asset into the analysis, considerable insight may be gained; as has been the case when it was incorporated into financial models (e.g., Hirschleifer investment model and Sharpe-Lintner-Mossin Capital Assets Pricing Model).

Similar to recent papers [Buser and Smith (1984), Doherty and Schlesinger (1983), Mayers and Smith (1983)] it is demonstrated that the insurance transaction should not be considered in isolation. We show that the amount of insurance is related to other exposures of the firm, requiring the insurance decision to be made as part of the management of all the risks of the firm. From the insured's point of view, insurance instruments compete with risk-free investments as well as other investments in the portfolio as a means of risk reduction. It is shown that the risk-free asset plays an important role in the analysis of the optimal insurance coverage.

By definition, the return on a risk-free asset is not correlated with returns from risky investments.

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