Expansion and cooling of a bright rubidium three-dimensional optical molasses

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We have studied the expansion of a bright rubidium three-dimensional optical molasses realized with an intense laser light. In this regime, in addition to the laser friction force, the atoms strongly repel each other due to the radiation trapping force, which is analogous to the Coulomb force. The experimental observations show that the cloud volume increases linearly in time and that, simultaneously, the cloud temperature decreases following the law \( (1 + \tau)^{-4/3} \). These time evolutions are explained using a physical picture of Coulomb explosion in a viscous medium.

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I. INTRODUCTION

For many years, laser cooling and laser trapping have been extensively studied, with various geometry configurations, with various laser polarizations, and with many atoms. In usual optical traps, the momentum exchange between the laser light and the atom leads to a friction force, which reduces the atom velocity and cools it down. In a magneto-optical trap (MOT), the confinement is done because of the presence of a magnetic field gradient which, combining to the light, induces a restoring force towards the zero-magnetic field region. These two forces are the main ingredients to understand the behavior of the atoms in MOT realized with laser at a low saturation parameter.

For large cloud densities, and, for intense trapping laser (large saturation parameter), these ingredients are not enough to explain the cloud properties and behavior. Some other processes have to be taken into account to describe correctly the properties of the atomic cloud, i.e., the absorption of the laser light by the cloud and the multiple scattering of photons in the cloud. The absorption of light induces a laser intensity gradient and therefore modifies the radiation pressure force [1]. The so-called absorption force is larger at the cloud border than at its center. Therefore, the absorption force compresses the cloud. The multiple scattering of photons in the atomic cloud induces the radiation trapping force [2]. As explained by Sesko, Walker and Wieman [3], the photons emitted by an atom can be reabsorbed by a neighboring atom and this photon exchange leads to a repulsive force between the atoms. The radiation trapping force is proportional to the reabsorption cross section, which depends not only on the emission spectra but also on the absorption of the neighbor. In the presence of an intense laser light, the emission and the absorption spectra are modified. The emission spectrum exhibits a structure with three components (the Mollow triplet) [4,5]. The blue component contains photons nearly at resonance, which can be efficiently absorbed, because the absorption spectrum [6] is asymmetric with its main component blue shifted. Consequently, the reabsorption cross-section is enhanced and also the radiation trapping force. In the weak-intensity regime, the radiation trapping force balances the absorption force and becomes larger in the multiple scattering regime.

In a MOT, many experimental manifestations are due to the absorption and to radiation trapping forces. For bright optical traps and for clouds with a large atom number, these forces are responsible for the limitation of the atomic density and for the cloud shape [7–9]. In the case of slightly misaligned laser beams, the forces explain the spatial structures observed in the atomic cloud [10]. In MOT and in optical molasses (OM), the fluctuations of these forces explain the observed temperatures and the limitation for lower temperatures in dense atomic clouds [9,11].

In this paper we analyze the manifestations of absorption and radiation trapping forces by another way. We realize a dynamical study of a bright OM. We first fill a MOT. Subsequently, we suppress the restoring force. Therefore, we get an OM, which expands only under the absorption and radiation trapping forces and the friction force. The laser light being intense, the effects of the absorption and radiation trapping forces dominate and contrary to previous studies [12,13], the atom momentum diffusion due to the photon exchange with the cooling laser becomes negligible. During the cloud expansion, we record the atom number, the size, the shape, and the temperature. The absorption and radiation trapping forces decrease as the cloud expands, whereupon we observe a decrease of the temperature simultaneously with the expansion. The observed expansion and the cooling are reproduced and explained using a Coulomb explosion model in a viscous medium.

As explained in Ref. [3] the radiation trapping force between two atoms is proportional to \( 1/r_{12}^2 \), where \( r_{12} \) is the distance between the atoms. The radiation trapping force is analogous to the Coulomb law and the expansion of the cloud can be viewed as a Coulomb explosion. To obtain a correct order of magnitude estimate for the radiation trapping force, the effective charge to put in the Coulomb law is about \( 10^{-5}e \), where \( e \) is the electron charge (\( e = 1.6 \times 10^{-19} \text{C} \)). According to Coulomb explosion models, usually used for charged molecules and clusters [14–15], the Coulomb explo-
The OM is characterized by a small charge $q$, a mass $m$, and $R_0$ representing the initial interparticle spacing. According to the Coulomb explosion time scales by a numerical factor of $10^{10}$ from 100 fs in large molecules and clusters of heavy atoms, e.g., Xe [14–15], to $\sim 1$ ms in the bright OM of Rb. In an OM, in addition to the Coulomb force, the atoms are submitted to the friction force due to the laser. The expansion of a bright OM is thus analogous to a Coulomb explosion in a viscous medium. The viscosity increases the time scale of the expansion. We benefit of this advantage to perform an experimental analysis of the cloud expansion, using an imaging method. Note also that the bright OM constitutes a simple experimental system for the interrogation of Coulomb explosion. In clusters and molecules, other phenomena are often in competition with the Coulomb explosion, for example, the fission, dissociation, or evaporation. In plasmas, the presence of positive and negative charges changes considerably the dynamics of the temporal evolution. In our study, we have no competitive processes and the Coulomb explosion parameters can be modified by the variation of laser parameters (intensity and frequency).

The presentation of the paper is the following. Section II is devoted to the experimental setup. We give the procedure for recording the cloud images and for analyzing the temperature versus time. In Sec. III, we present experimental results for the cloud expansion, the model for the Coulomb explosion of a viscous medium and a comparison with the experimental data. In Sec. IV, we present the cooling of the expanding cloud and we compare the experimental temperature to the model. Section V summaries our conclusions.

II. THE EXPERIMENT

A schematic view of the experimental setup is given in Fig. 1. The MOT is realized at the center of a non-magnetic stainless steel vacuum chamber evacuated by a 20 l/s ionic pump (Varian, 20 l/s). The achieved vacuum is about $10^{-9}$ Torr, measured by monitoring the ionic pump current. At the center of the chamber, in a volume of 8 cm$^3$, the magnetic fields are reduced to zero (less than 1 mG = $10^{-7}$ T), by using three pairs of Helmholtz coils.

The MOT magnetic-field gradient is produced at the center of the chamber by a pair of water cooled anti-Helmholtz coils with a magnetic gradient, $b_\perp$, along the coil axis, up to 25 gauss/cm. The MOT is realized by 3 retroreflected $\sigma^+$ or $\sigma^-$polarized laser beams in three orthogonal space directions. The laser beams are provided by laser diodes, which frequencies are locked to cross-over lines of the saturated absorption spectrum of the $5S_{1/2} \rightarrow 5P_{3/2}$ $^{85}$Rb transition. The cooling laser beam is provided by a 50 mW, Hitachi diode laser, 10 MHz line width, which frequency is locked to a cross-over resonance located at 92 MHz from the $5S_{1/2} F = 3 \rightarrow 5P_{3/2} F' = 4$ cooling transition. The cooling laser frequency detuning, $\delta$, measured relatively to the atomic resonance, is then tuned in the range $-1.5 \Gamma$ to $-7 \Gamma$, where $\Gamma$ is the natural width of the cooling transition ($\Gamma = 2 \pi \times 5.89$ MHz). This frequency detuning is accomplished via an acousto-optic modulator of a central frequency of 80 MHz. The laser beam is made round by using anamorphic prisms, and expanded to a 0.36 cm$^2$ size (the 1/e$^2$ radius being equal to 3.4 mm). At the entrance of the vacuum chamber up to 12 mW laser power is available. It allows to vary the ratio $I/I_s$ up to 20 ($I$ is the six-beam total intensity and $I_s$ is the saturation intensity, equal to 1.62 mW/cm$^2$ for $5S_{1/2} F = 3 \rightarrow 5P_{3/2} F' = 4$ $M' = 4$ of the Rb atom). Another laser diode provides the repumping beam. Its frequency is stabilized to the $5S_{1/2} F = 2 \rightarrow 5P_{3/2} F' = 2$ transition. About 5 mW of power is used. This beam overlaps the cooling laser beam in the three directions of space.

To produce the OM, switching on the magnetic field gradient and the lasers, we first fill the MOT for 4 s, which is longer than the trap filling time constant ($2 \tau$). The OM is obtained by rapidly switching off the magnetic field gradient, the laser parameters (intensity, frequency and polarization) remaining the same. The measured switching time of the anti-Helmholtz coil current is less than 500 $\mu$s. Some residual eddy currents limit the magnetic-field gradient switching time to 1 ms. The OM submitted to the already mentioned forces rapidly expands. At time $t$ the lasers are switched off via the acousto-optics devices. A mechanical shutter is added to eliminate any residual light. Just before switching off the lasers, the cloud image is formed onto a charge coupled device (CCD) camera. The CCD exposure is started by opening the shutter located in front of the detector and by triggering the camera electronics. The cloud fluorescence is collected by a 6-cm focal length anti reflection coated doublet lens with effective aperture 2.8 cm. The camera device is a commercial (Princeton Instrument) nitrogen cooled camera. Its CCD array contains 512 $\times$ 512, 24 $\times$ 24 $\mu$m size, pixels with 70% quantum efficiency. Stray ambient light is reduced by using an interferometric filter, with a 83% transmission at 780 nm. To avoid any CCD array saturation, the detector exposure begins just before the trapping laser beams are switched off. We obtain a cloud exposure corresponding to a cloud emission time of 1 ms.
When the lasers are off, the cloud falls down. A probe laser beam located 6 mm below the center of the trap is used to image the fluorescence of the cloud as it falls across. The probe beam is 2 mm diameter sized, -31 red detuned from resonance, by using another acousto-optic device. The beam is retro-reflected with orthogonal linear polarization (linear ⊥ linear configuration), in order to get a molasses effect in the detection. This detection configuration avoids any atom displacement in the probe beam due to radiation pressure force, and allows a long interaction time in the probe beam. A weak repumping beam (0.1 mW), provided by the above mentioned laser is superimposed on this beam. The probe beam light is switched on during 10 ms, when the atom packet is passing through. The detector exposure time is chosen to be long enough, in order to obtain the OM image and the cloud fluorescence in the probe beam with the same CCD record. Such a record is shown in Fig. 2. To study the dynamics of the cloud expansion we record such an exposure for different values of the expansion time \( t \).

The top region of each exposure is numerically integrated and gives the total number of counts, which is proportional to the OM atom number. For this measurement we assume a two level atom model for the calculation of the fluorescence rate. We do not take into account the beam polarization and the cloud position in the intersection of the light beams. Therefore the absolute atom number uncertainty is about 30%, but the cloud atom number variations are quantitatively analyzed. The OM image is numerically integrated over rows and over columns and gives the horizontal [Fig. 2(b)] and vertical cloud profiles. The cloud profiles are well fitted by gaussian profiles giving the 1/e² cloud radii, \( \sigma_x \) along the \( x \) axis corresponding to the magnetic field gradient \( b_x \) and \( \sigma_z \) in the vertical direction. The OM size is measured with a precision of 28 \( \mu m \).

The bottom region of each exposure is numerically integrated over 10 rows. The obtained profile is plotted in Fig. 2(c). Its width, \( \sigma_z \), depends on the atomic velocity distribution in the OM at the time \( t \). If the position and velocity distributions are not correlated, \( \sigma_z \) depends only on the OM size, \( \sigma_0 \), its temperature, \( T \), and the time of flight, \( t_f \), by the relation \( \sigma_z^2 = \sigma_0^2 + aTt_f^2 \), where \( a \) is a constant. If the position and the velocity distributions are correlated this relation is no longer valid. In our experiment, the radiation trapping force induces a strong correlation between velocity and position and the measurement of the cloud “temperature” requires some assumptions. Assuming the correlation described in Secs. II and IV, \( \sigma_z \) is related to \( \sigma_0 \) by the relation \( \sigma_z = \sigma_0 + \nu_\text{m}t_f \), where \( \nu_\text{m} \) is the mean radial velocity.

III. EXPANSION OF THE BRIGHT MOLASSES

A. Experimental results

The time evolution of the OM has been recorded for a laser intensity in the range of \( I/I_s = 15–20 \) and for a set of laser detuning \( (\delta I = -1.5, -2.5, -3.7, -5.4, -6.8) \). Figure 3 shows the experimental data for \( I/I_s = 17.5 \) and \( \delta I = -3.7 \).

We observe an exponential decrease of the atom number [Fig. 3(a)] with a time constant of \( \sim400 \) ms, which is large compared to the evolution time of the cloud size [Fig. 3(b)]. For the time range of our interest (up to 200 ms) we can thus consider that the atom number remains nearly constant during the cloud expansion.

The cloud expansion [Fig. 3(b)] is fast. The cloud size reaches twice its initial value on a time scale of about some milliseconds. The observation does not correspond to a momentum diffusion process, which is expected to occur on the time scale of seconds [12]. The millisecond time scale is in agreement with the scaling argument for the Coulomb explosion, as already mentioned in Sec. I.

According to Fig. 3(c), we observe a linear time dependence of the cloud volume. Consequently the mean cloud density rapidly decreases as shown in Fig. 3(d). Again, the observations rule out the momentum diffusion process, where the square of the cloud size evolves linearly with time [12].
We note that the experimental time scales for the expansion along the $x$ axis and the $z$ axis are somewhat different [Fig. 3(b)] with the characteristic time for the cloud expansion along the $z$ axis being longer. This is due to the initial shape of the cloud in the MOT. Because of the anisotropy of the quadrupolar magnetic field, the cloud is not completely spherical.

The data analysis is performed in Sec. III D using the model of Coulomb explosion in a viscous medium. Details about the model are given in Secs. III B and III C.

B. Coulomb explosion in a viscous medium

The main ingredients of the model are the absorption, the radiation trapping and the friction forces. We will first present them explicitly and write the corresponding Fokker-Planck equation for the time evolution of the cloud. Then, the model will be applied to a spherical problem. The variation laws for the time evolutions of the volume and the density will be established in the case of a homogeneous sphere.

The expressions of the absorption force, $F_A(\vec{r})$, and of the radiation trapping force, $F_R(\vec{r})$, have been established by Dalibard [1] and Sesko, Walker, and Wieman [3]. Their space dependencies depend on the atomic density by the relation:

\[ \vec{v} \cdot [F_A(\vec{r})] = -\sigma_L^2 I/c n(\vec{r}), \]

\[ \vec{v} \cdot [F_R(\vec{r})] = \sigma_R \sigma_L n(\vec{r}), \]

where $\sigma_L$ is the laser absorption cross section, $\sigma_R$ is the absorption cross section of the scattered light, $I$ is the laser light intensity, $c$ is the light velocity, and $n(\vec{r})$ is the atomic density of the cloud. The expressions are valid in the linear regime for the light absorption and the photons emitted by the atoms. Equation (1) are analogous to the well-known Maxwell equation and the Gauss’ theorem can be used to derive the expression of the forces. The total force, $\vec{F} = \vec{F}_A + \vec{F}_R$, is then driven by

\[ \vec{v} \cdot [\vec{F}(\vec{r})] = Kn(\vec{r}) \]

where $K$ is given by

\[ K = (\sigma_R - \sigma_L) \sigma_L I/c. \]

The coefficient $K$ depends only on the laser intensity and frequency. Details about the calculation of $\sigma_L$, $\sigma_R$ and $K$ are given in the Appendix for the two-level atom. In most cases, $\sigma_R$ is larger than $\sigma_L$ and according to Eqs. (2) and (3), the force $F$ expels the atoms out of the high-density region.

The friction force is assumed to be linearly dependent on the atom velocity $\vec{v}$ as

\[ \vec{F}_f = -\alpha \vec{v}, \]

where $\alpha$ is the friction coefficient. Analytical or numerical calculations of $\alpha$ have been done in many laser configurations [16–19], mostly for linear and linear polarizations of the lasers. For the $\sigma^- - \sigma^-$ configuration, in a low laser intensity regime, the results indicate dependence on the laser polariza-
tion and on the laser frequency [16–18]. For the sub-Doppler
‘cooling regime and in the case of low-saturation parameter
these results indicate also that α depends neither on the po-
position nor on the atom velocity. They show that for large
laser detuning, the friction parameter α varies as 1/δ. The
time scale corresponding to the damping of velocity, given
by $m/\alpha$, is in the microsecond range, which is shorter than
the observed expansion time scale [Fig. 3(b)]. That lets us to
assume that the atom velocity is rapidly damped.

The time evolution of the cloud is derived from the
Fokker-Planck equation including the friction force and the
total force. In our approach we neglect the momentum dif-
fusion term [1,12,13]. With this assumption the Fokker-
Planck equation for the normalized Wigner distribution
$w(\vec{r},\vec{v},t)$ is:

$$\frac{\partial w(\vec{r},\vec{v},t)}{\partial t} + \vec{v} \cdot \nabla [w(\vec{r},\vec{v},t)] = -\frac{1}{m} \vec{v} \cdot \left\{ [\vec{F}(\vec{r}) + \vec{F}_f(\vec{v})]w(\vec{r},\vec{v},t) \right\} + \frac{1}{m} \nabla \cdot [w(\vec{r},\vec{v},t) \nabla w(\vec{r},\vec{v},t)] = 1. \tag{5}$$

The atomic density, given by $n(\vec{r},t) = N \int w(\vec{r},\vec{v},t) d^3\vec{v}$ where
$N$ is the atom number, deduced from Eq. (5) by a velocity
integration, is driven by

$$\frac{\partial n(\vec{r},t)}{\partial t} = -N \nabla \cdot \left[ \int \vec{v} w(\vec{r},\vec{v},t) d^3\vec{v} \right]. \tag{6}$$

Because of the rapid damping of the atom velocity, an adia-
batic elimination of the velocity variables is possible [1]. The
atom located at the position $\vec{r}$ rapidly reaches a limit velocity
equal to $\vec{v} = \vec{F}(\vec{r})/\alpha$. Inserting this relation in the right-hand
side of Eq. (6) results in the time evolution of the density:

$$\frac{\partial n(\vec{r},t)}{\partial t} = -\frac{1}{\alpha} \nabla \cdot [\vec{F}(\vec{r})n(\vec{r},t)]. \tag{7}$$

Solving Eqs. (7) and (2) gives the time evolution of the cloud
density. Because the position and velocity distributions are
strongly correlated, the knowledge of the density determines
velocity distribution and therefore the mean velocity (see Sec. IV).

**C. Coulomb explosion in a spherical viscous medium**

In order to illustrate the time evolution of the cloud den-
sity we will consider some spherical problems. First, we will
give the solution for the Coulomb explosion in the case of an
initially homogeneous sphere, and then for an initially gauss-
ian shaped cloud.

Assuming spherical symmetry, the total force $\vec{F}$ is radial
and the density depends only on the radial variable $r$, dis-
tance to the center of the cloud. Eq. (7) is thus expressed as

$$\frac{\partial n(r,t)}{\partial t} = -\frac{1}{\alpha} \frac{\partial}{\partial r} \left[ r^2 F(r)n(r,t) \right]. \tag{8}$$

Equation (8), integrated by parts, allows to derive the evolu-
tion of the mean radius $\langle r \rangle = 1/N \int r n(r,t) r^2 dr$ in the form

$$\frac{\partial \langle r \rangle}{\partial t} = \frac{\langle F \rangle}{\alpha}, \tag{9}$$

where $\langle F \rangle$ is the mean force defined by $\langle F \rangle = 1/N \int F(r)n(r,t) r^2 dr$. Equation (9) relates the mean ra-
dial force $\langle F \rangle$ to the time evolution of the cloud size.

Now we assume that, at $t=0$, the cloud containing $N$ at-
omes is a homogeneous sphere of radius $R_0$. Its initial density is then

$$n(r,0)=n_0=\frac{N}{4/3\pi R_0^3} \text{ for } r\leq R_0. \tag{10}$$

Equations (2) and (8) then imply that the spatial distribution
stays spherically symmetric and homogeneous during the
time evolution. The density at time $t$ is then

$$n(r,t)=n(t)=\frac{N}{4/3\pi R(t)^3} \text{ for } r\leq R(t), \tag{11}$$

where $R(t)$ is the sphere radius at time $t$. A combination of
Eqs. (2) and (8) leads to

$$\frac{\partial n(t)}{\partial t} = -\frac{K}{\alpha} n(t)^2, \tag{12}$$

which is solved

$$\frac{1}{n(t)} = \frac{1}{n_0} + \frac{K}{\alpha} t. \tag{13}$$

Equation (13) shows the linear time evolution of the inverse
of the atomic density. The slope, $K/\alpha$, depends only on the
laser parameters and does not depend on the cloud param-
eters, e.g., the size or the atom number. Another form of Eq.
(13) is $4\pi R(t)^3/3 = 4\pi R_0^3/3 + Knt/\alpha$, which indicates a lin-
ear increase of the cloud volume versus time. The expansion
rate $Knt/\alpha$ is proportional to the cloud atom number and
depends on the laser intensity and detuning via the ratio
$K/\alpha$. Introducing the time constant $\tau = \alpha/(Kn_0)$, the relative
linear expansion of the cloud volume $V$ and the reduction of
its density $n$ can be expressed by

$$V/V_0 = 1 + t/\tau, \tag{14a}$$

$$\frac{n}{n_0} = (1 + t/\tau)^{-1}, \tag{14b}$$

$\tau$ represents the time scale within the atomic cloud reaches
twice its initial volume or half its initial density. The use of
Eqs. (2) and (9) leads to the evolution of the mean radius $\langle r \rangle$ as
In order to improve our description of the experimental conditions, we have performed an analysis of the Fokker-Planck equation with an initially gaussian shaped cloud density. The cloud shape at \( t = 0 \) is chosen as

\[
\frac{\langle r \rangle}{\sigma} = \frac{3}{4} R(t) = \frac{<r_0}(1 + t/\tau)^{1/3}.
\]

(15)

In order to improve our description of the experimental conditions, we have performed an analysis of the Fokker-Planck equation with an initially gaussian shaped cloud density. The cloud shape at \( t = 0 \) is chosen as

\[
n(r, t = 0) = n_0(r) = \frac{N}{(\pi/2)^{3/2} \sigma^3} e^{-2r^2/\sigma^2},
\]

(16)

where \( \sigma \) is the 1/\( e^2 \) radius of the distribution. The solution was performed numerically. As shown in Fig. 4(a), the cloud shape changes in time. The final shape is no longer a gaussian function. The knowledge of the density, \( n(r, t) \), gives the mean radius, \( \langle r \rangle \), of the cloud. The time evolution of \( \langle r \rangle /\sigma \) is given in Fig. 4(b) simultaneously with the relative mean volume \( (\langle r \rangle/\sigma)^3 \). The results show a linear evolution in time for the mean volume. The time constant is equal to \( \tau = 2.38a/[Kn_0(r=0)] \) where \( n_0(r=0) \) is the initial atomic density at the center of the cloud.

D. Experimental data analysis

The results of the previous section are used to fit the experimental data. The fitted curves are plotted in Fig. 3. With the experimental parameters, the cloud is not perfectly spherical, so we have introduced two time constants, one for the cloud evolution along the coil x axis and another along the z axis [Fig. 3(b)]. The data for \( V \) in Fig. 3(c) and for \( n \) in Fig. 3(d) are fitted for time less than 150 ms with the laws of Eq. (14). On this time scale, the atom number can be considered as constant. The latter condition is implicitly assumed in our model. The slope obtained from the linear fit in Fig. 3(c) is the expansion rate \( KN/\alpha \). With the measured atom number we deduce the ratio \( K/\alpha \). A fit of the density plot [Fig. 3(d)] by the Eq. (14b) gives another value of \( K/\alpha (3.210^{-15} \text{m}^3/\text{s}/\text{atom}) \) which differs from the previous one by only 10%.

In order to evaluate the quantity \( K/\alpha \), we have used a model based on the two-level atom interacting with an intense laser field to calculate \( K \) and the results of Ref. [16] to evaluate \( \alpha \). According to Eq. (3) an evaluation of \( K \) requires the knowledge of the absorption cross sections, \( \sigma_L \) for the laser light absorption, and \( \sigma_R \) for the scattering light re-absorption. All the details of the calculation are given in the Appendix. To calculate \( \sigma_R \) we have used the dressed atom approach [20], which leads to an analytical expression for \( \sigma_R \). The calculation performed by Sesko, Walker, and Wierman [3] uses an exact expression for emission and absorption spectra, and takes into account the space modulation of the laser light in the trap but does not include a multi-level description of the atom. For \( \delta/\Gamma = -3.7 \), and \( I/I_{s} = 17.5 \), calculations give \( \sigma_L/\sigma_0 = 0.014 \), \( \sigma_R/\sigma_0 = 0.042 \), and \( K = 8.3310^{-15} \text{Nm}^2/\text{atom} \). A rigor evaluation of \( \alpha \) requires a 3D calculation of the cooling process in the case of large saturation parameters and a multilevel atom. To our knowledge only 1D or low-saturation cases have been treated for the \( \sigma_L - \sigma \) configuration. Nevertheless, the results of Ref. [16] give a reasonable order of magnitude estimate for the friction parameter \( \alpha \). Formula (5.14) of Ref. [16] gives \( \alpha = 310^{-21} \text{Ns/m} \). We thus get \( K/\alpha = 2.810^{-15} \text{m}^3/\text{s}/\text{atom} \), which is in good agreement with the measurement.

As mentioned above, the ratio \( K/\alpha \) depends only on the laser characteristics (detuning and intensity). To check this property we have varied the cloud atom number, i.e., by changing the magnetic gradient of the magneto-optical trap. As shown in Fig. 5, the slope of the inverse density \( 1/n \) versus time is nearly independent of the initial atom number and initial density for magnetic field gradients in the range 11.5–21 Gauss/cm. Some discrepancies are observed for small magnetic gradients because the clouds are more dilute and larger.

Because coefficient \( K \) varies as \( 1/\delta^4 \) at large laser detuning (\( \delta \gg \Gamma \)), (see the appendix) and the friction coefficient \( \alpha \) varies as \( 1/\delta \), a variation of \( K/\alpha \) as \( 1/\delta^3 \) is expected. Figure 6 shows the time evolution of the density for a set of laser detunings. For \( \delta/\Gamma = -1.5 \), -3.7, -3.8 (not shown in the figure), -5.4 and -6.8 (not shown in the figure) the measurement of the \( K/\alpha \) coefficient is in a good agreement with the prediction, either for the magnitude or for the scaling with the detuning. For a laser detuning \( \delta/\Gamma = -2.5 \) (circles in Fig. 6) the expansion rate is more than ten times lower than the expected value. With this detuning the number of trapped atoms is at the maximum (about \( 8.10^7 \) to \( 10^8 \) atoms). The small expansion rate can be explained neither by collision effects as suggested by Ref. [7] nor
by a nonlinearity of the friction force \[12,16,18\], because of the order of magnitude and of the sign of these effects. A possible explanation is a maximum value of the friction coefficient. The evaluation of the friction coefficient we have used assumes a cooling process in the low-saturation regime. In our case, the laser light is intense and the friction coefficient could be larger, as it has been observed in Ref. \[21\].

This study, concerning a one-dimensional cooling experiment using intense laser light, shows an increase of the capture and the friction coefficient. The explanation is based on coworking of the Doppler cooling and Sisyphus cooling.

IV. COOLING OF THE EXPANDING BRIGHT MOLASSES

In the case of spherical symmetry, because of the small velocity damping time, the radial velocity of the atom located at the position \( r \) is given by \( \nu (r) = F(r)/\alpha \). Using Eq. (3) and Gauss’s theorem for the homogeneous spherical cloud, the force is \( F(r) = Knr/3 \). The force varies linearly with the position variable, thus the velocity distribution is homogeneous. The maximum value of the radial velocity is obtained at \( r = R(t) \) and is equal to

\[
\nu_M = \frac{K N}{4 \pi R(t)^3}.
\]

The mean square velocity is therefore: \( \langle \nu^2 \rangle = 3 \nu_M^2/5 \). Using the following definition of the temperature \( k_B T_{\text{MS}} = m \langle \nu^2 \rangle \), Eqs. (16) and (14), we obtain the time evolution of the cloud temperature as \( 1/R(t)^4 \) and

\[
T_{\text{MS}} = \frac{T_0}{(1 + t/\tau)^{4/3}},
\]

where \( T_0 \) is the temperature at \( t = 0 \), and \( \tau \) is the time constant defined in the previous section. We note that the quantity \( T_{\text{MS}} \nu_M^{4/3} \) is constant during the cloud expansion.

The laws obtained for the cloud expansion and the temperature have to be restricted to the multiple scattering regime. The momentum diffusion has been neglected. The treatment is valid as the mean force \( \langle F \rangle \) is larger than \( D/R(t) \), where \( D \) is the spatial diffusion constant [1,12]. The condition can be expressed as

\[
R(t) < \frac{3}{16 \pi} \frac{K N}{\alpha D}.
\]

In our experimental situations this condition is satisfied for clouds containing \( 10^7 \) atoms with radius less than a few millimeters.

Figure 7 shows \( T_{\text{MS}} \) obtained for \( \delta \Gamma = 3.7 \) and \( \hbar \Gamma_s = 20 \), \( N = 3.5 \times 10^6 \), and \( n_0 = 1.4 \times 10^{10} \) atom/cm\(^3\). The decrease of the temperature is observed as the cloud size increases. We check, by the temperature values, that the experiment is done in the sub-Doppler regime. The data curve is well fitted by Eq. (18), with \( \tau = 26 \) ms and \( T_0 = 133 \) \( \mu \)K. The corresponding evolution of the cloud density (not shown here) has been fitted by Eq. (14b) with a time constant equal to 15 ms.
With such a system, we have observed the expansion of the cloud. The phenomenon is characterized by a linear variation of the cloud volume in time. We have checked the variation of the time constant with the laser parameter, in particular with the laser detuning. Simultaneously to the expansion, the cloud is cooled down. The cooling process can be qualitatively understood by the escape of the “blue” photon out of the cloud. These photons, staying in the cloud, are responsible of the heating of the neighbor atoms. With these observations we can conclude that the expansion occurs with the conservation of the quantity $T_{\text{MS}}V^{4/3}$.

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APPENDIX

We present details of the calculation of absorption cross section $\sigma_L$, the reabsorption cross section $\sigma_R$ and the coefficient $K$, in the case of the two-level atom, and using the dressed atom approach.

We consider a two-level atom illuminated by an intense quasi resonant monochromatic light. The absorption cross section is

$$\sigma_L = \frac{\sigma_0}{1 + I/I_s + 4\delta^2/\Gamma^2},$$

where $\sigma_0$ is the absorption cross-section at resonance, $I$ is the laser intensity, $I_s$ is the saturation intensity of the transition, $\delta$ is the laser light detuning and $\Gamma$ is the natural width of the excited level. For the rubidium atom $D_2$ line, $\sigma_0 = 1.5 \times 10^{-9} \text{ cm}^2$, $I_s = 1.62 \text{ mW/cm}^2$, $\Gamma/2\pi = 5.89 \text{ MHz}$.

The reabsorption cross section is obtained by the evaluation of the absorption of the emitted light by the neighboring atoms. If $S(\omega)$ denotes the emission spectral density and $\sigma_A(\omega)$ the absorption spectra of the atom illuminated by the laser light, the re-absorption cross section is equal to

$$\sigma_R = \int \sigma_A(\omega)S(\omega)d\omega.$$

The fluorescence spectra, and the absorption spectra, are those of the atom in the presence of the intense laser light [4–6,20]. They can be expressed with the approach of Refs. [4–6] or by using the dressed atom approach [20]. In the dressed atom formalism, the normalized emission and absorption spectra are given by

$$S(\omega) = \frac{(c^2 - s^2)^2}{c^4 + s^4}L(\omega - \delta, 0) + \frac{4c^4s^4}{c^4 + s^4}L(\omega - \delta, \Gamma_p)$$

$$\times c^2s^2\{L(\omega - \delta - \Omega_G, \Gamma_c) + L(\omega - \delta + \Omega_G, \Gamma_c)\}.$$
and
\[ \sigma_A(\omega) = \sigma_0 \frac{\pi \Gamma}{c^2 + s^2} \times \left\{ c^4 L(\omega - \delta - \Omega_G, \Gamma_c) \right\} - s^4 L(\omega - \delta + \Omega_G, \Gamma_c), \]

where \( \Omega = \Gamma \sqrt{2L} \) is the Rabi frequency, \( \Omega_G = \sqrt{\delta^2 + s^2} \) is the generalized Rabi frequency, \( c^2 = \cos^2(\theta) \), \( s^2 = \sin^2(\theta) \), with the angle \( \theta \) defined by \( \tan(2\theta) = \Omega/\delta \). The photon frequency \( \omega \) is referred to the atomic transition. \( L(\omega - \omega', \Gamma) \) is the Lorentzian normalized function, centered at \( \omega = \omega' \) with a width equal to \( 2\Gamma \), and \( L(\omega - \omega', 0) \) its limit when \( \Gamma \) tends to zero, namely the delta function. \( \Gamma_p \) and \( \Gamma_c \) are given by
\[ \Gamma_p = \Gamma(c^4 + s^4) \]
and
\[ \Gamma_c = \Gamma \frac{1 + 2c^2s^2}{2}. \]

These formula are valid if the generalized Rabi frequency is much larger than the natural width of the transition. In our case the condition is not fulfilled at small laser detuning and weak laser intensity.

The reabsorption cross section is then easily deduced and is equal to:

\[ \sigma_R = \sigma_0 \frac{\Gamma(e^2 - s^2)^2}{2(c^4 + s^4)} \left( \frac{c^4 - s^4}{\Omega^2 + \delta^2 + \Gamma_c^2} \right) \times \left\{ \frac{\Gamma(e^2 + s^2)}{2(c^4 + s^4)} \left( \frac{\Gamma_c}{\Gamma_c + \Gamma_p} \right) \frac{(\Gamma_c + \Gamma_p)^2}{2} + \frac{c^2s^2}{\Omega^2 + \delta^2 + \Gamma_c^2} \right\}. \]

The coefficient \( K \) is given by \( K = (\sigma_R - \sigma_L)/c \). Plots of \( \sigma_R \) and \( K \), versus the laser detuning are given in Fig. 8, for a set of laser intensities. Calculations done with the Mollow’s approach [4–6] give the same results for large values of laser detuning and for large intensities. Discrepancies are observed for very small laser detunings \( (|\delta| < \Gamma) \) which is achieved our experiments.

Asymptotic laws for previous formula are given. For large laser detuning, \( |\delta| \gg |\Omega| \) one gets \( \theta \approx |\Omega/(2\delta) | \) and the previous relations give
\[ \sigma_L = \sigma_0 \frac{\Gamma^2}{4\delta^2}, \quad \sigma_R - \sigma_L = \sigma_0 \frac{\Omega^2}{8\delta^2}, \quad \text{and} \quad K = \frac{\sigma_0^2 \lambda_s}{c} \frac{\Omega^4}{16\delta^2}. \]

The other limit is obtained for large laser intensity \( \Omega \gg \delta \gg |\Gamma| \). In this case, one has \( \cos(2\theta) \approx \Omega |\Omega| \) and \( \sin(2\theta) \approx 1 - \delta^2/(2\Omega^2) \) and a Taylor’s expansion leads to
\[ \sigma_L = \sigma_0 \frac{\Gamma^2}{2\Omega^2}, \quad \sigma_R - \sigma_L = \sigma_0 \frac{\delta^2}{6\Omega^2}, \quad \text{and} \quad K = \frac{\sigma_0^2 \lambda_s}{c} \frac{\delta^2}{6\Omega^2}. \]