CHILDREN, TIME ALLOCATION, AND CONSUMPTION INSURANCE

Not for Publication Online Appendix*

1 Appendix 1: Derivation of MRS equations

In this appendix we show how to derive equations (7) and (8) in the main text. The same derivation applies to equations (9) and (10).

First, take the first order conditions of the problem in equations (4) in the text, using the utility function in (6). Assuming interior solution, the marginal rate of substitution (MRS) between L_2 and L_1 can be written (omitting household and time subscripts for simplicity) as:

$$L_2 = \left(\frac{W_1}{W_2} \frac{\exp\left(\tilde{\phi}_{L_2}\right)(1/\varphi_{L_2} - 1)}{\exp\left(\tilde{\phi}_{L_1}\right)(1/\varphi_{L_1} - 1)} L_1^{1/\varphi_{L_1}} \right)^{\varphi_{L_2}}, \tag{A1}$$

while the MRS between L_2 and $\tilde{C}$ can be written as:

$$L_2 = \left[\frac{\exp\left(\tilde{\phi}_C\right)}{\exp\left(\tilde{\phi}_{L_2}\right)(1/\varphi_{L_2} - 1)} \tilde{C}^{-1/\eta} \left(\exp\left(\tilde{\phi}_{L_1}\right) L_1^{1-1/\varphi_{L_1}} + \exp\left(\tilde{\phi}_{L_2}\right) L_2^{1-1/\varphi_{L_2}} \right)^{\rho_L} W_{2,t} \chi_t (1-\mu_t) Y_t^{-\mu_t} \right]^{-\varphi_{L_2}}. \tag{A2}$$

Taking logs of both sides of (A1) and of (A2) yields:

$$l_2 = \varphi_{L_2}(w_1 - w_2) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \varphi_{L_2}\left[\tilde{\phi}_{L_2} - \tilde{\phi}_{L_1}\right] + \varphi_{L_2} \log\left(\frac{(1/\varphi_{L_2} - 1)}{(1/\varphi_{L_1} - 1)} \right) \tag{A3}$$

$$\begin{aligned} l_2 = {} & -\varphi_{L_2} w_2 + \mu \varphi_{L_2} y + \frac{\varphi_{L_2}}{\eta} c - \varphi_{L_2} \rho_L \log\left(\exp\left(\tilde{\phi}_{L_1}\right) L_1^{1-1/\varphi_{L_1}} + \exp\left(\tilde{\phi}_{L_2}\right) L_2^{1-1/\varphi_{L_2}} \right) \\ & -\varphi_{L_2}\left(\tilde{\phi}_C - \tilde{\phi}_{L_2}\right) + \varphi_{L_2} \log(1/\varphi_{L_2} - 1) - \varphi_{L_2} \log\left[\chi(1-\mu)\right] \end{aligned} \tag{A4}$$

*Blundell: Department of Economics, University College London, and Institute for Fiscal Studies, Gower Street, London WC1E 6BT, UK (email: r.blundell@ucl.ac.uk); Pistaferri: Department of Economics, Stanford University, Stanford, CA 94305 (email: pista@stanford.edu); Saporta-Eksten: The Eitan Berglas School of Economics, Tel Aviv, Israel, and Department of Economics, University College London (email: itaysap@post.tau.ac.il).

where lower case letters are used for logs, and $c = \log \tilde{C}$.

As explained in the text, we assume a loglinear form for the (scaled) preference shifters (see main text for details):

$$\tilde{\phi}_x = \phi_x^{nk} + \phi_x^k z + \varepsilon_x \tag{A5}$$

To ease notation, write $\phi_x^z = \phi_x^{nk} + \phi_x^k z$. This implies that equation (A3) can be written as

$$l_2 = K_0 + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \varphi_{L_2}(w_1 - w_2) + \varphi_{L_2}(\varepsilon_{L_2} - \varepsilon_{L_1}), \tag{A6}$$

where

$$K_0 = \varphi_{L_2}\left(\phi_{L_2}^z - \phi_{L_1}^z\right) + \varphi_{L_2} \log\left(\frac{(1/\varphi_{L_2} - 1)}{(1/\varphi_{L_1} - 1)}\right)$$

Given a set of instruments which are uncorrelated with $(\varepsilon_{L_2} - \varepsilon_{L_1})$, this delivers moment condition (7) in the main text.

To see how to obtain equation (8) in the main text, plug (A5) into (A4) to get

$$l_2 = Q - \varphi_{L_2} w_1 + \mu\varphi_{L_2} y + \frac{\varphi_{L_2}}{\eta} c - \varphi_{L_2}\rho_L M - \varphi_{L_2}(\varepsilon_C - \varepsilon_{L_2}), \tag{A7}$$

where

$$\begin{aligned} Q &= -\varphi_{L_2}\left(\phi_C^z - \phi_{L_2}^z\right) - \varphi_{L_2}\log\left[\chi(1-\mu)\right] + \varphi_{L_2}\log\left(1/\varphi_{L_2} - 1\right) \\ M &= \log\left(\exp\left(\tilde{\phi}_{L_1}\right) L_1^{1-1/\varphi_{L_1}} \left(1 + \frac{\exp\left(\tilde{\phi}_{L_2}\right)}{\exp\left(\tilde{\phi}_{L_1}\right)} \frac{L_2^{1-1/\varphi_{L_2}}}{L_1^{1-1/\varphi_{L_1}}}\right)\right) \end{aligned}$$

To complete the derivation we need to simplify $\frac{\exp(\tilde{\phi}_{L_2})}{\exp(\tilde{\phi}_{L_1})}$ in M. To do that, note that we can use (A1) again, where $\frac{\exp(\tilde{\phi}_{L_2})}{\exp(\tilde{\phi}_{L_1})} = \frac{(1/\varphi_{L_1}-1)W_{2,t}}{(1/\varphi_{L_2}-1)W_{1,t}} \frac{L_1^{-1/\varphi_{L_1}}}{L_2^{-1/\varphi_{L_2}}}$. Hence:

$$\begin{aligned} M &= \log\left(\exp\left(\tilde{\phi}_{L_1}\right) L_1^{1-1/\varphi_{L_1}} \left(1 + \frac{(1/\varphi_{L_1} - 1) W_2 L_2}{(1/\varphi_{L_2} - 1) W_1 L_1}\right)\right) \\ &\approx \phi_{L_1}^z + \varepsilon_{L_1} + \left(1 - 1/\varphi_{L_1}\right) l_1 + \frac{\varphi_{L_2}\left(1 - \varphi_{L_1}\right) W_2 L_2}{\varphi_{L_1}\left(1 - \varphi_{L_2}\right) W_1 L_1} \end{aligned}$$

Substitute M back into (A7) to obtain:

$$l_2 \approx K_1 - \varphi_{L_2} w_2 + \mu\varphi_{L_2} y + \frac{\varphi_{L_2}}{\eta} c + \frac{\varphi_{L_2}}{\varphi_{L_1}}\rho_L\left(1 - \varphi_{L_1}\right) l_1 - \varphi_{L_2}\rho_L \frac{\varphi_{L_2}\left(1 - \varphi_{L_1}\right)}{\varphi_{L_1}\left(1 - \varphi_{L_2}\right)} \frac{W_2 L_2}{W_1 L_1} - \varphi_{L_2}\left(\varepsilon_C - \varepsilon_{L_2} - \rho_L \varepsilon_{L_1}\right),$$

where

$$K_1 = Q - \varphi_{L_2}\rho_L \phi_{L_1}^z$$

Given a set of instruments which are uncorrelated with $(\varepsilon_C - \varepsilon_{L_2} - \rho_L \varepsilon_{L_1})$, this delivers moment condition (8) in the main text.

2 Appendix 2: Model Solution

We need to solve the optimization problem described in equation (4) in the main text. In recursive formulation, our state variables include A, F_1, F_2, u_1, u_2 and age. Presence of young kids is a deterministic function of age. Leisure preference shifters for the wife (ε_{L_2}) are modeled as two types, where each household draws a type with probability 0.5, and the type is constant over the life cycle (hence we solve the model twice, once for each type). Choice variables include next period assets A', as well as L_1, L_2, T_1 and T_2. Labor supply is then implicitly defined by $H_j = \bar{L} - L_j - T_j$. Because of the presence of fixed costs on female employment, the labor supply choice incorporates a discrete employment decision for the wife.

We discretize the state space using a 30 point grid for assets, 11 points grids for F_j and 3 points grids for the transitory shocks u_j.[1] Because the model includes two unit root processes (F_1 and F_2), their grids are non-stationary, and the value of grid points is increasing over age according to the variances of permanent shocks to the husband and to the wife.

To solve the policy functions for each point on the grids we rely on the fact that at each age, within a discrete choice decision, the objective function is concave, and the choice set is convex.[2] This implies that for each of the two discrete options (wife employed/wife not-employed) we can use Kuhn-Tucker (KT) conditions to solve for the optimal policy. Solving the KT system ensures that we correctly solve for corners (in labor supply for each earner, and in assets, given the liquidity constraint). We solve the model by backward induction, starting from the last period, comparing the value under each discrete choice option, and choosing the policy functions associated with the discrete choice that gives higher utility.

Using the solution to the value function, we simulate the choices of 100,000 households for which we calculate the moments described in section (5.3) in the paper, and we match these moments to the data moments. The SMM estimation procedure is conducted using a diagonal weighting matrix with the inverse of the variance of the moments from the data on the main diagonal. For the calculation of standard errors, we apply numerical derivatives to the moments. We calculate each derivative on two sides of the point estimates, and for two magnitudes (5% and 10%) and take the derivative to be the average of the four calculations.

[1] Results with 5 point grids for u_j are indistinguishable from results with 3 point grids.

[2] With the discrete choice, one cannot prove anymore that the expectation of the value function is concave. However, the model has 4 stochastic components in the wage process at the household level, smoothing the value function, so that in practice this is not an issue in the solution.

3 Appendix Table

Table 1: **Robustness for MRS Estimates**

	(1)	(2)	(3)
φ_{L_1}	0.097 (0.018)	0.095 (0.018)	0.095 (0.018)
φ_{L_2}	0.109 (0.013)	0.112 (0.014)	0.112 (0.014)
ρ_L	0.265 (0.098)	0.231 (0.097)	0.236 (0.098)
η	0.924 (0.102)	1.130 (0.212)	1.059 (0.184)
Instruments	lagged wages, lagged diff. of leisure times and of consumption	lagged wages, lagged diff. of leisure times and of consumption	lagged wages, lagged diff. of leisure times and of consumption dummy for low liq. assets, non-labor income
Controls	None	Education and Cohort	Education and Cohort
Observations:	4,982	4,982	4,982

Notes: The parameters are estimated by GMM. Standard errors clustered by household in parenthesis.