Abstract

Economies routinely experience a variety of sector-specific supply and demand shifts. Yet, the distributional welfare consequences of these shifts are not well understood. We address this gap by developing an analytical framework that jointly integrates supply-side and demand-side heterogeneity without imposing specific functional forms on consumption and production. This enables us to identify the key forces that shape the distributional welfare impact of sector-specific supply and demand shifts—in terms of consumer preferences and sectoral production functions. We estimate key parameters and quantify the heterogeneous welfare effects of sectoral shifts, revealing significant variation in their impact.
1. Introduction

Economies routinely experience a myriad of sector-specific supply and demand shifts. Prime examples include sector-specific technical change (as in Baumol cost disease), preference shifts (e.g., Covid "preferences" for durables) and changes to public sector expenditures (e.g., military buildup). While the importance of such supply and demand shifts is well established, their distributional welfare consequences are not well understood.

In this paper, we propose a unified framework that characterizes the determinants of distributional welfare effects of such sectoral shifts across workers of different skills. By analyzing theoretically and quantifying empirically the interplay between consumer preferences and sectoral production functions within a general equilibrium framework, we shed light on the mechanisms through which sectoral shifts affect wages, goods prices, and hence, ultimately, welfare outcomes.

Our framework, which does not impose any functional form restrictions on preferences and production, integrates supply-side and demand-side heterogeneity, which, as we show below, is crucial in understanding welfare effects. The framework identifies analytically four key factors influencing the welfare impact of sectoral change: income elasticities of consumption goods, consumption substitution patterns, consumption shares of the different goods, and the vector of sectoral skill intensities. Our empirical findings indicate that sectoral shifts impact high and low-skilled workers’ welfare differently, and these differential effects are contingent upon the aforementioned factors.

We begin our analysis in Section 2 where we develop a general multi-sectoral model featuring workers of heterogeneous skills. Workers derive utility from consuming a bundle of goods and can have non-homothetic preferences. On the production side, goods are produced in different sectors which vary in their skill intensity. This model is then applied to study the distributional impact of sectoral technical change (Sections 3 and 4), changes to public sector demand (Section 5.1), and sectoral demand shifts driven by preference changes (Section 5.2). In what follows, we describe these applications in detail.

Our first application considers the impact of sectoral technical changes, which have long been acknowledged as pivotal for economic growth. As vividly described in Harberger’s 1998 AEA Presidential Address (Harberger (1998)), growth exhibits characteristics similar to that of "mushrooms" rather than "yeast"; That is, variation in sectoral total factor productivity (TFP) across different
We begin our analysis by asking in Section 3 what determines the distributional welfare impact of sectoral technical change. Our first theorem shows that this welfare impact can be decomposed into two distinct effects. The first effect, which we refer to as the "Engel effect", stems from the fact that with non-homotheticities, consumers of different income levels consume different bundles with different expenditure shares. As a result, price changes stemming from sectoral technical changes, benefit deferentially agents with different income. This effect is in the spirit of the inflation inequality literature (for a survey see, e.g., Jaravel (2021a)).

The Relative Wage effect, our second effect, arises because sectoral technical change shifts demand for goods. Solving analytically for the equilibrium elasticity of the relative wage with respect to a sectoral technical change, our second theorem shows that the relative wage is determined by the extent to which demand moves towards or away from high skilled sectors. Put differently, when technical change causes the economy to be more high-skill intensive the high-skill wage relative rises. What determines how demand shifts across sectors following sectoral technical change? Our third theorem shows that income elasticities of consumption goods, the patterns of consumption substitution in the economy, sectoral expenditure shares, and the relative skill intensity of the sector experiencing technical change are the key determinants of how demand shifts and hence of the Relative Wage effect.

The key determinants of the welfare effect derived in our third theorem can be empirically estimated. Hence, our framework allows us to directly calculate the differential welfare effect using these model primitives in Section 4. As discussed above, our framework allows to do so without imposing structural restrictions on the consumption and production sides; the analytical results highlight the importance of allowing for flexibility in key model primitives, and especially substitution patterns in the demand system.

Relying on the analytical results, we use data from the Consumer Expenditure Survey (CEX) to estimate an Almost Ideal Demand System (AIDS) from which we recover the key price and income elasticities. We use the Current Population Survey (CPS) to recover labor shares by skill level, and KLEMS data for sectoral productivity dynamics. Finally, we use an input-output matrix to map the measures obtained from the CPS and KLEMS to consumption categories.

Armed with these data, we conduct two empirical exercises using two different granulation of consumption categories. The first examines the welfare consequences of sectoral technical
changes using the consumption categorization common in the structural transformation literature (see, e.g., Buera and Kaboski (2012), Herrendorf, Rogerson and Valentinyi (2014), and Comin, Lashkari and Mestieri (2021)) – Agriculture, Manufacturing, and Services. Then, given our interest in sectoral technical change and our analytic results emphasizing the importance of substitution patterns, in our second empirical exercise we use a finer goods categorization to capture more nuanced substitution patterns.

We discuss our five main quantitative findings in Section 4.3. First, sectoral technical change affects welfare of high and low-skilled workers differently. Second, this differential effect varies across sectors in both magnitude and sign: using the finer goods categorization, the differential welfare effect between high and low skilled workers ranges between a negative 20% and a positive 70%.

Third, while both the Relative Wage effect and the Engel effect are quantitatively important in determining the overall distributional impact of technical change, it is the former that plays a more important role.

Fourth, our estimates show that gross complementarities are prominent in the substitution matrix. Consistent with our theoretical analysis regarding the role of complementarities, we find a general pattern by which, in response to a positive productivity change in low-skill intensive sectors the high-skilled relative wage increases, while in contrast, a positive productivity change in high-skill intensive sectors reduces the high-skilled relative wage.

In our final quantitative analysis, we calculate the overall differential welfare impact of sectoral technical change on high versus low workers given the realized empirical changes in sectoral TFP over different decades. We find that the relative welfare gain of high- versus low-skilled workers varies over these decades depending on the realization of the sectoral TFP changes.

As discussed above, our analytical framework is general enough to be applicable to a broad range of scenarios. To illustrate this, we analyze two cases of demand-driven sectoral shifts: changes in public sector expenditure as well as changes in consumer preferences.

In Section 5.1 we augment our baseline model to include a public sector and derive analogous analytical results to those in Section 3. We show that the distributional welfare impact of shifts in public sector expenditures operates only through the Relative Wage effect – i.e. there is no Engel effect. This stems from the fact that unlike in the case of technical change, shifts in public sector expenditures do not affect directly the relative cost of good production, and thus all relative
price changes are due to changes in relative wages. Relative to the case of technical change, in the case of public expenditures an additional factor determining the change in relative wages (and hence the distributional effect) is the skill-intensity of the public sector relative to the equilibrium skill intensity of the private sector. We then illustrate the distributional welfare impact of public expenditures for different sizes of the public sector and varying levels of public sector skill intensity.

In Section 5.2 – our third and final application – we examine the distributional welfare impact of exogenous changes in consumer preferences across goods. As in the case of public expenditures, we show that this distributional impact operates only through the relative wage effect. We then analytically derive the primitives that determine the distributional consequences of such preference changes. Furthermore, we show that the skill intensity of the sectors to which demand shifts as a result of the preference change is again an important factor in determining the welfare consequences. Guided by this analysis, we then estimate distributional welfare consequences of preference shifts.

Related Literature Our framework identifies and quantifies the mechanisms through which sectoral demand and supply shifts affect the distributional welfare in the economy. As such it is linked to five main strands of literature.

Within this scope, the segment of our study that examines the distributive effects of sector-specific technological change is particularly aligned with three of these literature streams. First it is related to the structural change literature, which analyzes how technical change at the aggregate level causes sectoral reallocation of economic activity between manufacturing, services, and agriculture (for recent contributions, see, e.g., Kongsamut, Rebelo and Xie (2001), Buera and Kaboski (2012), Herrendorf, Rogerson and Valentinyi (2013), Matsuyama (2019), Baqee and Burstein (2021), Comin, Lashkari and Mestieri (2021), Alder, Bopart and Muller (2022) as well as the review in Herrendorf, Rogerson and Valentinyi (2014)). In contrast to this literature, which focuses on the determinants of structural change, we provide a general framework that uncovers the general equilibrium mechanisms governing the differential welfare impact of sectoral technical changes. Clearly, our framework also enables an analysis of the distributional welfare impact of aggregate technical change – by considering the scenario where all sectoral technical changes are equal. Within the structural change literature, closest to our paper is Buera et al. (2022), which explores the implications of structural change to the skill premium. In addition to the focus on a different motivating question, the Buera et al. (2022) paper and our paper take a very different modelling approach. While they employ specific functional forms – both on the preferences and on the
production side – our analytical framework is functional-form free. This allows us to derive analytical results that clarify the forces driving changes to relative wages in the economy in terms of model primitives; among others, these include the crucial role of flexible complementarity and substitutability patterns in demand across goods and their interaction with heterogeneity in skill-intensity across sectors.

Second, the part of the analysis regarding sectoral technical change is naturally related to the "Baumol cost disease" literature as first discussed by Baumol and Bowen (1965). Using industry level data, Nordhaus (2008) verifies a key prediction of the Baumol effect, whereby technologically stagnant sectors exhibit rising relative prices and declining relative real outputs. These papers focus on a reduced form empirical analysis of the Baumol effect. In contrast, we analytically analyze and empirically estimate the heterogeneous welfare implications of technical change.

Our section dedicated to sectoral technological change relates to a third body of literature. This particular stream examines the impact of differential price changes on the welfare distribution in contexts where non-homothetic preferences lead consumers with different income levels to have differential consumption baskets. The inflation inequality literature, for instance, delves into how distinct inflation rates across goods, combined with diverse consumption baskets among households, shape welfare outcomes (see Jaravel (2021a) for a comprehensive survey). Similarly, the international trade literature examines the influence of trade on the relative prices of exported and imported goods and its implications on welfare when accounting for heterogeneous consumption baskets among households (for such a mechanism see Fajgelbaum and Khandelwal (2016)). Our analysis shows that sectoral technical change also impacts the welfare distribution through such a price mechanism: Sectoral technical change affects the relative prices of goods which, given differential consumption shares, will affect the welfare distribution. As explained above, our decomposition result also exhibits an important second mechanism through which sectoral technical change affects the welfare distribution – namely through its impact on relative wage levels. As such, our findings emphasize that understanding the overall consequences of sectoral changes on the welfare distribution mandates a model that integrates both supply-side and demand-side heterogeneity.

Moving beyond the realm of sectoral technological change, our analysis also intersects with two additional literature strands, each distinct in its focus and contribution. First, a substantial body

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1Hartwig (2011) shows similar results for Europe.
of empirical literature investigates the effects of government expenditure changes on the economy (See Ramey (2016), for a comprehensive survey). Recent studies employing HANK models have delved deeper into analyzing the distributional welfare impacts of fiscal stimulus. Our analysis emphasizes that understanding the distributional effects of changes in government expenditure requires considering not just the expenditure level but also its composition, as this affects labor demand and consequently, relative wages in the economy. Second, consumer responses during the COVID-19 pandemic have spurred interest in sector-specific demand shifts (See for example Cox et al. (2020) and Beraja and Wolf (2021)). Our analysis underscores the distributional welfare impacts of such preference-driven demand shifts.

2. Model

We utilize a static multi-sector model, which enables us to emphasize the fundamental economic mechanisms that shape the distributional welfare impacts of sectoral technical change. We impose minimal constraints on preferences and production, enabling us to identify the essential forces at play without confining ourselves the analysis to restrictive parametric forms. This approach allows us not only to demonstrate the underlying factors driving the distributional welfare impact of sectoral technical change, but also clearly pinpoints the elements that need to be estimated.

2.1. Setup

The general structure of the model is as follows. Workers derive utility from the consumption of a bundle of \( N \) different goods. The production of each of the \( N \) goods is performed by perfectly competitive firms that use two inputs – high- and low-skilled labor – and maximize profits. Finally, we assume that workers can move freely across sectors. Consequently, there are only two wages in the economy, one for each skill level. In what follows we formally present the model.

2.1.1. Production

The model is comprised of \( N \) sectors, each producing a different good. Sector \( i \) produces \( Y_i \) goods using the following constant returns to scale production function:

\[
Y_i = A_i F^i(L_i, H_i), \forall i \in N,
\]
where $L_i$ and $H_i$ are low- and high-skilled labor inputs respectively, and $A_i$ is Hicks neutral productivity parameter. The representative firm in sector $i$ solves:

$$\max_{L_i, H_i} P_i Y_i - W_L L_i - W_H H_i.$$ 

Crucially, the production function $F$ is indexed by $i$ as well, allowing for differential production elasticities of the two inputs across sectors.

### 2.1.2. Workers

There are two types of workers, $l$ and $h$. Types are fixed (a worker cannot switch type), with a mass of $L$ and $H$ in the economy of $l$ and $h$ type workers respectively. We normalize the population so that $L + H = 1$.

Workers derive utility from the bundle of goods and supply work inelastically. The maximization problem for an individual of type $j \in \{l, h\}$ is thus:

$$\max_{C_{j1}, \ldots, C_{jN}} U(C_{j1}, \ldots, C_{jN})$$

$$\text{s.t. } \sum_{i=1}^{N} P_i C_{ji} = W_j,$$

where $P_i$ is the price of good produced by sector $i$, and $C_{ji}$ is the consumption of good $i$ by a worker of type $j$. Throughout, we normalize the price of the sector experiencing the technical change to 1. Importantly, we do not restrict the utility function to be homothetic. Thus, expenditure shares of each type $j \in \{l, h\}$ worker for good $i$ are denoted by $s_i^j \equiv s_i(W_j, \mathbf{P})$, and depend on the worker’s wage, $W_j$, and the price vector $\mathbf{P}$.

### 2.1.3. Equilibrium

In equilibrium both firms and workers behave optimally and all markets clear. Given the lack of frictions, wages are equated across all markets, and hence workers are indifferent over which market to work in.

Formally, the market for low-skilled labor, denoted as $L$, is in equilibrium when the total supply
of low-skilled labor input matches its demand across the $N$ sectors, i.e.,

$$\sum_{i=1}^{N} L_i = L.$$ 

Analogously, for the high-skilled labor market $H$, equilibrium is achieved when

$$\sum_{i=1}^{N} H_i = H.$$ 

Furthermore, for a low-skilled worker, the budget constraint is satisfied when the expenditure on consumption across all goods equals the wage, i.e.,

$$\sum_{i=1}^{N} P_i C_{l,i} = W_L,$$

where $C_{l,i}$ denotes the consumption of good $i$ by a low-skilled individual. A similar equation holds for a high-skilled individual, where

$$\sum_{i=1}^{N} P_i C_{h,i} = W_H.$$ 

Lastly, for any sector $i \in N$, the production of its good must be equal to the sum of the demand from both low and high-skilled workers, i.e.,

$$LC_{l,i} + HC_{h,i} = Y_i.$$ 

Henceforth, we define $\alpha_i$ the equilibrium $L$ labor share of sector $i$: $\alpha_i = \frac{W_L L_i}{P_i Y_i}$.

### 2.2. Welfare

Our welfare measure is a variant of the Equivalent Variation measure. Specifically, given any change that affects equilibrium prices and wages, the induced welfare change is defined as the incremental income necessary to obtain the post-change utility at the pre-change prices and wages. Denoting the expenditure function given prices $p$ and utility level $u$ to be $e(p, u)$, then the change in welfare for type $j$ is given by:

$$\tilde{EV}^j = e(p_0, u_1^j) - W_0^j,$$
where $p_0$ are the pre equilibrium prices, $u_1^j$ is the post equilibrium utility for type $j$, and $W_0^j$ is the pre-change wage for type $j$.

Using Roy’s identity, a well-known result is that, locally, the welfare change can be decomposed into changes in wages and changes in good prices:

$$\widetilde{EV}^j = dW^j - \Sigma_i C_i(p_0, W_0^j) dp_i.$$  

Normalizing the welfare change by the pre-change wage, and using a circumflex to denote percent deviations from pre-change levels, we obtain that the percent change in welfare is given by:

$$EV^j := \frac{\widetilde{EV}^j}{W_0^j} = \widetilde{W}^j - \Sigma_i s_i(p_0, W_0^j) \tilde{p}_i.$$  \hspace{1cm} (1)  

Thus, the change in welfare equals the change in the real wage of type $j$ with a type-specific price index, where the weights are given by type-specific consumption shares $s_i(p_0, W_0^j)$.

It follows then from Equation (1), that the difference between types in the welfare change is given by:

$$\Delta EV := EV^H - EV^L = \left(\frac{W_H^j}{W_L^j}\right) - \Sigma_i \left(s^H_i(p_0, W_0^j) - s^L_i(p_0, W_0^j)\right) \tilde{p}_i.$$  \hspace{1cm} (2)  

That is, Equation (2) shows that the change in the welfare distribution over types can be decomposed into a change in the relative wage of high-versus low-skill workers, $\frac{W_H^j}{W_L^j}$, and the difference in the type-specific, share-weighted price changes.

3. Sectoral Technical Change: Analytical Results

Our key objective in this section is to study the distributional welfare effects of sectoral technical change. In what follows, we consider a Hicks neutral change in productivity to a given sector, $k \in 1 \ldots N$, denoted by $\Lambda_k$ (while holding all other productivities constant). We employ $\eta_{WH/WL, \Lambda_k}$ to express the elasticity of the relative wage in relation to the aforesaid change in sectoral productivity. Our analysis begins with the establishment of the following theorem.

**Theorem 1.** For a productivity change in any sector $k$ (while holding all other productivities constant), the difference between types in the elasticity of welfare to the productivity change is given by
\[ \Delta \eta_{EV, A_k} := \frac{EV^H}{A_k} - \frac{EV^L}{A_k} = (s^H_k - s^L_k) + \eta_{WH/WL, A_k} \left\{ 1 + \sum \alpha_i (s^H_i - s^L_i) \right\}, \] (3)

The proofs for this theorem, as well as all subsequent proofs, can be found in Appendix A.1.

Equation (3) shows that \( \Delta \eta_{EV, A_k} \), which measures the distributional welfare impact of the productivity change, can be decomposed into two main objects.

The first object, \( s^H_k - s^L_k \), stems from non-homotheticity, reflected in the difference in consumption shares over types. A positive technical change to sector \( k \) is directly reflected in a price decline for good \( k \), so that the type that has a larger consumption share of that good will experience a larger welfare gain. In what follows, we refer to this effect as the Engel effect.

The second object is comprised of two factors: the elasticity of the equilibrium relative wage w.r.t. productivity, denoted by \( \eta_{WH/WL, A_k} \), and an extra term that captures the covariance between the labor share and the difference over types in consumption shares.\(^2\) It is easy to show that the covariance term is positive, and hence the sign of the second term is determined by \( \eta_{WH/WL, A_k} \).\(^3\) Furthermore, as we discuss below, empirically, the covariance term is very close to 0, implying that the magnitude of the second term is dominated by the relative wage elasticity.

3.1. Signing the Skill-Premium Elasticity

In what follows we analyze the determinants of \( \eta_{WH/WL, A_k} \). Our motivation stems from the significant role this elasticity plays in dictating the distributional welfare outcomes of sectoral technical change as seen in Equation 3. We begin our analysis with the following lemma.

**Lemma 1.** The set of equilibrium conditions of the model, as outlined in Section 2.1.3, can be consolidated into a single market-clearing condition for high-skilled individuals. This condition is solely a function of the exogenous productivities and the relative wage:

\[ \mathcal{H}(A, \frac{W_H}{W_L}) = 0. \] (4)

\(^2\)Throughout, the term “relative wage elasticity” refers to the elasticity of the high-skill vs. low-skill wages, i.e. \( \eta_{WH/WL, A_k} \).

\(^3\)To see that the covariance term is positive, note that:

\[ 1 + \sum \alpha_i (s^H_i - s^L_i) = 1 + \sum \alpha_i s^H_i - \sum \alpha_i s^L_i \geq 1 + \min \{ \alpha_i \} - \max \{ \alpha_i \} > 0, \]

where the last inequality stems from the fact that \( \alpha_i \in (0, 1) \forall i. \)
This single equation inherently encompasses all the equilibrium conditions of the other markets, and therefore functions as an excess demand equation specifically for the high-skilled labor market.

Equation 4 serves two key purposes. First, it enables us to ascertain what influences the sign of the relative wage’s elasticity with respect to technical change, which we establish in Theorem 2. Second, it helps us understand how this elasticity is determined by the model’s primitives—preferences and production functions—which is established in Theorem 3. This last theorem allows for an analytical characterization of the equilibrium based on model primitives and provides guidance regarding the elements we need to estimate.

As a first step, Theorem 2 signs the relative wage elasticity:

**Theorem 2.** The sign of the relative wage elasticity w.r.t a technical change in sector k (while holding all other productivities constant) is given by

\[
\text{sign} \left( \eta_{W_H/W_L,A_k} \right) = -\text{sign} \left\{ \frac{d}{dA_k} \sum_{i=1}^{N} \alpha_i \left( S_{H_i}^H + S_{L_i}^L \right) \right\},
\]

Where \( S_{H} = \frac{W_{H}}{W_{H} + W_{L}} \) and \( S_{L} = \frac{W_{L}}{W_{H} + W_{L}} \) represent the aggregate expenditure shares of the high-skilled and low-skilled individuals in the economy, respectively. Similarly, \( s_{H_i}^H \) and \( s_{L_i}^L \) stand for the consumption share of a sector \( i \) as a fraction of the total expenditures for the high-skilled and low-skilled individuals, respectively.

Recall that \( \alpha_i \) signifies the labor share of the low-skilled in sector \( i \). Thus, Equation 5 illustrates the importance of whether demand moves either toward or away from the high-skilled sector in influencing the relative wage of high- versus low-skill workers. Specifically, when technical change reduces the share weighted \( \alpha \) (i.e., when \( \sum_{i=1}^{N} \alpha_i \left( S_{H_i}^H + S_{L_i}^L \right) \) decreases), the economy shifts towards being more high-skill intensive. As might be anticipated, equation (5) indicates that in such a case, the high-skilled relative wage increases.

### 3.2. The Determinants of the Relative Wage Elasticity

The next theorem characterizes the relative wage elasticity—a key determinant of the distributional welfare impact of sectoral technical change—in terms of model primitives. Applying the Implicit Function Theorem to equation (4), and writing the result in terms of an elasticity, we show that:

**Theorem 3.** The elasticity of the equilibrium relative wage w.r.t. a technical change in sector k...
(while holding all other productivities constant) is given by

\[
\eta_{WH/WL, A_{k}} = \frac{-\eta_{\mathcal{H}, A_{k}}(A, \frac{W_{H}}{W_{L}})}{\eta_{\mathcal{H}, W_{H}/W_{L}}(A, \frac{W_{H}}{W_{L}})} = -\sum_{i=1}^{N} \alpha_{i} (S_{LSi,L} \eta_{Si,L} + S_{HSi,H} \eta_{Si,H}) \frac{Q(\alpha, \sigma, S_{L}, S_{H}, S_{L}, S_{H}, \eta_{C,P}^{L}, \eta_{C,P}^{H}, \eta_{C,W}^{L}, \eta_{C,W}^{H})}{},
\]

where \( \eta_{\mathcal{H}, A_{k}} \) and \( \eta_{\mathcal{H}, W_{H}/W_{L}} \) denote the partial elasticity of \( \mathcal{H} \) w.r.t \( A_{k} \) and \( W_{H}/W_{L} \), respectively. \( \sigma \) is the vector of elasticity of substitutions of the N production functions. \( \eta_{Si,L} \) is the uncompensated elasticity of the consumption share of good \( i \) w.r.t \( P_{k} \) for the L types. Additionally, \( \eta_{C,P}^{L} \) denotes the uncompensated price elasticity matrix with element \( \{i, j\} \) capturing uncompensated price elasticity of good \( i \) to price \( j \) for the L types. \( \eta_{C,W}^{L} \) is the vector of Engel elasticities for the L types. \( \eta_{Si,H} \), \( \eta_{C,P}^{H} \) and \( \eta_{C,W}^{H} \) are defined analogously for the H types. Finally, \( Q \) is a real-valued function and it is explicitly defined as:

\[
Q = \frac{S_{L}}{W_{L}} (\alpha \circ P)' Slutsky^{L} (\alpha \circ P) + S_{H} \frac{W_{H}}{W_{L}} (\alpha \circ P)' Slutsky^{H} (\alpha \circ P)
\]

\[
- \sum_{i=1}^{N} \sigma_{i} (1 - \alpha_{i}) \alpha_{i} (S_{LSi,L} + S_{HSi,H})
\]

\[
- \sum_{i=1}^{N} \alpha_{i} \left( \frac{\partial E_{i,H}}{\partial W} - \frac{\partial E_{i,L}}{\partial W} \right)
\]

where \( E_{i,H} \) and \( E_{i,L} \) are expenditures of high- and low-skilled workers, respectively, and \( Slutsky^{H} \) and \( Slutsky^{L} \) are the Slutsky matrices for high- and low-skilled workers, respectively.

In Equation (6), the numerator captures the partial equilibrium effect of technical change on excess demand for high-skilled labor while holding constant the relative wage. We discuss below how its sign is determined by the model’s primitives.

Following the Implicit Function Theorem intuition, the denominator reflects the response required in the skill premium in order to maintain excess demand at zero given the change in demand induced by the technical change. It comprises three components. The first term represents consumer substitution patterns across goods due to price changes from relative wage adjustments, holding income constant; As a weighted sum of quadratic forms using the low- and high-skilled Slutsky.
matrices, this term is negative. This is intuitive – when the relative wage of the high-skilled increases, the relative price of high-skilled intensive goods rises. The substitution effects, as captured by the Slutsky matrix, imply that consumers shift away from these goods, leading to a decline in the relative demand for high-skilled workers. The second term denotes the substitution effect in production between low- and high-skilled workers when their relative wages change; it remains negative as long as at least one production function is not of the Leontief type, indicated by \( \sigma_i \geq 0 \). The third term reflects the varied income effects on demand due to non-homothetic preferences between low- and high-skilled workers. As the high-skilled relative wage increases, income effects operate in the opposite direction for the low- and high-skilled, and hence the sign of this effect is ambiguous. For example, if demand of high-skilled worker shift towards high-skilled intensive (low-\( \alpha \)) sectors at a faster rate than the low-skilled shift away from these sectors, overall demand for high-skilled sectors will rise, and this third term will have a positive sign.

Overall, our estimation of \( Q \) is routinely negative. This outcome primarily arises because \( Q \) can only be positive if the third term, stemming from varied income effects between low- and high-skilled workers is positive. In cases of homothetic preferences, where this term is zero, \( Q \) is always negative. For non-homothetic preferences, we find that in all tested specifications, the sum of the first two negative terms significantly outweighs the third term, explaining the negative sign we observe in \( Q \).

Returning to the numerator, equation (6) points again to the importance of the share-weighted \( \alpha \) in determining the relative wage elasticity. However, we note that the weights are defined as functions of preference parameters and specifically, the price elasticities. As such, given a sectoral technical change to a specific sector, the degree to which goods are complements or substitutes dictates the behavior of the share-weighted \( \alpha \). Consider, for example, a decline in the \( P_k \) price due to a positive \( A_k \) technical change. Aggregate expenditure shares of all goods \( i \), both for the low- and the high-skilled workers will change according to the price elasticity w.r.t. good \( k \). The numerator then captures how these demand shifts are reflected in changes to the share-weighted \( \alpha \).

The results from Theorem 3 allows us to revisit Theorem 2 and establish the condition that determines the sign of the relative wage elasticity in terms of model primitives. This is summarized in the following lemma:

**Lemma 2.** A sufficient condition for the elasticity of the relative wage \( \eta_{W_H/W_L}\cdot A_k \) to be positive is
that for each type $T \in \{H, L\}$:

$$\sum_{i=1}^{N} \alpha_i s_i^T \eta_i^T > 0$$

First, we note that the numerator in equation (6) is constructed from the expression in the lemma, for the low and high skilled workers. For intuition, it is useful to rewrite this condition (see proof for this equivalence in Appendix A.1) as

$$\sum_{i=1}^{N} \alpha_i s_i^T \eta_i^T = s_k^T \alpha_k - s_k^T \sum \alpha_i w_i^T > 0,$$ with $w_i^T = \frac{s_i^T \eta_i^T \eta_i^T \eta_i^T P_k}{\sum_i s_i^T \eta_i^T \eta_i^T P_k}$

Thus, this lemma can be understood from a decomposition of the numerator in equation (6) into two components. This decomposition stems from the fact that for $i \neq k$, $\eta_i^T \eta_i^T \eta_i^T P_k = \eta_i^T \eta_i^T \eta_i^T P_k$, while for $i = k$, $\eta_i^T \eta_i^T \eta_i^T P_k = 1 + \eta_i^T \eta_i^T \eta_i^T P_k$, where the latter stems from the fact that a price change of good $k$ has a direct effect on the expenditure share of good $k$. The impact on the expenditure share weighted alpha ($s_k^T \alpha_k - s_k^T \sum \alpha_i w_i^T$) is thus affected by two forces. The first element ($s_k^T \alpha_k$) captures the mechanical direct effect on the share-weighted $\alpha$: holding constant demand, the expenditure share on good $k$ declines simply because the price of that good declined. The second element ($s_k^T \sum \alpha_i w_i^T$) captures how the share-weighted $\alpha$ changes due to the demand responses of all goods (including $k$) to the price change.

To see the intuition of this lemma, consider then a positive technical change to sector $k$ inducing a decline in $P_k$, which we denote as $\widehat{P}_k < 0$. The condition in this lemma becomes:

$$\widehat{P}_k s_k^T \alpha_k - \widehat{P}_k s_k^T \sum \alpha_i w_i^T < 0$$

(7)

The mechanical direct effect, $\widehat{P}_k s_k^T \alpha_k$, is always negative – prior to any change in demand, the price decline leads to a decline in the expenditure share in good $k$. Hence, for the condition in Lemma 2 to hold, the demand effect ($\widehat{P}_k s_k^T \sum \alpha_i w_i^T$) cannot be too negative.

To delve deeper into the interplay between the $\alpha$s and the substitution patterns in determining the relative wage elasticity, Corollary 1 summarizes the case where all goods are complements.

**Corollary 1.** Assume that $Q < 0$, and that all goods are gross-complements. Then: $\eta_{WH}/W_{LA}$ will be positive when the lowest skill intensity sector (max($\alpha_i$)) experiences positive technical change, and negative when the highest skill intensity sector (min($\alpha_i$)) experiences positive technical change.

---

4This is relevant for many preference specifications. See, for example, the preferred estimates of the non-Homothetic CES preferences in Comin, Lashkari and Mestieri (2021).
When all goods are complements, \( w_i^T > 0, \forall i \), i.e. all weights in Lemma 2 are positive. As such, when the sector experiencing the positive technical change has the highest \( \alpha \), \( \alpha_k > \sum_i \alpha_i w_i \) and so (7) always holds. Put differently, in this situation the direct effect always dominates. Intuitively, when all goods are complements, positive technical change to a sector increases demand for all other sectors. Therefore, a positive technical change to the lowest-skill sector (highest \( \alpha_i \)), shifts demand away from this sector, thereby increasing overall demand for high-skilled labor in the economy. This reduces the equilibrium share-weighted \( \alpha \) in the economy, and increases the high-skilled relative wage, as stated in the corollary. The reverse holds for the lowest \( \alpha \).

Corollary 1 discusses the impact of complementary between goods. However, as discussed above, in general the effect of technical change on the share-weighted \( \alpha \) and on the relative wage elasticity will depend on the degree to which sectors are either substitutes or complements to the sector experiencing the productivity change. Consider for example the case of the highest \( \alpha \) sector again, but without assuming that all goods are complements. Under which conditions is the relative wage elasticity negative? That is, when does the high-skilled relative wage decline? From (7), for this to occur it is sufficient that \( \alpha_k < \sum_i \alpha_i w_i \), i.e. that there exists at least one sufficiently large \( w_i \). Following Corollary 1, this cannot occur when all goods are gross-complements and hence all \( w_i \in [0,1] \), implying that in order to have a sufficiently large \( w_i \) it is necessary that at least some of the weights (\( w_i \)) are negative, i.e. some goods are substitutes. Let’s consider a prime example where this can happen. Suppose that good \( k \) with the highest \( \alpha \) is also very elastic, with \( \eta_{ck}, P_k < -1 \). This implies that when \( P_k \) declines, the share of good \( k \) increases, in which case it can be shown that \( w_k > 1 \). Because the weights must sum to 1, this in turn implies that there must exist at least one good with a negative \( w_k \) – i.e. a good which is substitutable with good \( k \). In such a case, the decline in \( P_k \) implies an increase in demand for the low-skilled good \( k \), and thus an increase in the share-weighted \( \alpha \) and a decrease in the high-skilled relative wage.

---

\( ^5 \)With further restrictions on preferences, these results extend beyond the highest and lowest \( \alpha \) sectors. Indeed, if preferences are Non-homothetic CES and all goods are gross-complements (as in Comin, Lashkari and Mestieri (2021)), there exist \( \overline{\alpha} > \alpha \) such that:

\[
\begin{align*}
\forall \alpha_k > \overline{\alpha} : & \quad \eta_{WH/WL, A_k} > 0 \\
\forall \alpha_k < \overline{\alpha} : & \quad \eta_{WH/WL, A_k} < 0
\end{align*}
\]

Furthermore, if preferences are homothetic CES and all goods are gross-complements, then \( \overline{\alpha} = \alpha \).
3.3. Taking Stock

To summarize, this section provides an analytic framework that characterizes the distributional impact of sectoral technical change and its underlying mechanisms. We started by showing in Theorem 1 that the distributional welfare effects are driven by two components – the Engel effect and the impact of sectoral technical change on the relative wage. The price effect is driven by the non-homotheticities and the resultant differential expenditure shares of high- and low-skilled individuals. Engel curves will thus play an important role in determining the size of this effect. Theorem 2 highlights that the relative wage’s impact stems from the change in the share-weighted $\alpha$. Meanwhile, Theorem 3 emphasizes the role of consumption price elasticities in shaping this share-weighted $\alpha$ (through changes in demand), which in turn influences both the relative wage and the welfare distribution.

4. A Quantitative Evaluation of Sectoral Technical Change

In the previous section we characterized analytically the distributional impact of sectoral technical change. Notably, Theorems 1 and 3 have another important implication: they provide a framework to assess the welfare effects of sectoral productivity changes by estimating key preference and production parameters, thereby circumventing the need for assuming specific functional forms.

As implied by our analytical approach, the set of parameters required for estimation fall into two broad categories – one pertaining to consumption and the other to production. On the consumption side we need to estimate for both high and low-skilled workers: (1) aggregate expenditure shares, (2) expenditure shares by good categories, (3) the matrix of uncompensated elasticities of consumption goods w.r.t prices, and (4) the Engel elasticities. On the production side, we need to estimate for each category (1) the equilibrium labor shares, and (2) the elasticity of substitution of the N production functions. Finally, we need to measure for each category its specific technical change. In what follows, we discuss the empirical strategy for obtaining these parameters.
4.1. Estimation Framework

4.1.1. Consumption parameters

Given our analytical results, we do not need to constrain our analysis to a particular utility function and instead estimate an almost ideal demand system (AIDS) following Deaton and Muellbauer (1980). We can then recover the required price and income elasticities from this flexible estimated demand system. In contrast, committing to a particular utility function would involve the cost of placing restrictions on price and income elasticities, which we have shown are key in determining the welfare impact of sectoral technical change.

4.1.2. Production parameters

The first set of production parameters consist of the equilibrium labor shares by good categories. As we describe below, we measure these good-category labor shares directly from the data using industry-level labor shares and a mapping between good-categories and industries. The second set of production parameters are the sectoral elasticity of substitutions.

4.2. Data

For the consumption side of the data we use the Consumer Expenditure Survey (CEX). The CEX is a dataset produced by the U.S. Bureau of Labor Statistics that provides detailed information on the spending habits, income, and household characteristics of U.S. consumers. It is commonly used as a primary source of information for understanding and analyzing patterns of consumer expenditures. We use the dataset provided by Comin, Lashkari and Mestieri (2021), and keep their sample of urban households with a present household head aged between 25 and 64 for the years 1999-2010 and four CEX interviews. We further drop households with extreme shares in a product category. We define low- and high-skilled workers in the CEX based on the household head’s education level, with high-skilled defined as those with a BA degree or above.

As in Comin, Lashkari and Mestieri (2021) we combine the CEX data with regional quarterly price series by consumption category from the BLS’s urban CPI (CPI-U). To aggregate prices for a given good category, we use region-by-quarter aggregate expenditure shares.

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6Their data construction is based on Aguiar and Bils (2015).
Our analysis requires income shares at the goods-category level. To obtain these data, we first calculate income shares by skill at the industry level using earnings data from the Current Population Survey (CPS) over the years 1999-2010. We then use an input-output matrix to map industry cost shares to consumption category cost shares (similar to Bils, Klenow and Malin (2013)). To obtain the income share of a given goods-category, we calculate the weighted-average of the income shares of the industries associated with this goods-category, with the weights equal to the input costs. To obtain the vector of elasticities of substitution, we use variation across sectors in the growth rates of the ratio of high- to low-skilled workers (see Appendix A.2.3 for details). Appendix Table A1 reports the descriptive statistics for our sample.

Finally, we use KLEMS U.S. data on industry-level Total Factor Productivity (TFP). We aggregate these TFP measures to a goods-category measure of TFP using an analogous process to that described for income shares.

4.3. Results

In what follows we report the results from two quantitative analyses. In each we estimate the required underlying set of preferences and production function parameters as outlined in Section 3. We then use the results from the analytical analysis to calculate the distributional welfare impact of sectoral productivity changes.

The first analysis examines the welfare consequences of sectoral productivity changes using the consumption categorization employed in the structural transformation literature (e.g. Buera and Kaboski (2012) and Comin, Lashkari and Mestieri (2021)). As in this literature, we aggregate consumption to three broad categories–Agriculture, Manufacturing and Services–and estimate the underlying price and income elasticities using an AIDS.

Given our interest in sectoral technical changes, there is a concern that a three-sector categorization is too coarse and does not adequately capture substitution patterns between goods. Hence, in our second analysis, we extend the number of sectors, looking at a finer categorization. This finer categorization naturally gives rise to more complex substitution patterns, which the AIDS formulation easily captures.

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7See Appendix A.2 for a detailed discussion of how we link the various data sets.
4.3.1. Three Sectors Case

Theorem 3 shows that estimating the distributional impact of sectoral change requires recovering price and income elasticities. To do so we estimate an AIDS to recover the required elasticities. To maintain comparability to the structural transformation literature we estimate the demand system using similar controls and instruments as in Comin, Lashkari and Mestieri (2021). The controls include household age (by age groups), household size, and number of earners. Household expenditures are instrumented with after-tax annual household income and the income quintile of the household. Regional prices for each goods-category are instrumented using the average price of the good in the other regions, weighting these prices by the goods’ regional expenditure share in the households’ region.

We thus estimate separately the vectors of parameters of the demand system \((\theta: \gamma, \beta, \delta)\) for the low- and high-skilled types \(T \in \{H, L\}\) using a standard AIDS specification:

\[
s_{i,j} = \beta^{T}_{i} + \gamma^{T}_{i}' \log P_{j} + \delta^{T}_{i} \left[ \log C_{j} - b \left( \log P_{j}, \theta^{T} \right) \right] + u_{i,j}
\]

\[
b \left( \log P_{j}, \theta^{T} \right) = \beta^{T}_{0}' X_{j} + \beta^{T}' \log P_{j} + 0.5 \log P_{j}' \Gamma^{T} \log P_{j},
\]

where there is an equation \(i\) for each sector, \(s_{i,j}\) stands for the expenditure share of good \(i\) for household \(j\) in a specific period, \(P_{j}\) is the vector of goods prices, and \(X_{j}\) are household level controls.

The Differential Welfare Effects of Sectoral Technical Changes

Figure 1 provides a first look at the differential welfare impact of sectoral technical change. For each of the three sectors – Services, Manufacturing, and Agriculture – the figure depicts the welfare elasticity w.r.t. technical change in sector \(k\) for high- and low-skilled workers, \(\eta_{EV^{H,k}}\) and \(\eta_{EV^{L,k}}\), respectively, as implied by our estimates. For example, the figure shows that a 10% positive technical change in Services leads to an approximately 5.8% welfare increase for high-skilled workers.

There are three key messages to the figure. First, technical change across industries affects workers differently. Naturally, a productivity increase in the largest sector, Services, has a larger impact on welfare. Second, within each sector, technical change affects welfare of high- and low-
skilled workers differently. Third, this differential effect between high- and low-skilled workers varies across sectors in both magnitude and sign. In fact, while for Services the welfare elasticity w.r.t. technical change is 4.5 p.p smaller for the high-skilled, for Manufacturing and Agriculture the welfare elasticity is 5 and 4.5 p.p greater for the high-skilled, respectively. These p.p differences imply that while positive technical change in services leads to a 7% smaller welfare increase for high-skilled workers, in Manufacturing this number is about 20%, and in Agriculture it is 40%.

**Decomposing the Differential Welfare Effects** Theorem 1 of Section 3 allows us to decompose the differential welfare change, \( \Delta \eta_{EV,A_k} \), into its two underlying components – the Engel effect and the change in welfare stemming from the change in the relative wage. The results are shown in Figure 2.

Panel A depicts the three industries according to two attributes: their degree of low skill intensity, \( \alpha \), and their Engel elasticity. As implied by Section 3 and as will be discussed further shortly, these two attributes are directly linked to the channels underlying the welfare analysis: (1) **the Engel elasticity** determines the difference in expenditure shares between high- and low-skilled workers. Higher Engel elasticities imply that the difference in expenditure shares between high and low-skilled workers is larger (i.e., \( s_H - s_L \) rises), which means that the Engel effect stemming from positive technical change will rise. (2) **The relative skill intensity of each sector**. As shown in Section 3, the relative skill intensity of a sector, \( \alpha \), plays an important role in determining the relative wage elasticity.

Panels B, C, and D of the figure depict the decomposition of the welfare impact of technical change into the Engel effect and the relative wage elasticity for Services, Manufacturing, and Agriculture respectively. Following Equation 3, the decomposition is conducted as follows:

\[
\Delta \eta_{EV,A_k} = (s_H^k - s_L^k) + \eta_{WH/WL,A_k} \left\{ 1 + \Sigma_i \alpha_i (s_i^H - s_i^L) \right\}.
\]  

(8)

**The Engel Effect** Consider first technical change in the Manufacturing and Agricultural sectors, depicted in Panels C and D. These are necessity sectors (Engel elasticity is smaller than one) and are low-skill intensive relative to the average in the economy. Because they are necessity sectors,
expenditure shares of low-skilled workers are higher than that of high-skilled workers. As such, when these sectors experience a positive technical change, the associated price decrease benefits low skill workers by more. This is reflected in the negative Engel effect in panels C and D. In contrast to Manufacturing and Agriculture, Services is a luxury sector. In this sector expenditure shares of high-skilled workers are thus higher than that of low-skilled workers, implying that the Engel effect of technical change in this sector benefits high-skilled workers relatively more (a positive Engel effect in Panel B).

The Relative Wage Effect  

Turning to the effect of technical changes on the relative wage, consider first a positive productivity change in the Agriculture sector, which lowers its relative price. Panel D shows that in this case the high-skilled relative wage rises, thereby benefiting the high-skilled workers more than the low-skilled. To understand why this occurs, note that Agriculture is the sector with the lowest intensity of high-skilled workers (i.e., the highest \( \alpha \) sector). If both Manufacturing and Services are complements to Agriculture, then following the results in Corollary 1, the high-skilled relative wage must rise. As explained above, when the Agriculture price declines as a results of the productivity change, complementarities imply that demand will shift towards Manufacturing and Services – the more high-skilled intensive sectors – increasing the relative demand for high skilled workers. Now, as reported in Appendix Table A2 while Services and Agriculture are indeed estimated to be complements, there is evidence for substitution between Manufacturing and Agriculture. This makes the sign of the effect on the relative wage an empirical question. However, because Services is the dominant sector in the economy, and because it is complement to Agriculture, it is not surprising that the high-skilled relative wage rises consistent with the case where all sectors are complements. Indeed, when plugging in our estimates in the numerator of Equation 3, the price elasticity stemming from services together with its large consumption share swamps their counterparts stemming from Manufacturing.

Turning to the sector at the opposite extreme of skill intensity – Services (the high-skill intensity sector) – the predicted results are unambiguous. This is because we estimate complementarity between Services and both of the other two sectors (see Appendix Table A2). Because Services are the highest-skill intensive sector (lowest \( \alpha \)), Corollary 1 directly implies that the high-skilled relative wage must decrease, as indeed shown in Panel B.

Finally, the skill intensity of Manufacturing lies between that of Services and that of Agriculture,
but overall higher when compared to the mean in the economy. As panel C shows, the effect of a price decline due to technical change in this sector on the high-skilled relative wage is positive. As discussed above, it is the estimated complementarity between Services (the largest sector in the economy) and Manufacturing that is responsible for the increase in the high-skilled relative wage.

Taken together, Panels B through D show that the Relative Wage effect is the dominant force in determining the overall distributional impact of technical change. Across the three sectors, in absolute value, the relative magnitude of the Relative Wage effect is on average almost five times that of the direct effect.

Further, the results show that the Relative Wage effect is larger than the total effect of technical change; this is because in the three sector case, the Engel effect is always in the opposite directions to the Relative Wage effect. This, in turn, is explained by the fact that these three sectors lie in quadrants two and four in the $\alpha$-Engel space. As we show below, once we examine sectors using a more disaggregated goods-category classification, sectors will appear in other quadrants as well – implying that the relative wage and direct effects are not always in opposite directions.

4.3.2. Extending the Number of Sectors

Given our interest in sectoral technical changes, we extend our analysis to seven sectors to adequately capture substitution patterns between sectors. This finer categorization naturally gives rise to more complex substitution patterns, which the AIDS formulation easily captures.

The Differential Welfare Effects of Sectoral Technical Changes

Similar to Figure 1, Figure 3 provides the welfare elasticity w.r.t. technical for the seven-sector case. The figure provides two key takeaways. First, we see a differential effect of technical change on the welfare of high-versus low-skilled workers, as in the case of three sectors. In fact, in three of the seven sectors, technical change benefits low-skilled workers relatively more, while in the other four sectors, technical change benefits more the high-skilled workers. Second, the differential welfare effect of positive technical change varies in magnitude across sectors, with the relative differences in elasticities between high- and low-skilled workers exhibiting a wider range compared to that in the three-sector case. Across sectors, the implied differential effects of a positive productivity change range between a 20% smaller welfare increase for high-skilled workers (as compared to low-skilled workers) to a 70% larger welfare increase for the high-skilled.
Figure 4 decomposes the overall effect into the Engel effect and the Relative Wage effect (using equation (8)). While overall the Relative Wage effect remains the driving force in explaining the total welfare effect, with this finer category classification, the direct effect plays a sizeable role. In fact, in two of the seven sectors (Transportation and Other Services) the direct effect is in opposite direction to the Relative Wage effect and dominates it. This larger direct effect is in the spirit of Jaravel (2021b) which shows that with finer goods categorization, there are more pronounced differences between expenditure shares across income groups.

Finally, Figure 5 depicts the seven sectors based on their degree of high skill intensity, $\alpha$, and their Engel elasticity. Our findings point to a general pattern whereby low-skilled (high-$\alpha$) sectors exhibit positive Relative Wage effects, while high-skilled (low-$\alpha$) sectors exhibit negative Relative Wage effects. This is similar to the pattern exhibited in the three sector case, suggesting that complementarities are the dominant pattern in the data. In fact, as shown in Appendix Tables A3 and A4, for low skilled workers our point estimates show that approximately 66% of the cross price elasticities are negative (i.e. complement goods) while for the high-skilled this number is 55%.

The Overall Effect of TFP Changes In the prior section, we analyzed the differential welfare elasticities to sectoral technical change and showed that there is a significant variation in the magnitude of these elasticities. It is therefore of interest to calculate the overall differential welfare impact on high- versus low-skilled workers given the empirical changes in sectoral TFP observed in the data. To do so, we calculate for each sector in a given time period, the product of the TFP growth and the difference in the welfare elasticities as calculated above, then summing over all sectors. Our data on TFP spans 1987 to 2019. We therefore analyze TFP growth rates over three different time periods: 1987–1997, 1997–2007, and 2010–2019. Results indicate that the overall welfare impact over high- and low-skilled workers differ by time period. Incorporating realized TFP growth in each sector together with the sectoral differential welfare elasticities, between 1987 and 1997, high-skilled workers enjoyed an overall 18% greater welfare increase compared to low-skilled workers. This differential impact declines to 12% between 1997 and 2007, and to -5% in the last period.

10The third period excludes 2008–2010, the period of the "Great Recession", but results are qualitatively similar when analyzing 2007–2019.
5. Beyond Productivity Shocks: Two Additional Applications

In this section, we demonstrate the versatility of our framework, emphasizing its application beyond the analysis of the welfare impact of sectoral technical change. Specifically, we explore two demand-driven changes—a shift in public sector expenditures and a change to consumer preferences.

We note that the actual change in the welfare level following these demand changes is bound to be smaller than the one in the case of sectoral technical change. In the latter case, the supply change increases the economy’s production capacity, while demand shifts do not. Yet, due to shifts in consumption across sectors and the variation of skill intensity across them, changed in demand will have distributional consequences as shown below.

5.1. Public Sector Demand Shifts

So far, our multi-sector model included only private-sector goods. In this section, we extend our model to include a public-sector good—for example, this sector could be thought of as government expenditure on health care.

To do so, we consider an $N+1$ sector which produces a public sector good $G$, using a constant returns to scale production function $G = A_G F(L_G, H_G)$. For a given level of $G$, inputs are chosen optimally, taking the equilibrium wages as given. Furthermore, the total demand for the public sector good $G$ is exogenously determined and does not enter the household maximization problem. The public sector is funded by a proportional tax on labor, implying that the flat tax rate $\tau$ satisfies:

$$\tau = \frac{G}{\left(W_L L + W_H H\right)}$$

where without loss of generality, we assume that the public good is the numeraire.

What is the distributional welfare impact of an exogenous change in the level of public sector expenditure, $G$? This is given by the difference in welfare elasticities between high- and low-skilled workers with respect to $G$:

$$\Delta EV_G = \tilde{G} \left[ \frac{\eta_{W^H/W^L, G}}{1 + \sum_i \alpha_i \left(s_i^H - s_i^L\right)} \right].$$

Contrasting with Equation (3), it is apparent that the Engel effect is absent. This is because unlike in the case of productivity changes, a shift to the size of the public sector has no direct effect on goods prices; It affects prices only indirectly through changes in the composition of demand.
for private sector goods and through the direct effect on the demand for labor in the public sector. These effects are captured in the impact on the relative wage.

Following Equation (9), we continue by analyzing the elasticity of the relative wage with respect to the public sector expenditure level, \( \eta_{WH/WL,G} \). It is straightforward to show that Equation (4), which encompasses all equilibrium conditions in the case of sectoral technical change, can be reformulated as

\[
\mathcal{H}(G, \frac{WH}{WL}) = 0, \tag{10}
\]

where we replace the productivity vector, \( A \), with the level of public expenditure, \( G \). As in the productivity case, the \( \mathcal{H} \) function can be thought of as representing an excess demand for high skilled workers, with the adjustment that demand for such workers also incorporates those required to produce the public good.

Similar to the technical change case, we apply the implicit function theorem to equation (10) to get:

**Theorem 4.** The elasticity of the equilibrium relative wage w.r.t. the level of public sector expenditure \( G \) is given by

\[
\eta_{WH/WL,G} = \frac{\tau \eta_{\tau,G} \alpha_{N+1} + (1 - \tau) \eta_{(1-\tau),G} \sum_{i=1}^{N} \alpha_i \left( S_{LS_i} \eta_{C_i,L} W_L + S_{HS_i} \eta_{C_i,H} W_H \right)}{\tilde{Q} \left( \alpha, \sigma, S_L, S_H, \alpha, \eta_{C,P}^L, \eta_{C,P}^H, \eta_{C,W}^L, \eta_{C,W}^H, \tau \right)} \tag{11}
\]

\[
= \frac{\tau \left[ \alpha_{N+1} - \sum_{i=1}^{N} \alpha_i \left( S_{LS_i} \eta_{C_i,L} W_L + S_{HS_i} \eta_{C_i,H} W_H \right) \right]}{\tilde{Q} \left( \alpha, \sigma, S_L, S_H, \alpha, \eta_{C,P}^L, \eta_{C,P}^H, \eta_{C,W}^L, \eta_{C,W}^H, \tau \right)} \tag{12}
\]

where \( \eta_{\tau,G} \) and \( \eta_{(1-\tau),G} \) are partial elasticities of \( \tau \) and \( 1 - \tau \) w.r.t \( G \), respectively, holding the relative wage constant. \( \tilde{Q} \) is a real-valued function with arguments as in Theorem 3, which is provided explicitly in Appendix A.1.

As in the case of technical change, we estimate the denominator to be negative.\(^{11}\) The numerator in (11) is quite intuitive, in that it captures the effect of a change in \( G \) on the economy-wide share-weighted \( \alpha \). As \( G \) increases two forces are at play. First, the economy shifts towards a larger public sector and a smaller private sector. This is captured by the term \( \tau \eta_{\tau,G} \) which multiplies the low skilled-share of the public sector, \( \alpha_{N+1} \), and the term \( (1 - \tau) \eta_{(1-\tau),G} \) which multiplies the share weighted \( \alpha \) in the private sector (clearly, as \( G \) increases, the former is positive and the latter

\(^{11}\)The argument is similar to that in Theorem 3 – when excess demand for high-skilled labor declines, the high-skilled relative wage rises (see section 3.2).
is negative). Second, as $G$ rises, after-tax wages of both high- and low-skilled workers decline, which will impact the share-weighted $\alpha$ in the private sector due to non-homotheticities.

Re-writing (11) as (12) provides an additional insight. The skill intensity of the public good sector, $\alpha_{N+1}$, relative to the equilibrium (i.e. post-change) skill intensity of the private sector, $\sum_{i=1}^{N} \alpha_i (S_{L} s_{i,L} \eta c_{i,L} W_{L} + S_{H} s_{i,H} \eta c_{i,H} W_{H})$, determines the sign of the impact on the skill premium. For example, if the private sector is intensive in high-skilled workers (low $\alpha_{N+1}$), then as $G$ rises, the economy becomes more high-skill intensive and the high-skilled relative wage rises (i.e. the numerator in (12) will be negative). The next corollary formalizes this result for two extreme cases of the skill intensity of the public sector.

**Corollary 2.** Assume all goods are normal, and that $\tilde{Q} < 0$, then: $\eta_{W} / W_{L} G$ will be negative when $\alpha_{N+1} \geq \max_{i=1\ldots N}(\alpha_i)$ and positive when $\alpha_{N+1} \leq \min_{i=1\ldots N}(\alpha_i)$.

Relying Theorem 4, Figure 6 depicts the differential welfare impact stemming from changes in public sector expenditure level $G$ over the size of the public sector (pre-change) for three different levels of public sector skill intensity. The three skill intensity levels are the minimum, mean, and maximum skill intensity over the seven sectors used in Section 4.3.2. The different elements required for the calculation of equations (9) and (12) are as in the case of technical change.

As can be seen in the figure, and reflecting the insights from the theoretical analysis, the distributional impact of government expenditure varies according to whether the public sector is high- or low-skill intensive. The high-skilled benefit from increases in $G$ when the public sector is high-skilled intensive, while the low skilled benefit when it is low-skilled intensive. As expected, the figure also shows that the distributional welfare impact increases with the pre-change public sector’s size.

As an example, consider two types of public sector expenditures. Military personnel as a fraction of private consumption in the U.S. is approximately 2.25% and its skill intensity is similar to the average in the economy, hence the distributional impact of an increase in public expenditure in this sector is relatively small (see Figure 6). In contrast, consider the public health expenditures. This sector’s low-skill intensity is approximately 0.55, and as a fraction of private consumption in the U.S. its share is approximately 12%. In this sector, for high-skilled workers, the elasticity of welfare to an expenditure change is 0.018, while for the low-skilled it is -0.01, resulting in a differential elasticity favoring the high-skilled of 0.028 as depicted in Figure 6.

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12See Appendix A.2.4 for a discussion of the construction of these public sector measures.
5.2. Changes to Consumer Preferences

In this section we analyze demand shifts driven by changes to consumer preferences. For ease of exposition, we return to the model without a public sector. All proofs in this section are analogous to the relevant proofs for the case of technical change.\textsuperscript{13}

Let $\beta_k$ parameterize the demand shifter for the good produced by sector $k$. Then, the difference in welfare elasticities between high- and low-skilled workers with respect to $\beta_k$ is given by

$$\Delta \eta_{EV,\beta_k} = \frac{EV^H_{\beta_k}}{\beta_k} - \frac{EV^L_{\beta_k}}{\beta_k} = \eta_{W^H/W^L,\beta_k} \left[ 1 + \sum_i \alpha_i \left( s^H_i \beta_{\text{post}} - s^L_i \beta_{\text{post}} \right) \right],$$

where we evaluate welfare using ex-post preferences denoted by $\beta_{\text{post}}$, and $s_i \beta_{\text{post}}$ denotes the consumption share of good $i$ when using the ex-post preferences.\textsuperscript{14} Similar to the public sector case, the Engel effect is absent. Again, this is because unlike in the case of productivity changes, preference shifters have no direct effect on the price. Hence, the focal point is the elasticity of the relative wage with respect to the sectoral preference shifter, $\eta_{W^H/W^L,\beta_k}$. Following the same strategy as in the technical change case, we show

**Theorem 5.** The elasticity of the equilibrium relative wage w.r.t. the preference demand shifter, $\beta$, is

$$\eta_{W^H/W^L,\beta_k} = \frac{\sum_{i=1}^N \alpha_i \left( S_{L} S_{I} \eta_{S_{I},\beta_k} + S_{H} S_{I} \eta_{S_{I},H,\beta_k} \right)}{Q \left( \alpha, \sigma, S_{L}, S_{H}, S_{L}, S_{H}, \eta_{C,P}^L, \eta_{C,P}^H, \eta_{C,W}^L, \eta_{C,W}^H \right)},$$

where $Q$ is the same real-valued function defined in Theorem 3.

It is instructive to note the similarity here with Equation (6). The denominator, $Q$, is in fact identical, since it continues to capture the required response of the relative wage in order to maintain excess demand at zero. In contrast, the numerator in Equation (6) features the share elasticities with respect to prices, which allowed for a direct estimation of price elasticities without requiring an explicit model specification. This stands in stark contrast to the current case of demand shifters. Indeed, Equation (14) features the share elasticities with respect to the sectoral demand shifter, $\beta_k$. This implies that a specific model for demand is a prerequisite for any quantifiable analysis – one that explicitly takes a position on how the preference shifters affect demand.

\textsuperscript{13}Proofs are available upon request.
\textsuperscript{14}The quantitative results are almost identical to when we use the ex-ante values.
Recognizing this fundamental difference between the two cases (exogenous changes to supply and preference changes), in our analysis of this case we assume that demand follows the Non-homothetic CES demand model as in Comin, Lashkari and Mestieri (2021). Specifically, the demand for good $k$ is given by

$$C_k = (\beta_k U_k^{\nu_k}) \left( \frac{W}{P_k} \right)^{\frac{\nu}{1-\nu}} \epsilon_k^\nu,$$  

(15)

where $W$ is the individual’s income, and $U$ is implicitly defined as

$$\sum_{i=1}^{N} (\beta_i U_i^{\nu_i})^{\frac{1}{\nu}} C_i^{\frac{\nu-1}{\nu}} = 1.$$  

(16)

Under this functional form, the $\beta_k$ term acts as a demand shifter for consumption good $k$. Hence, the elasticities of the consumption shares with respect to a sectoral change in demand, denoted as $\eta_{s_L, \beta_k}$ and $\eta_{s_H, \beta_k}$ in the numerator, can be expressed as follows. First, for the L types, the elasticity of the share of good $k$ with respect to its own demand change, $\beta_k$, is given by $\eta_{s_L, \beta_k} = 1 - \frac{\epsilon_k}{\sum_{i=1}^{N} \epsilon_i}$. Second, for the L types, the elasticity of the share of good $j \neq k$ with respect to $\beta_k$ is given by $\eta_{s_L, \beta_k} = -\frac{\epsilon_j}{\sum_{i=1}^{N} \epsilon_i}$. Analogous expressions apply to the H type. Therefore, by following the estimation approach outlined in Comin, Lashkari and Mestieri (2021), once we have the values for $\epsilon$ and $\nu$, we can proceed to evaluate Equation (14).

As discussed above, under this preferences formulation all sectors are either complements or all substitutes. Given the prominence of complementarities in the three sector categorization, we estimate this demand system using the categorization of Section 4.3.1.

Our estimates on the differential welfare impact of sectoral demand change are intuitive. When a sector that is more low skill intensive experiences an increase in its demand, the high-skilled relative wage declines. Because in the case of demand changes, the relative wage is the only operational channel, this directly maps into the welfare of the two groups: the welfare of the low-skill rises, while that of the high-skill decreases. In contrast, when a high-skill intensity sector experiences an exogenous increase in demand the reverse holds: the high-skilled relative wage rises, increasing the welfare of the high-skilled while reducing the welfare of the low-skilled.

Our quantitative results are depicted in Figure 7. We find that when the Agriculture or Manufacturing sectors (both low-skill intensive relative to the aggregate economy) experience a positive demand change, the low-skill welfare gain is approximately 60% of the high-skill welfare loss. In contrast, when the Services sector (high-skill intensive) experiences a positive demand change, the high-
skill gain is approximately 170% of the low-skill loss.

6. Conclusions

In this paper we develop a general analytical framework that analyzes the distributional welfare implications of sectoral supply and demand changes. Integrating supply-side and demand-side heterogeneity into our model, we show that the distributional welfare impact of sectoral shifts can be decomposed into two effects: an "Engel effect" and a "Relative Wage effect." The former is driven by non-homothetic consumption patterns, wherein consumers at varied income levels allocate their expenditure differently across goods, leading to differential gains when prices change due to sectoral productivity changes. The Relative Wage effect is driven by how sectoral supply and demand shifts change the composition of demand across sectors.

In our theoretical analysis, we show how the patterns of consumption substitution and their interaction with skill intensities across sectors determine the Relative Wage effect. By not confining the analysis with specific functional forms for the utility and production functions we are able to identify four key forces that shape the distributional welfare impact of sectoral supply and demand shifts: (i) income elasticities of consumption goods, (ii) consumption substitution patterns, (iii) shares of different consumption sectors, and (iv) the relative skill intensity of the sector experiencing the change.

Importantly, these key factors are all objects that can be estimated in the data, allowing us to quantify the welfare effects of sectoral shifts. We apply our estimation framework to three different sectoral shifts: sectoral technical change, public sector demand shifts, and changes in consumer preferences. By combining various data sources, our quantitative results reveal significant welfare disparities between high- and low-skilled workers as a consequence of sectoral demand and supply changes. For example, in the case of sectoral technical change, the differential welfare effect between high- and low-skilled workers ranges between negative 20% and positive 70%, depending on the sector experiencing the technical change. Moreover, our findings emphasize the prominence of the Relative Wage effect in influencing the overall distributional impact of technical change. We then apply our framework to the case of sectoral demand shifts arising from either exogenous changes in the size of the public sector or changes in sectoral preferences.

In summary, the key contribution of this framework is its flexibility, which enables researchers
to precisely identify the critical forces shaping the welfare impact of sectoral shifts. It clarifies the fundamental elements involved and provides guidance on the pivotal parameters on which to focus.
References


Figure 1: Differential Welfare Impact of Sectoral Technical Change

Notes: Sectors are sorted by their aggregate share.

Figure 2: Decomposing the Welfare Impact of Sectoral Technical Change: 3 Sectors
Figure 3: Differential Welfare Impact of Sectoral Technical Change - 7 Sectors

Notes: Sectors are sorted by their aggregate share.

Figure 4: Decomposing the Welfare Impact of Sectoral Technical Change: 7 Sectors
Figure 5: $\alpha$-Engel Space: 7 Sectors

Figure 6: Differential Welfare Impact of Public Sector Shifts
Figure 7: Differential Welfare Impact of Sectoral Preference Change

- AGR
- MAN
- SER

Elasticities to sectoral shocks

High-Skill
Low-Skill
A. Appendix

A.1. Theoretical Derivations

Theorem 1

Before providing the proof for Theorem 1, it is useful to derive the following result:

Lemma A.1. $\sigma_i \left( \eta F_{iL}(1, \frac{H_i}{L_i}, \frac{H_i}{L_i}) \right) = 1 - \alpha_i$, and $\sigma_i \left( \eta F_{iH}(1, \frac{H_i}{L_i}, \frac{H_i}{L_i}) \right) = -\alpha_i$, where $\sigma_i = \eta_{\frac{H_i}{L_i}, \frac{W_i}{W_H}}$ is the elasticity of substitution in sector $i$ at the point $(\frac{H_i}{L_i}, \frac{W_i}{W_H})$, and $\alpha_i$ is the low-skill labor share in sector $i$. $F_{iL}(1, \frac{H_i}{L_i})$ is the marginal product of $F^i(1, \frac{H_i}{L_i})$ w.r.t. the first element (low-skilled), and $F_{iH}(1, \frac{H_i}{L_i})$ is the marginal product w.r.t. the second element (high-skilled). $\eta_{x,y}$ denotes the elasticity of $x$ w.r.t. $y$, and all elasticities are calculated at the pre-change equilibrium values.

Proof. We have that

$$\sigma_i \left( \eta F_{iL}(1, \frac{H_i}{L_i}, \frac{H_i}{L_i}) \right) = (\sigma_i) \left( \frac{\partial F_{iL}(1, \frac{H_i}{L_i})}{\partial \frac{H_i}{L_i}} \frac{H_i}{L_i} \right)$$

$$= \left( \frac{F_{iL}^L(L_i, H_i) F_{iH}^i(L_i, H_i)}{F_{iLH}^i(L_i, H_i) F^i(L_i, H_i)} \right) \left( \frac{\partial F_{iL}(1, \frac{H_i}{L_i})}{\partial \frac{H_i}{L_i}} \frac{H_i}{L_i} \right)$$

$$= \left( \frac{F_{iH}(L_i, H_i)}{F_{iLH}^i(L_i, H_i) F^i(L_i, H_i)} \right) \left( \frac{\partial F_{iL}(1, \frac{H_i}{L_i})}{\partial \frac{H_i}{L_i}} \frac{H_i}{L_i} \right)$$

$$= \left( \frac{F_{iH}(L_i, H_i)}{F_{iLH}^i(L_i, H_i) F^i(L_i, H_i)} \right) \left( \frac{\partial F_{iL}(1, \frac{H_i}{L_i})}{\partial \frac{H_i}{L_i}} \frac{H_i}{L_i} \right)$$

where $F_{iLH}$ stands for the cross-derivative. The first line is from the definition of the elasticity, and the second line stems from the fact the production is constant returns to scale (see for example footnote 6 on page 308 of Nicholson and Snyder (2012)). $F_{iLH}$ is homogeneous of degree $-1$,
hence:
\[
\sigma_i \left( \eta_{F_H^i(1, \frac{L_i}{H_i})} \right) = \left( \frac{F_H^i(L_i, H_i)}{F_H^L(L_i, H_i) F_L^i(L_i, H_i)} \right) \left( L_i F_H^i(L_i, H_i) H_i \right)
\]
\[
= \left( \frac{F_H^i(L_i, H_i)}{F_L^i(L_i, H_i)} \right) (H_i)
\]
\[
= \left( \frac{p_i A_i F_H^i(L_i, H_i)}{p_i A_i F_L^i(L_i, H_i)} \right) (H_i)
\]
\[
= \left( \frac{W_H H_i}{p_i A_i F_L^i(L_i, H_i)} \right) = 1 - \alpha_i \equiv \text{H-labor share},
\]

where the last two lines use the first order conditions of the firm.

Similarly, it is straightforward to show that:
\[
\sigma_i \left( \eta_{F_H^i(1, \frac{L_i}{H_i})} \right) = -\alpha_i
\]

Armed with this lemma, we can now turn to the proof of Theorem 1:

**Proof.** Consider first the L type. From equation (1):
\[
EV^L = \tilde{W}^L - \sum_i s^L_i(p_0, W_0^L) \tilde{p}_i
\]
\[
= \sum_i s^L_i(p_0, W_0^L) \tilde{W}^L - \sum_i s^L_i(p_0, W_0^L) \tilde{p}_i
\]
\[
= \sum_i \left( \frac{W_i^L}{p_i} \right) s^L_i(p_0, W_0^L)
\]
\[
= \sum_i \left( A_i F_L^i(1, \frac{H_i}{L_i}) \right) s^L_i(p_0, W_0^L)
\]
\[
= \sum_i \left( F_L^i(1, \frac{H_i}{L_i}) \right) s^L_i(p_0, W_0^L) + \sum_i \tilde{A}_i s^L_i(p_0, W_0^L),
\]

where the penultimate line stems from the first order condition for the firm. We write each term
\( F^L_i \) as a function of \( \frac{W_L}{W_H} \), hence:

\[
EV^L = \sum_i \left( \eta_{F^L_i(\frac{W_L}{W_H})} \frac{W_L}{W_H} \right) s^L_i(p_0, W^L_0) + \frac{\sum_i \tilde{A}_i}{\frac{W_L}{W_H}} s^L_i(p_0, W^L_0) = \\
= \sum_i \left( \eta_{F^L_i(\frac{W_L}{W_H})} \sigma_i \frac{W_L}{W_H} \right) s^L_i(p_0, W^L_0) + \frac{\sum_i \tilde{A}_i}{\frac{W_L}{W_H}} s^L_i(p_0, W^L_0) = \\
= \sum_i \left( \eta_{F^L_i(\frac{W_L}{W_H})} \left(-\sigma_i\right) \frac{W_H}{W_L} \right) s^L_i(p_0, W^L_0) + \frac{\sum_i \tilde{A}_i}{\frac{W_L}{W_H}} s^L_i(p_0, W^L_0) = \\
= \left( \frac{W_H}{W_L} \right) \sum_i \left( \alpha_i s^L_i(p_0, W^L_0) - 1 \right) + \frac{\sum_i \tilde{A}_i}{\frac{W_L}{W_H}} s^L_i(p_0, W^L_0).
\]

Consider now a change to the productivity of sector \( k \), \( \tilde{A}_k \), holding productivity in all other sectors constant, then:

\[
\frac{EV^L}{\tilde{A}_k} = \frac{\eta_{W_H/W_L} \sum_i \alpha_i s^L_i(p_0, W^L_0) - 1}{\tilde{A}_k} + s^L_k(p_0, W^L_0)
\]

Applying a similar derivation for the H type implies:

\[
\frac{EV^H}{\tilde{A}_k} = \frac{\eta_{W_H/W_L} \sum_i \alpha_i s^H_i(p_0, W^H_0) + s^H_k(p_0, W^H_0)}{\tilde{A}_k}
\]

Taking the difference:

\[
\Delta \eta_{EV,A_k} := \frac{EV^H}{\tilde{A}_k} - \frac{EV^L}{\tilde{A}_k} = s^H_k(p_0, W^H_0) - s^L_k(p_0, W^L_0) + \eta_{W_H/W_L} \sum_i \alpha_i \left[ s^H_i(p_0, W^H_0) - s^L_i(p_0, W^L_0) \right].
\]

Lemma 1

Proof. Market clearing for sector \( i \) implies:

\[
LC_i(W_L, P) + HC_i(W_H, P) = \gamma_i
\]
\[ LP_i C_i(W_L, P) + HP_i C_i(W_H, P) = P Y_i = W_L L_i + W_H H_i \]

\[ = L_i(W_L + W_H \frac{H_i}{L_i}), \]

where the last two equations stem from production being CRS. We thus have:

\[ L_i = \frac{LP_i C_i(W_L, P) + HP_i C_i(W_H, P)}{W_L(1 + \frac{W_H H_i}{W_H L_i})} \tag{A.1} \]

Using firms’ first order conditions:

\[ \frac{W_H}{W_L} = \frac{F_H^i(1, \frac{H_i}{L_i})}{F_L^i(1, \frac{H_i}{L_i})} \]

so that we can define functions \( \Omega_i \) with:

\[ \frac{H_i}{L_i} = \Omega_i \left( \frac{W_H}{W_L} \right) \tag{A.2} \]

i.e., in equilibrium, each \( \frac{H_i}{L_i} \) is a function solely of the relative wage \( \frac{W_H}{W_L} \).

Now from equation (A.2), we have that \( H_i = L_i \Omega_i \left( \frac{W_H}{W_L} \right) \), and so from labor market clearing for the high and low types we get:

\[ H = \sum_{i=1}^{N} H_i = \sum_{i=1}^{N-1} \frac{H_i}{L_i} L_i + \frac{H_N}{L_N} \left( L - \sum_{i=1}^{N-1} L_i \right) \]

\[ = \sum_{i=1}^{N-1} \frac{H_i}{L_i} L_i + \frac{H_N}{L_N} \left( L - \sum_{i=1}^{N-1} L_i \right) \]

Pluggin equation (A.1) we then have:

\[ H = \sum_{i=1}^{N-1} \frac{H_i}{L_i} \left( \frac{LP_i C_i(W_L, P) + HP_i C_i(W_H, P)}{W_L(1 + \frac{W_H H_i}{W_H L_i})} \right) + \frac{H_N}{L_N} \left( L - \sum_{i=1}^{N-1} \left( \frac{LP_i C_i(W_L, P) + HP_i C_i(W_H, P)}{W_L(1 + \frac{W_H H_i}{W_H L_i})} \right) \right) \]

which after some manipulation yields:
\[
\frac{H}{L} = \frac{H_N}{L_N} + \sum_{i=1}^{N-1} \left( \frac{H_i}{L_i} - \frac{H_N}{L_N} \right) \left( \frac{s_i(W_L, P) + \frac{H}{L} \frac{W_H}{W_L} s_i(W_H, P)}{1 + \frac{W_H}{W_L} \frac{H_i}{L_i}} \right)
\]  
(A.3)

Since for all \(i\) we have that \(\frac{H_i}{L_i}\) are functions of \(\frac{W_H}{W_L}\), it is left to show that \(W_L, W_H,\) and \(P\) are also functions of only \(\frac{W_H}{W_L}\) and \(A\). This is seen from firms’ FOCs. Normalizing the price of the Nth good to one, exploiting the fact that the marginal products are homogenous of degree zero, and using the Nth sector’s first order condition, we have that

\[
W_L = A_N F^N_L (L_N, H_N) = A_N F^N_L (1, \frac{H_N}{L_N})
\]

\[
W_H = A_N F^N_H (L_N, H_N) = A_N F^N_H (1, \frac{H_N}{L_N})
\]

In regards to the price vector, \(P\), using the FOCs again we obtain:

\[
P_i = \frac{W_L}{A_i F^i_L (1, \frac{H_i}{L_i})} = \frac{A_N F^N_L (1, \frac{H_N}{L_N})}{A_i F^i_L (1, \frac{H_i}{L_i})}.
\]

Hence, we have shown that (A.3) is a market-clearing condition which consolidated goods and labor market clearing conditions as a funcion of only \(\frac{W_H}{W_L}\) and \(A\).

\[\square\]

**Theorem 2**

*Proof.* We start from equation (A.3), and for convenience ommit the explicit reference of the share function \((s^L_i\) and \(s^H_i)\) to their arguments \((W\) and \(P)\):

\[
\frac{H}{L} = \frac{H_N}{L_N} + \sum_{i=1}^{N-1} \left( \frac{H_i}{L_i} - \frac{H_N}{L_N} \right) \left( \frac{s^L_i + \frac{H}{L} \frac{W_H}{W_L} s^H_i}{1 + \frac{W_H}{W_L} \frac{H_i}{L_i}} \right).
\]

Recalling that \(\alpha_i\) is the labor share of the low-skilled, multiplying both sides by \(\frac{L_W}{H_W}\), and denoting \(S_H = \frac{H_W}{L_W + H_W}\) and \(S_L = \frac{L_W}{L_W + H_W}\), we obtain:
\[
\frac{W_H}{W_L} = \frac{L}{H} \frac{W_H H_N}{W_L L_N} + \frac{L}{H} \sum_{i=1}^{N-1} \frac{W_H H_i}{W_L L_i} - \frac{W_H H_N}{W_L L_N} \left( \frac{s_i^L + H \frac{W_H}{W_L} s_i^H}{1 + \frac{W_H}{W_L} L_i} \right)
\]
\[
= \frac{L}{H} \left( 1 - \frac{\alpha_N}{\alpha_N} \right) + \frac{L}{H} \sum_{i=1}^{N-1} \left( 1 - \frac{\alpha_i}{\alpha_N} \right) \left( \frac{S_H^L}{S_L^H} s_i^L + \frac{S_H}{S_L} s_i^H \right)
\]
\[
= \frac{L}{H} \left( 1 - \frac{\alpha_N}{\alpha_N} \right) + \frac{L}{H} \sum_{i=1}^{N} \left( 1 - \frac{\alpha_i}{\alpha_N} \right) \left( \frac{S_H^L}{S_L^H} s_i^L + \frac{S_H}{S_L} s_i^H \right)
\]
\[
= \frac{1}{\alpha_N} \frac{L}{H} + \frac{L}{H} \frac{S_H}{S_L} - \frac{L}{H} \sum_{i=1}^{N} \left( \frac{\alpha_i}{\alpha_N} \right) \left( s_i^L + \frac{S_H}{S_L} s_i^H \right)
\]
\[
= \frac{1}{\alpha_N} \frac{L}{H} + \frac{W_H}{W_L} - \frac{1}{\alpha_N} \frac{L}{S_L} \frac{L}{H} \sum_{i=1}^{N} \alpha_i \left( s_i^L + \frac{S_H}{S_L} s_i^H \right)
\]

And therefore:
\[
\frac{1}{\alpha_N} \frac{L}{H} + \frac{W_H}{W_L} - \frac{1}{\alpha_N} \frac{L}{S_L} \frac{L}{H} \sum_{i=1}^{N} \alpha_i \left( s_i^L + \frac{S_H}{S_L} s_i^H \right) = 0.
\]

Additional manipulation thus yields that equation (A.3) can be written as:
\[
S_L - \sum_{i=1}^{N} \alpha_i \left( s_i^L + \frac{S_H}{S_L} s_i^H \right) = 0.
\]

Since \( S_L = \frac{L W_H}{L W_L + H W_H} \), this is equivalent to:
\[
W_H \frac{W}{W_L} = \frac{L}{H} \left[ \frac{1}{\sum_{i=1}^{N} \alpha_i \left( s_i^L + \frac{S_H}{S_L} s_i^H \right)} - 1 \right],
\]

which directly implies that in equilibrium, the relative wage, \( \frac{W_H}{W_L} \), is inversely related to \( \sum_{i=1}^{N} \alpha_i \left( s_i^L + \frac{S_H}{S_L} s_i^H \right) \).

Given an exogenous change in productivity in the \( k \)th sector, \( A_k \), we therefore have:
\[
\text{sign}(\eta_{W_H W_L} A_k) = -\text{sign} \left\{ \frac{d}{dA_k} \left[ \sum_{i=1}^{N} \alpha_i \left( s_i^L + \frac{S_H}{S_L} s_i^H \right) \right] \right\}.
\]

\[\square\]

**Theorem 3**
Proof. For simplicity, and without loss of generality, we refer to the sector that experiences the technical change as sector \( N \), and assume that this sector is the numeriare, i.e. \( P_N = 1 \).

We begin with equation (A.3) in the proof of Lemma 1, which we repeat here for convenience,

\[
H(A, W_H, W_L) = H_N L_N + \sum_{i=1}^{N-1} \left( \frac{H_i}{L_i} - \frac{H_N}{L_N} \right) \left( \frac{s_i(W_L, P) + H \frac{W_H}{W_L} s_i(W_H, P)}{(1 + \frac{W_H}{W_L} \frac{H_i}{L_i})} \right) - \frac{H}{L} = 0.
\]

This equation implicitly defines the relative wage as a function of the exogenous productivity vector \( A \) while taking into account all the equilibrium conditions. Using the implicit function theorem to calculate the elasticity of the relative wage with respect to a change in the productivity of sector \( N \) (while holding other productivities constant), we have:

\[
\eta_{W_H, W_L, A} = -\frac{\eta_{A_N, W_H, W_L}}{\eta_{A_N}}.
\]

where as discussed in the text, \( \eta_{A_N} \) denotes the partial elasticity of \( H \) with respect to \( A_N \) (i.e. while holding the relative wage constant) and where \( \eta_{W_H, W_L} \) denotes the partial elasticity of \( H \) with respect to the relative wage.

**Numerator of Implicit Function Theorem**

For simplicity, define

\[
\gamma_i \left( \frac{W_H}{W_L}, A_N \right) := \frac{L_i}{L} = \frac{s_i(W_L, P) + H \frac{W_H}{W_L} s_i(W_H, P)}{(1 + \frac{W_H}{W_L} \frac{H_i}{L_i})}
\]  

(A.4)

where the equality follows from equation (A.1). We note that \( \gamma_i, L_i, H_i \) are all functions of \( \left( \frac{W_H}{W_L}, A_N \right) \), i.e. the relative wage and the the exogenous productivity; For ease of notation and to streamline the exposition, we suppress explicit dependence of these functions on their arguments.

Hence, it follows that

\[
\eta_{A_N} = \frac{H_N}{L_N} \eta_{W_H, W_L, A_N} + \sum_{i=1}^{N-1} \left( \frac{H_i}{L_i} \gamma_i \right) \eta_{L_i, \gamma_i, A_N} - \frac{1}{L} \sum_{i=1}^{N-1} \left( \frac{H_N}{L_N} \gamma_i \right) \eta_{L_i, \gamma_i, A_N}
\]

which following some algebraic manipulations yields

\[
\eta_{A_N} = \frac{1}{H} \sum_{i=1}^{N-1} \left[ \frac{H_i}{L_i} - \frac{H_N}{L_N} \right] \eta_{L_i, A_N} L_i
\]
Hence, we are required to calculate $\eta_{L_i,AN}$. Given equation (A.4), it follows that:

$$\eta_{L_i,AN} = \frac{Ls_i(W_L, P)}{L_i(1 + \frac{WH_i}{WL_i})} \eta_{S_i(W_L, P),AN} + \frac{H \frac{WH_i}{WL_i} s_i(W_H, P)}{L_i(1 + \frac{WH_i}{WL_i})} \eta_{S_i(W_H, P),AN}.$$  

For future reference we note that combining the last two equations it follows that

$$\eta_{K,AN} = \frac{1}{H} \sum_{i=1}^{N-1} \left[ \frac{H_i}{L_i} - \frac{H_N}{L_N} \right] \left( \frac{Ls_i(W_L, P)}{L_i(1 + \frac{WH_i}{WL_i})} \eta_{S_i(W_L, P),AN} + \frac{H \frac{WH_i}{WL_i} s_i(W_H, P)}{L_i(1 + \frac{WH_i}{WL_i})} \eta_{S_i(W_H, P),AN} \right)$$  

(A.5)

Now, the elasticity of the share function with respect to the productivity of sector $N$, $\eta_{S_i(W_L, P),AN}$, is given by

$$\eta_{S_i(W_L, P),AN} = \eta_{\frac{PC_i(W_L, P)}{W_L},AN} = \eta_{P,AN} - \eta_{W_L,AN} + \eta_{C_i(W_L, P),AN}$$  

(A.6)

Recalling that sector $N$ is the numeraire, then from the first order conditions as shown in the proof of Lemma 1 it follows that

$$W_L = ANF_L^N(1, \frac{H_N}{L_N})$$  

(A.7)

and for each sector $i \neq N$,

$$P_i = \frac{WL}{ANF_i^i(1, \frac{H_i}{L_i})} = \frac{ANF_L^N(1, \frac{H_N}{L_N})}{ANF_i^i(1, \frac{H_i}{L_i})^N}.$$  

(A.8)

As in Theorem 2, we note from the firms’ first order conditions that the ratios $\frac{H_i}{L_i}$ are solely a function of the relative wage. This implies:

$$\eta_{W_L,AN} = \eta_{ANF_L^N(1, \frac{H_N}{L_N}),AN} = 1,$$

and also:

$$\eta_{P,AN} = \eta_{ANF_L^N(1, \frac{H_N}{L_N}),AN} = 1.$$
Plugging this in into Equation (A.6), we obtain

\[ \eta_{s_i}(W_L, p)_{AN} = \eta_{C_i}(W_L, p)_{AN} \]  

(A.9)

Hence, we are now required to calculate the partial elasticity of the uncompensated demand to \( A_N \).

To do so, note that this demand is a composite function \( C_i(W_L, p) = f_2 \circ f_1 \) with:

- \( f_1 : \frac{W_H}{W_L} \rightarrow \left( \begin{array}{c} P(A_N, \frac{W_H}{W_L}) \\ W_L(A_N, \frac{W_H}{W_L}) \end{array} \right)_{N \times 1} \), with dimensions due to having \( N - 1 \) prices. Given Equations (A.7)-(A.8), and recalling that the price vector has only \( N - 1 \) elements, it is easy to see that the elasticity matrix is given by:

\[ \begin{bmatrix} 
\eta_{P,A_N} \\ \eta_{W_L,A_N} 
\end{bmatrix}_{N \times 1} = 
\begin{bmatrix} 
1 \\ 1 \\ 1 \\
1 
\end{bmatrix}_{N \times 1} \]

- \( f_2 : \left( \begin{array}{c} P \\ W_L \end{array} \right)_{N \times 1} \rightarrow C_i(W_L, p)_{1 \times 1} \). Thus, the elasticity matrix is

\[ \left( \begin{array}{c} \eta_{C_i,p_j} \\ \eta_{C_i,W_L} \end{array} \right)_{1 \times N} \]

Using the multivariate chain rule, we thus have:

\[ \eta_{C_i(W_L,p),A_N} = \left( \begin{array}{c} \eta_{C_i,p_j} \\ \eta_{C_i,W_L} \end{array} \right)_{1 \times N} \ast (1)_{N \times 1} \]

Plugging this into Equation (A.9), we obtain

\[ \eta_{s_i}(W_L, p)_{AN} = \eta_{C_i(W_L, p)_{AN}} \]

\[ = \left( \begin{array}{c} \eta_{C_i,p_j} \\ \eta_{C_i,W_L} \end{array} \right)_{1 \times N} \ast (1)_{N \times 1} = (\Sigma^{N-1} \eta_{C_i,p_j} \eta_{p_j,A_N}) + \eta_{C_i,W_L} \eta_{W_L,A_N} \]

\[ = (\Sigma^{N-1} \eta_{C_i,p_j}) + \eta_{C_i,W_L} \]

\[ = -\eta_{C_i(W_L, p)_{P_N}} \]
where the last line stems from the homogeneity of degree zero of the uncompensated demand function $\eta_{C,i} + \sum_{j=1}^{N} \eta_{C,j} P_j = 0$. Following a similar line of reasoning, it is possible to show that:

$$\eta_{s_j(W_H,P),A_N} = -\eta_{C_i(W_H,P),P_N}.$$ 

Taking these two results and plugging them into Equation (A.5) it follows that:

$$-\eta_{i, A_N} = -\sum_{i=1}^{N-1} \left( \frac{H_i}{H} - \frac{H_N L_i}{H L_i} \right) \left( \frac{L S_i(W_L,P)}{L_i(1 + \frac{W_{pH} H_i}{W_{pL} L_i})} (-\eta_{C_i(W_L,P),P_N}) + \frac{H W_{L} S_i(W_H,P)}{L_i(1 + \frac{W_{pH} H_i}{W_{pL} L_i})} (-\eta_{C_i(W_H,P),P_N}) \right)$$

which after some algebraic manipulation yields:

$$-\eta_{i, A_N} = \sum_{i=1}^{N-1} \left( \frac{\alpha_N - \alpha_i}{\alpha_N} \right) \left( \frac{1}{S_{H}^{0}} \right) \left( S_{L}^{0} S_i(W_L,P) \eta_{C_i(W_L,P),P_N} + S_{H}^{0} S_i(W_H,P) \eta_{C_i(W_H,P),P_N} \right). \quad (A.10)$$

Multiplying by $\alpha_N$, we have:

$$-\alpha_N \eta_{i, A_N} = \sum_{i=1}^{N-1} \left( \alpha_N - \alpha_i \right) \left( \frac{1}{S_{H}^{0}} \right) \left( S_{L}^{0} S_i(W_L,P) \eta_{C_i(W_L,P),P_N} + S_{H}^{0} S_i(W_H,P) \eta_{C_i(W_H,P),P_N} \right)$$

$$= \frac{1}{S_{H}^{0}} \sum_{i=1}^{N} \left( \alpha_N - \alpha_i \right) \left( S_{L}^{0} S_i(W_L,P) \eta_{C_i(W_L,P),P_N} + S_{H}^{0} S_i(W_H,P) \eta_{C_i(W_H,P),P_N} \right)$$

$$= \alpha_N \frac{1}{S_{H}^{0}} \sum_{i=1}^{N} \left( S_{L}^{0} S_i(W_L,P) \eta_{C_i(W_L,P),P_N} + S_{H}^{0} S_i(W_H,P) \eta_{C_i(W_H,P),P_N} \right)$$

$$- \frac{1}{S_{H}^{0}} \sum_{i=1}^{N} \alpha_i \left( S_{L}^{0} S_i(W_L,P) \eta_{C_i(W_L,P),P_N} + S_{H}^{0} S_i(W_H,P) \eta_{C_i(W_H,P),P_N} \right)$$

$$= \alpha_N \frac{1}{S_{H}^{0}} \left[ -S_{L}^{0} S_{L,N} - S_{H}^{0} S_{H,N} \right] - \frac{1}{S_{H}^{0}} \sum_{i=1}^{N} \alpha_i \left( S_{L}^{0} S_i(W_L,P) \eta_{C_i(W_L,P),P_N} + S_{H}^{0} S_i(W_H,P) \eta_{C_i(W_H,P),P_N} \right)$$

where the last line relies on Cournot aggregation whereby

$$S_{L}^{0} \left\{ \sum_{i=1}^{N} \left( s_i(W_L,P) \eta_{s_i(W_L,P),P_N} \right) - s_N(W_L,P) \right\} = -S_{L}^{0} s_N(W_L,P)$$

and for an analogous equation for the H types. Finally, some additional algebraic manipulation yields:

$$-\alpha_N \eta_{i, A_N} = -\frac{1}{S_{H}^{0}} \sum_{i=1}^{N} \alpha_i \left( S_{L}^{0} S_i(W_L,P) \eta_{s_i(W_L,P),P_N} + S_{H}^{0} S_i(W_H,P) \eta_{s_i(W_H,P),P_N} \right) \quad (A.11)$$
Denominator of Implicit Function Theorem

Beginning again from the definition of $\mathcal{H}$, we have:

$$
\eta_{\mathcal{H}, \frac{w_H}{w_L}} = \frac{H_N}{L_N} \eta_{\frac{w_H}{w_L}} + \sum_{i=1}^{N-1} \frac{H_i}{L_i} \frac{w_H}{w_L} \eta_{\frac{w_H}{w_L}} - \frac{1}{H_L \sum_{i=1}^{N-1} \frac{w_H}{w_L}} \left( \frac{w_H}{w_L}, A_N \right) \eta_{\frac{w_H}{w_L}} \frac{w_H}{w_L}
$$

Recalling that $\eta_{\frac{w_H}{w_L}}$ is the elasticity of substitution between $L_i$ and $H_i$, i.e. $\sigma_i$, we have

$$
\eta_{\mathcal{H}, \frac{w_H}{w_L}} = \frac{1}{H} \left\{ \sum_{i=1}^{N} \left\{ -H_i \sigma_i + H_i \eta_{L_i \frac{w_H}{w_L}} \right\} - \frac{H_N}{L_N} \sum_{i=1}^{N} \left\{ L_i \sigma_i + L_i \eta_{H_i \frac{w_H}{w_L}} \right\} \right\}
$$

where the second line again stems from algebraic manipulation. We proceed by calculating $\eta_{L_i \frac{w_H}{w_L}}$.

Recalling that $L_i = \frac{L_i (W_L P) + H_i \frac{w_H}{w_L} L_i (W_H P)}{1 + \frac{w_H}{w_L} L_i}$, it follows that...
\[ \eta_{L, \frac{w}{W}} = \eta_{L_s(W_L, P) + H \frac{w}{W}, s(W_H, P), \frac{w}{W}} - \eta_{(1 + \frac{w}{W})L_s(W_L, P), \frac{w}{W}} = \]
\[= \frac{L_s(W_L, P)}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \eta_{s(W_L, P), \frac{w}{W}} + \frac{H \frac{w}{W} s(W_H, P)}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \left( 1 + \eta_{s(W_H, P), \frac{w}{W}} \right) - \frac{w_H}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \left( 1 + \eta_{L_s(W_L, P), \frac{w}{W}} \right) \]
\[= \frac{L_s(W_L, P)}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \eta_{s(W_L, P), \frac{w}{W}} + \frac{H \frac{w}{W} s(W_H, P)}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \left( 1 + \eta_{s(W_H, P), \frac{w}{W}} \right) - (1 - \alpha) (1 - \sigma) \]
\[= \frac{L_s(W_L, P)}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \eta_{s(W_L, P), \frac{w}{W}} + \frac{H \frac{w}{W} s(W_H, P)}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \left( 1 + \eta_{s(W_H, P), \frac{w}{W}} \right) - (1 - \alpha) (1 - \sigma) \]

Thus, we need to calculate \( \eta_{s(W_L, P), \frac{w}{W}} \) and \( \eta_{s(W_H, P), \frac{w}{W}} \). Starting from the first term we have:

\[ \eta_{s(W_L, P), \frac{w}{W}} = \eta_{P, \frac{w}{W}} + \eta_{C, (W_L, P), \frac{w}{W}} = \]
\[= \frac{1}{L_s(W_L, P) + H \frac{w}{W} s(W_H, P)} \eta_{C, (W_L, P), \frac{w}{W}} \]
\[= - \eta_{F, (1 + \frac{w}{W}), \frac{w}{W}} + \eta_{C, (W_L, P), \frac{w}{W}} \]
\[= - \eta_{F, (1 + \frac{w}{W}) \frac{w}{W}, \frac{w}{W}} + \eta_{C, (W_L, P), \frac{w}{W}} \]
\[= \eta_{F, (1 + \frac{w}{W}) \frac{w}{W}, \frac{w}{W}} \sigma + \eta_{C, (W_L, P), \frac{w}{W}} \]
\[= (1 - \alpha) + \eta_{C, (W_L, P), \frac{w}{W}} \]

where the last line follows from Lemma A.1. We therefore turn to calculating \( \eta_{C, (W_L, P), \frac{w}{W}} \). We note that both \( W_L \) and \( P \) are functions of the relative wage, and thus we need to calculate first their elasticities – i.e. \( \eta_{W_L, \frac{w}{W}} \) as well as \( \eta_{P, \frac{w}{W}} \).
Because $W_L = A_N F_L^N (1, \frac{H_N}{L_N})$:

$$\eta_{W_L \frac{W_H}{W_L}} = \eta_{A_N F_L^N (1, \frac{H_N}{L_N}) \frac{W_H}{W_L}}$$

$$= \eta_{F_L^N (1, \frac{H_N}{L_N}) \frac{W_H}{W_L}}$$

$$= \eta_{F_L^N (1, \frac{H_N}{L_N}) \frac{H_N}{L_N} \frac{W_H}{W_L}}$$

$$= \frac{1 - \alpha_N}{\sigma_N} \cdot \eta_{\frac{H_N}{L_N} \frac{W_H}{W_L}}$$

$$= \frac{1 - \alpha_N}{\sigma_N} \cdot (-\sigma_N) = \alpha_N - 1,$$

where the penultimate line again stems from Lemma A.1. Further,

$$\eta_{P_i \frac{W_H}{W_L}} = \eta_{\frac{W_L}{A_N F_L^N (1, \frac{H_N}{L_N})} \frac{W_H}{W_L}}$$

$$= \eta_{W_L \frac{W_H}{W_L}} - \eta_{F_L^N (1, \frac{H_N}{L_N}) \frac{W_H}{W_L}}$$

$$= \alpha_N - 1 - \left( \eta_{F_L^N (1, \frac{H_N}{L_N}) \frac{H_N}{L_N} \frac{W_H}{W_L}} \right)$$

$$= \alpha_N - 1 - (\alpha_i - 1)$$

$$= \alpha_N - \alpha_i$$

To calculate the elasticity of the uncompensated demand $\eta_{C_i(W_L, P) \frac{W_H}{W_L}}$, note now that this demand is a composite function $C_i(W_L, P) = f_2 \circ f_1$ with:

- $f_1 : \frac{W_H}{W_L} \rightarrow \left( P(A_N, \frac{W_H}{W_L}) \begin{pmatrix} W_L(A_N, \frac{W_H}{W_L}) \end{pmatrix} \right)_{N \times 1}$, with dimensions due to having $N - 1$ prices. Given the formulas above for $\eta_{W_L \frac{W_H}{W_L}}$ and $\eta_{P_i \frac{W_H}{W_L}}$, and recalling that the price vector has only $N - 1$ elements, the elasticity matrix is:

$$\begin{pmatrix} \eta_{P_i \frac{W_H}{W_L}} \\ \vdots \\ \eta_{W_L \frac{W_H}{W_L}} \end{pmatrix}_{N \times 1} = \begin{pmatrix} \alpha_N - \alpha_i \\ \vdots \\ \alpha_N - 1 \end{pmatrix}_{N \times 1}$$
\[
\bullet f_2: \begin{pmatrix} P \\ W_L \end{pmatrix}_{N \times 1} \rightarrow C_i(P, W_L)_{1 \times 1}. \text{ So that the elasticity matrix is}
\]
\[
\begin{pmatrix}
\cdots 
\eta_{C_i, p_j} & \eta_{C_i, W_L} 
\end{pmatrix}_{1 \times N}
\]

Therefore, using the multivariate chain rule on the composite function \( C_i(W_L, P) \), the elasticity \( \eta_{C_i(W_L, P), W_L} \) satisfies:

\[
\eta_{C_i(W_L, P), SP} = \begin{pmatrix}
\cdots 
\eta_{C_i, p_j} & \eta_{C_i, W_L} 
\end{pmatrix}_{1 \times N} \begin{pmatrix}
\alpha_N - \alpha_j \\
\cdot \\
\alpha_N - 1
\end{pmatrix}_{N \times 1}
\]

Plugging this into the equation for \( \eta_{s_i(W_L, P), W_L} \) above, we have that

\[
\eta_{s_i(W_L, P), W_L} = 1 - \alpha_i + \begin{pmatrix}
\cdots 
\eta_{C_i, p_j} & \eta_{C_i, W_L} 
\end{pmatrix}_{1 \times N} \begin{pmatrix}
\alpha_N - \alpha_j \\
\cdot \\
\alpha_N - 1
\end{pmatrix}_{N \times 1} \tag{A.14}
\]

By an analogous line of reasoning, it is possible to show that

\[
\eta_{s_i(W_H, P), W_L} = -\alpha_i + \begin{pmatrix}
\cdots 
\eta_{C_i, p_j} & \eta_{C_i, W_H} 
\end{pmatrix}_{1 \times N} \begin{pmatrix}
\alpha_N - \alpha_j \\
\cdot \\
\alpha_N
\end{pmatrix}_{N \times 1} \tag{A.15}
\]

Plugging these two last equations into (A.13) and plugging that into (A.12) some algebraic manipulation,
\[ \eta_{\mathcal{H}, \frac{W_H}{W_L}} = \sum_{i=1}^{N} \left( \frac{\alpha_N - \alpha_i}{\alpha_N} \right) \frac{L}{H} \frac{W_L}{W_H} \left\{ s_{i,L} (1 - \alpha_i) + \left( s_{i,L} \eta_{C_i,P_j} \cdot s_{i,L} \eta_{C_i,W_L} \right)_{1 \times N}^* \right\}^{N \times 1} \]

\[ + \frac{W_H}{W_L} \frac{H}{L} \left\{ s_{i,H} (1 - \alpha_i) + \left( s_{i,H} \eta_{C_i,P_j} \cdot s_{i,H} \eta_{C_i,W_H} \right)_{1 \times N}^* \right\}^{N \times 1} \]

\[ - 1 + \sum_{i=1}^{N} \left( \frac{\alpha_i}{\alpha_N} \frac{W_H}{W_H} \left( 1 - \sigma_i \right) \right) \]

Combining the expressions for the numerator \((\eta_{\mathcal{H}, A_N})\) and the denominator \((\eta_{\mathcal{H}, \frac{W_H}{W_L}})\) of the implicit function theorem and further algebraic manipulations yields Theorem 3 in the main body of the paper:

\[ \eta_{\frac{W_H}{W_L}} = -\sum_{i=1}^{N} \alpha_i \left( S_{L,i,L} \eta_{S_{i,L,P_N}} + S_{H,S_{i,H}} \eta_{S_{i,H},P_N} \right) \frac{G(\alpha, \sigma, S_L, S_H, S_L, S_H, \eta_{C,P}, \eta_{C,P}, \eta_{C,W}, \eta_{C,W})}{G(\alpha, \sigma, S_L, S_H, S_L, S_H, \eta_{C,P}, \eta_{C,P}, \eta_{C,W}, \eta_{C,W})}, \]

with

\[ G := S_H \left[ \sum_{i=1}^{N} (\alpha_N - \alpha_i) \frac{S_L}{S_H} \Omega_i - \alpha_N + \sum_{i=1}^{N} \left( \alpha_i \frac{W_H}{W_H} (1 - \sigma_i) \right) \right] \]
\[
\Omega_i := \begin{bmatrix}
  s_{i,L} (1 - \alpha_i) + (s_{i,L} \eta^L_{C_i,P_j} \cdot s_{i,L} \eta^L_{C_i,W})_{1 \times N} & \cdot \\
  \cdot & \cdot \\
  \cdot & \cdot
\end{bmatrix}^* \begin{pmatrix}
  \alpha_N - \alpha_j \\
  \alpha_N - 1 \\
  \alpha_N
\end{pmatrix}_{N \times 1}
\]

\[
+ \frac{S_H}{S_L} \begin{bmatrix}
  s_{i,H} (1 - \alpha_i) + (s_{i,H} \eta^H_{C_i,P_j} \cdot s_{i,H} \eta^H_{C_i,W})_{1 \times N} & \cdot \\
  \cdot & \cdot \\
  \cdot & \cdot
\end{bmatrix}^* \begin{pmatrix}
  \alpha_N - \alpha_j \\
  \alpha_N - 1 \\
  \alpha_N
\end{pmatrix}_{N \times 1}
\]

Lemma 2

Proof. Because the denominator in Equation 6 is negative, a sufficient condition for \(\eta \frac{w_H}{w_L} A_k\) to be positive is that for each type \(T \in \{H, L\}\):

\[0 < \sum_{i=1}^{N} \alpha_i s_{i,T} \eta_{s_{i,T} P_k}\]  \hspace{1cm} (A.16)

Since \(s_i = \frac{C_i P_i}{W_i}\), for all \(i \neq k\) we have that \(\eta_{s_{i,T} P_k} = \eta_{C_i,T P_k}\), whereas for \(i = k\) we have \(\eta_{s_{i,T} P_k} = \eta_{C_i,T P_k} + 1\). Therefore:

\[
\sum_{i=1}^{N} \alpha_i s_{i,T} \eta_{s_{i,T} P_k} = \alpha_k s_{k,T} + \sum_{i=1}^{N} \alpha_i s_{i,T} \eta_{C_i,T P_k}
\]

\[
= \alpha_k s_{k,T} - s_{k,T} \sum_{i=1}^{N} \alpha_i \frac{s_{i,T} \eta_{C_i,T P_k}}{-s_{k,T}}
\]

\[
= \alpha_k s_{k,T} - s_{k,T} \sum_{i=1}^{N} \alpha_i \frac{s_{i,T} \eta_{C_i,T P_k}}{\sum_{j=1}^{N} s_{j,T} \eta_{C_j P_k}}
\]

where the last line stems from Cournot aggregation.

Defining \(w_{i,T} := \frac{s_{i,T} \eta_{C_i,T P_k}}{\sum_{i} s_{i,T} \eta_{C_i,T P_k}}\) and plugging into Inequality (A.16), we have that a sufficient condition for the elasticity \(\eta \frac{w_H}{w_L} A_k\) to be positive is that for each type \(T \in \{H, L\}\):

\[0 < \alpha_k s_{k,T} - s_{k,T} \sum_{i=1}^{N} \alpha_i w_{i,T}\]

which completes the proof. \(\square\)
Corollary 1

Proof. From Lemma 2 it is easy to see that since the denominator in Equation (6) is negative, \( \eta w_{W, T, A_k} \) will be positive when for each type \( T \in \{H, L\} \):

\[
\sum_{i=1}^{N} \alpha_i w_{i, T} < \alpha_k
\]

Now, if all goods are gross complements, all \( \{w_{i, T}\}_{i=1}^{N} \) are non-negative. To see this note that

\[
w_{i, T} := \frac{s_{i, T} \eta_{C, T} p_k}{\sum_i s_i \eta_{C, T} p_k} = \frac{s_{i, T} (-\eta_{C, T} p_k)}{-\sum_i s_i \eta_{C, T} p_k} = \frac{s_{i, T} (-\eta_{C, T} p_k)}{s_{k, T}} \geq 0
\]

where the last equality again stems form Cournot aggregation, and the inequality stems from the fact that goods are gross complements.

Because \( \{w_{i, T}\}_{i=1}^{N} \) sum to one (by their definition) and are non-negative, under the conditions of Corollary 2 they are weights. As such, when the lowest-skill sector experiences the technical change – i.e. \( \alpha_k = max\{\alpha_i\} \) – then the inequality \( \sum_{i=1}^{N} \alpha_i w_{i, T} < \alpha_k \) will hold for \( T \in \{H, L\} \), and \( \eta w_{W, T, A_k} \) will therefore be positive.

By an analogous argument it is easy to show that when the highest-skill sector experiences the technical change – i.e. \( \alpha_k = min\{\alpha_i\} \) – then \( \eta w_{W, T, A_k} \) will be negative.

\[\square\]

A.2. Empirical Analysis

A.2.1. Data Construction and Estimation Details

The figure below provides a schematic representation of our data linking process.
A.2.2. Appendix Tables

Table A1: Descriptive Statistics

<table>
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<tr>
<th></th>
<th>Low skill</th>
<th>High skill</th>
<th>Low-skilled labor share</th>
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<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
<td>Mean</td>
</tr>
<tr>
<td>A. Expenditure shares: 3 goods</td>
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<td></td>
<td></td>
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<td>Agriculture</td>
<td>0.13</td>
<td>0.05</td>
<td>0.11</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.27</td>
<td>0.09</td>
<td>0.25</td>
</tr>
<tr>
<td>Services</td>
<td>0.6</td>
<td>0.1</td>
<td>0.64</td>
</tr>
<tr>
<td>B. Expenditure shares: 7 goods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food home</td>
<td>0.13</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>Housing</td>
<td>0.35</td>
<td>0.11</td>
<td>0.37</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.07</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.16</td>
<td>0.1</td>
<td>0.15</td>
</tr>
<tr>
<td>Food away, ent., apparel</td>
<td>0.14</td>
<td>0.07</td>
<td>0.15</td>
</tr>
<tr>
<td>Other services</td>
<td>0.09</td>
<td>0.07</td>
<td>0.1</td>
</tr>
<tr>
<td>Durables</td>
<td>0.06</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>C. Household aggregates and controls</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal household expenditures</td>
<td>37,644</td>
<td>20,188</td>
<td>48,420</td>
</tr>
<tr>
<td>Nominal household after tax income</td>
<td>53,578</td>
<td>29,974</td>
<td>73,766</td>
</tr>
<tr>
<td>Age</td>
<td>45.5</td>
<td>10.4</td>
<td>44.3</td>
</tr>
<tr>
<td>Number of family members</td>
<td>3</td>
<td>1.5</td>
<td>2.7</td>
</tr>
<tr>
<td>A two earner household</td>
<td>0.6</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Number of households</td>
<td>18,045</td>
<td></td>
<td>9,226</td>
</tr>
<tr>
<td>Number of observations</td>
<td>56,018</td>
<td></td>
<td>29,509</td>
</tr>
</tbody>
</table>

Notes: Descriptive Statistics for CEX sample used in the estimation of the AIDS.
Table A2: Expenditure and price elasticities: 3 categories

A. Low-skilled:

<table>
<thead>
<tr>
<th></th>
<th>Expenditure Elasticity</th>
<th>Price Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Agriculture</td>
</tr>
<tr>
<td>Agriculture</td>
<td></td>
<td>-0.274*</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.151)</td>
</tr>
<tr>
<td></td>
<td>[0.307,0.36]</td>
<td>[-0.523,-0.019]</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.874***</td>
<td>0.128***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
</tr>
<tr>
<td></td>
<td>[0.848,0.902]</td>
<td>[0.1,0.154]</td>
</tr>
<tr>
<td>Services</td>
<td>1.194***</td>
<td>-0.207***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.026)</td>
</tr>
<tr>
<td></td>
<td>[1.177,1.203]</td>
<td>[-0.248,-0.16]</td>
</tr>
</tbody>
</table>

B. High-skilled:

<table>
<thead>
<tr>
<th></th>
<th>Expenditure Elasticity</th>
<th>Price Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Agriculture</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.442***</td>
<td>-0.579***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.21)</td>
</tr>
<tr>
<td></td>
<td>[0.394,0.482]</td>
<td>[-0.914,-0.243]</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.856***</td>
<td>0.135***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.027)</td>
</tr>
<tr>
<td></td>
<td>[0.812,0.908]</td>
<td>[0.091,0.18]</td>
</tr>
<tr>
<td>Services</td>
<td>1.163***</td>
<td>-0.134***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.033)</td>
</tr>
<tr>
<td></td>
<td>[1.138,1.184]</td>
<td>[-0.181,-0.077]</td>
</tr>
</tbody>
</table>

Notes: Expenditure and uncompensated price elasticities implied by the AIDS estimates. Bootstrapped standard errors in parentheses, and 90% confidence intervals in square brackets.
Table A3: Expenditure and price elasticities: 7 categories, low-skilled

<table>
<thead>
<tr>
<th>Expenditure Elasticity</th>
<th>Food home</th>
<th>Housing</th>
<th>Utilities</th>
<th>Transportation</th>
<th>Food away, ent., apparel</th>
<th>Other services</th>
<th>Durables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food home</td>
<td>0.524***</td>
<td>-0.673***</td>
<td>-0.355*</td>
<td>0.236***</td>
<td>-0.091</td>
<td>0.028</td>
<td>0.109</td>
</tr>
<tr>
<td>(0.011)</td>
<td>(0.255)</td>
<td>(0.189)</td>
<td>(0.078)</td>
<td>(0.08)</td>
<td>(0.175)</td>
<td>(0.066)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>[0.508,0.542]</td>
<td>[-1.081,-0.238]</td>
<td>[-0.661,-0.048]</td>
<td>[0.110,0.357]</td>
<td>[-0.221,0.033]</td>
<td>[-0.237,0.306]</td>
<td>[-0.001,0.228]</td>
<td>[0.127,0.334]</td>
</tr>
<tr>
<td>Housing</td>
<td>1.046***</td>
<td>-0.190***</td>
<td>-0.374***</td>
<td>-0.085***</td>
<td>0.035</td>
<td>-0.418***</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.07)</td>
<td>(0.114)</td>
<td>(0.032)</td>
<td>(0.042)</td>
<td>(0.059)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>[1.031,1.06]</td>
<td>[-0.313,-0.088]</td>
<td>[-0.55,-0.19]</td>
<td>[-0.132,-0.026]</td>
<td>[-0.044,0.098]</td>
<td>[-0.516,-0.334]</td>
<td>[-0.102,0.039]</td>
<td>[-0.012,0.082]</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.664***</td>
<td>0.455***</td>
<td>-0.323*</td>
<td>-0.734***</td>
<td>0.051</td>
<td>-0.065</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.155)</td>
<td>(0.169)</td>
<td>(0.088)</td>
<td>(0.088)</td>
<td>(0.128)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>[0.645,0.683]</td>
<td>[0.203,0.691]</td>
<td>[-0.569,-0.009]</td>
<td>[-0.881,-0.594]</td>
<td>[-0.106,0.184]</td>
<td>[-0.293,0.15]</td>
<td>[-0.048,0.152]</td>
<td>[-0.219,0.029]</td>
</tr>
<tr>
<td>Transportation</td>
<td>1.06***</td>
<td>-1.141**</td>
<td>0.069</td>
<td>-0.006</td>
<td>-0.775***</td>
<td>-0.028</td>
<td>-0.102**</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.063)</td>
<td>(0.089)</td>
<td>(0.035)</td>
<td>(0.079)</td>
<td>(0.057)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>[1.036,1.083]</td>
<td>[-0.243,-0.045]</td>
<td>[-0.09,0.205]</td>
<td>[-0.069,0.046]</td>
<td>[-0.888,-0.626]</td>
<td>[-0.122,0.069]</td>
<td>[-0.179,-0.029]</td>
<td>[-0.132,-0.026]</td>
</tr>
<tr>
<td>Food away, ent., apparel</td>
<td>1.245***</td>
<td>-0.067</td>
<td>-1.104***</td>
<td>-0.068</td>
<td>-0.063</td>
<td>0.33**</td>
<td>-0.059</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.161)</td>
<td>(0.145)</td>
<td>(0.059)</td>
<td>(0.067)</td>
<td>(0.155)</td>
<td>(0.073)</td>
</tr>
<tr>
<td>[1.222,1.265]</td>
<td>[-0.312,0.192]</td>
<td>[-1.348,-0.895]</td>
<td>[-0.174,0.032]</td>
<td>[-0.172,0.051]</td>
<td>[0.091,0.591]</td>
<td>[-0.16,0.07]</td>
<td>[-0.315,-0.124]</td>
</tr>
<tr>
<td>Other services</td>
<td>1.269***</td>
<td>0.062</td>
<td>-0.204</td>
<td>0</td>
<td>-0.222***</td>
<td>-0.096</td>
<td>-0.883***</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.096)</td>
<td>(0.164)</td>
<td>(0.047)</td>
<td>(0.084)</td>
<td>(0.114)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>[1.235,1.302]</td>
<td>[-0.096,0.237]</td>
<td>[-0.475,0.071]</td>
<td>[-0.075,0.073]</td>
<td>[-0.362,-0.09]</td>
<td>[-0.254,0.107]</td>
<td>[-1.082,-0.684]</td>
<td>[-0.009,0.158]</td>
</tr>
<tr>
<td>Durables</td>
<td>0.986***</td>
<td>0.392***</td>
<td>0.179</td>
<td>-0.119</td>
<td>-0.191**</td>
<td>-0.438***</td>
<td>0.129*</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.131)</td>
<td>(0.166)</td>
<td>(0.077)</td>
<td>(0.086)</td>
<td>(0.127)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>[0.961,1.007]</td>
<td>[0.199,0.615]</td>
<td>[-0.043,0.48]</td>
<td>[-0.246,0.009]</td>
<td>[-0.334,-0.056]</td>
<td>[-0.661,-0.238]</td>
<td>[0.012,0.247]</td>
<td>[-1.052,-0.801]</td>
</tr>
</tbody>
</table>

Notes: Expenditure and uncompensated price elasticities implied by the AIDS estimates. Bootstrapped standard errors in parentheses, and 90% confidence intervals in square brackets.
Table A4: Expenditure and price elasticities: 7 categories, high-skilled

<table>
<thead>
<tr>
<th>Expenditure Elasticity</th>
<th>Food home</th>
<th>Housing</th>
<th>Utilities</th>
<th>Transportation</th>
<th>Food away, ent., apparel</th>
<th>Other services</th>
<th>Durables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expenditure Elasticity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| **Food home** | 0.603*** | -0.525* | -0.181 | -0.125 | 0.122 | 0.152 | 0.092 | -0.138*
| (0.014) | (0.315) | (0.238) | (0.082) | (0.087) | (0.214) | (0.083) | (0.073) |
| [0.58,0.626] | [-0.996,0.017] | [-0.557,0.216] | [-0.26,-0.006] | [-0.006,0.284] | [-0.198,0.489] | [-0.031,0.227] | [-0.261,-0.014] |
| **Housing** | 1.09*** | -0.119 | -0.393*** | -0.015 | -0.078 | -0.613*** | 0.019 | 0.108***
| (0.012) | (0.079) | (0.128) | (0.034) | (0.055) | (0.103) | (0.083) | (0.055) |
| [1.069,1.109] | [-0.246,0.017] | [-0.585,-0.178] | [-0.069,0.041] | [-0.171,0.013] | [-0.741,-0.494] | [-0.089,0.101] | [0.047,0.159] |
| **Utilities** | 0.639*** | -0.257 | 0.074 | -0.668*** | 0.039 | 0.383** | 0.055 | -0.264***
| (0.018) | (0.167) | (0.208) | (0.111) | (0.111) | (0.177) | (0.089) | (0.101) |
| [0.606,0.667] | [-0.532,-0.014] | [-0.248,0.412] | [-0.85,-0.49] | [-0.141,0.221] | [0.113,0.689] | [-0.08,0.209] | [-0.434,-0.1] |
| **Transportation** | 0.95*** | 0.055 | -0.135 | -0.003 | -0.733*** | 0.066 | -0.231*** | 0.031
| (0.023) | (0.069) | (0.132) | (0.044) | (0.114) | (0.085) | (0.072) | (0.051) |
| [0.912,0.989] | [-0.043,0.185] | [-0.375,0.072] | [-0.075,0.067] | [-0.928,0.531] | [-0.063,0.202] | [-0.36,-0.116] | [-0.048,0.109] |
| **Food away, ent., apparel** | 1.267*** | 0.046 | -1.587*** | 0.119 | 0.021 | 0.278 | -0.087 | -0.057
| (0.02) | (0.176) | (0.192) | (0.072) | (0.089) | (0.189) | (0.089) | (0.07) |
| [1.233,1.295] | [-0.243,0.323] | [-1.918,-1.281] | [-0.01,0.246] | [-0.116,0.162] | [-0.067,0.581] | [-0.221,0.073] | [-0.164,0.063] |
| **Other services** | 1.083*** | 0.053 | 0.072 | 0.006 | -0.371*** | -0.1 | -0.721*** | -0.022
| (0.033) | (0.1) | (0.212) | (0.053) | (0.109) | (0.129) | (0.162) | (0.062) |
| [1.055,1.139] | [-0.098,0.216] | [-0.332,0.362] | [-0.078,0.098] | [-0.559,0.202] | [-0.293,0.129] | [-0.971,-0.42] | [-0.115,0.105] |
| **Durables** | 0.957*** | -0.306** | 0.67*** | -0.268*** | 0.074 | -0.088 | -0.022 | -1.017***
| (0.023) | (0.138) | (0.203) | (0.096) | (0.124) | (0.162) | (0.098) | (0.116) |
| [0.918,0.995] | [-0.545,0.069] | [0.314,0.965] | [-0.431,-0.11] | [-0.117,0.265] | [-0.34,0.192] | [-1.65,0.179] | [-1.212,-0.83] |

Notes: Expenditure and uncompensated price elasticities implied by the AIDS estimates. Bootstrapped standard errors in parentheses, and 90% confidence intervals in square brackets.
A.2.3. Calculating Sectoral EOS

In our analysis, we aim for an average elasticity of substitution of 1.4, as indicated by the estimates in Krusell et al. (2000). Under our model’s assumption that wages specific to different skill levels are uniform across all sectors, the relationship between any two sectors elasticities of substitution equals the ratio of the sector-specific changes in the relative proportions of low-to-high skilled inputs. This relationship, combined with our target elasticity of 1.4 and the respective sectoral weights, allows us to calculate the varying degrees of labor substitution across different sectors. The specifics of these substitution degrees, reflecting how easily labor can switch between sectors, are detailed in Table A1, where we report the calculated values.

A.2.4. Data Construction for Public Sectors

The military personnel measures are taken from The center for strategic and budgetary assessment. In order to construct the skill intensity of this sector we calculate the share of workers that are low-vs-high skill in this sector. We then attribute to them the average low and high skill wages in the economy.

For the public health sector we note that the overall expenses on Medicare and Medicaid are approximately 12% of private consumption. We then calculate from the CPS the skill intensity of the health sector.