Concept of Non-exactness in Science Education

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Abstract

Historically, human beings, living in a complex and unknown world, learnt to survive by using rules, which were essentially simpler than the reality around them. The fact that human beings have survived for many hundreds of years in this complex and changing world, speaks for their ability to simplify rules for functioning in the surrounding reality according to their perception .

Different people simplify the surrounding reality in different ways. A subjective component is present in the perception of any scientific fact [1]. Therefore there exists a phenomenon of so-called "subjective science". This phenomenon contradicts the idea of the objective character of scientific knowledge, which is a basic principle of science education in an industrial society. We assume that, in the postindustrial society, the role of the subjective component in the scientific knowledge grows. New methods and contents of science education appear to use non-exact and insufficiently formalized models of reality .

We base our study on the fact that during any scientific study, a scientist is affected both by the reality and by the subjective perception of the reality, existing in a complex combination. If different scientists study one event, it is important to understand how much various versions of the event differ from one another .

In the following study, we suggest to a group of subjects various datasets, and ask them to derive some logical rule governing the sets. Each subject arrives to his/her own version about rules governing the dataset. Comparative study of the different versions is the object of our research. The study allows understanding of the manner in which people may analyze a system, what the common characteristics are of the obtained versions, and "the degree of difference" from the true rules of the system's functioning.

1 .Definitions

We will present some basic definition of Non-Exactness. We are discussing an understanding of some perceived environment - a dataset. Our dataset will be denoted **W** - the **World** perceived. The goal is to find a simple representation of W which encompasses all its points - a set of **Rules** denoted **R**. Non Exactness states some Rules (**R**) derived from a World (**W**) may not match all members of that World : .In other words, a non-exact set of Rules (**R**) are rules that may encompass some of the dataset **W**, but not necessarily all of it. Our definition leads us to these expected observations:

a. For a given W, there could be an **R** that does not accurately describe all items of **W**, in essence ignoring certain data points.

b. Our premise is that **R** should provide us with a simpler representation of **W**. Some work has been done on quantifying simplicity [2]. For the purposes of this paper, we rely on an intuitive sense of simplicity.

c. We expect that rules derived from **W** will have some level of redundancy: the rule will describe items that are not necessarily members of **W**.

d. A set of Rules **R** derived from **W** is not necessarily unique – there could be more than one derived set **R** for given **W**.

2 .Research description

Our goal is to study the principle of the non-exactness in Boolean concept learning - specifically, to study how people recognize reality in a case where this reality is describable by Boolean models. Throughout our research we present datasets to human subjects, and ask them to generalize each set into some rule. It is important to insure that each dataset does not have an obvious rule (for instance - a dataset of identical objects), but demands a degree of creativity on the part of the subject. On the other hand, each dataset needs to allow, on our part, an analysis of all its characteristics as well as the process by which a rule is derived - each dataset has to have a degree of compactness.

For these reasons we chose puzzles from the Bongard Problem set as our dataset [3]. Bongard Problems are a compilation of 100 problems. Each problem presents two sets of relatively simple diagrams, BPleft and BPright. All the diagrams from BPleft have a common characteristic, which does not exist in BPright. The problem is to find and formulate a convincing distinction rule between both sides[4].

For this research, with each Bongard Problem, the subject is told to describe some common attribute for BPleft and BPright, such that both sides are dissimilar. In addition, each subject is asked to describe avenues attempted prior to reaching his final conclusion. The point is to understand not only the distinction defined by the subject, but also to trace what other items in the BP were perceived. In this fashion we hope to retrace the structures evident to the subject and understand his simplification process using Boolean analysis. Our analysis uses the BP solution index available in Foundalis' doctoral dissertation [6.[

It is important to note that this research focuses on the manner in which people form rules in a situation where they decide what variables participate in the rule - we do not tell people what variable should be considered in forming the rule, instead allowing them the freedom to decide what variables to consider in the process of rule forming. Further research should be performed in the case where the variables are pre-recognized, and just the rule forming is given to the subjects.

3 .Preliminary Findings

We will present examples of BP's and the distinction rules given by individual subjects. As stated above, each subject was asked to provide a distinction rule and a description of other aspects of the problem they pursued. We will present our analysis of the final distinction rules given, in light of the different aspects of the BP. The idea is to create a Boolean model for aspects of the BP as perceived by the subject, and analyze the method by which a specific aspect (or complex grouping of aspects) is chosen as the distinction rule for the BP.

1 .The following is BP #2:

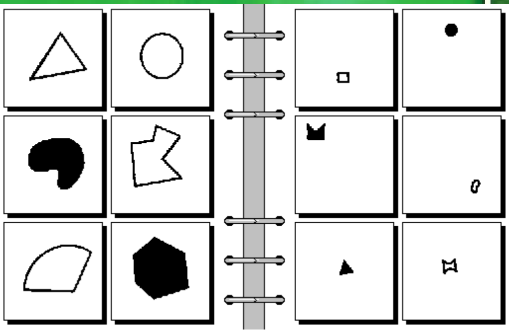


Fig 1. BP #2

Out of 6 subjects, 4 concluded that the distinction between BP_{left} and BP_{right} was size: BP_{left}=large; BP_{right}=small. This is the distinction recorded by Foundalis**[6]**. With one subject, a prior distinction was attempted on the basis of positioning (centered, not centered). We now have 2 evident Boolean variables:

a – centeredness (0=non-centered, 1=centered(

b – size (0=small, 1=large(

Reviewing items on both sides in terms of a,b, we find the following distinction:

BP_{left}: - items are both centered and large.

BP_{right}: - items can be centered or not centered, but always small.

Both sides of the BP can now be expressed as a single function, by introducing a new variable 's', denoting the side of the BP: 0=BP_{left}; 1=BP_{right}. The resulting expression is :

We can express these equations graphically using a Boolean cube as such**[5**:**[**

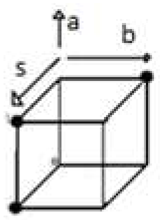


Fig. 2 - Boolean cube representation of BP #2

In this representation, the front panel reflects the situation in BP_{right} and the back panel reflects the situation in BP_{left}. All items in the back panel exist in the function ; i.e. fulfill both centeredness and are large. All items in the front panel can exist anywhere on the left axis - they are small and may be centered or non-centered .

There are two available distinctions between BP_{right} and BP_{left}:

BP_{right} =small; BP_{left} =large

BP_{right} =centered or non-centered; BP_{left} =centered

As is evident from the cube representation, the simpler distinction is that of size, as it clearly separates both sides of the BP. Adopting this distinction, a new cube representation emerges.

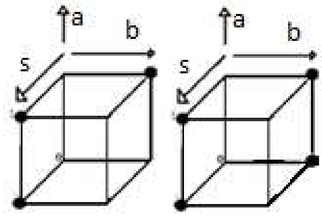


Fig. 3 (left) Original problem space; (right) Problem space using new distinction rule.

It is important to note that this solution has a drawback. Distinction on the basis of size assumes that BP_{left} could include items that are large but not centered, thus ignoring the fact that all items in BP_{left} are centered. We call this 'redundancy' – our simple solution covers unnecessary cases in BP_{left}. There may be over-redundant distinctions: distinctions that ignore seemingly pertinent information. In such cases, subjectivity is a major factor – one may prefer simplicity for the price of redundancy, whereas another may feel that the simplicity does not justify the insertion of redundancy into the model. Simplicity over accuracy (Non-Redundancy) demands (consciously or unconsciously) a form of choice, and inserts subjectivity to the analysis process.

2 .The following image (BP #4) was presented to 6 subjects :

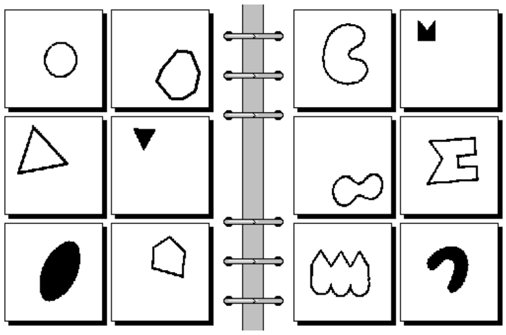


Fig. 4 BP #4

Most subjects immediately defined a distinction between both sides on the basis of convexity (BP_{left} =Convex; BP_{right}=Concave). That is the distinction presented by Foundalis[6] in his solution list. We examined other rules previously attempted by subjects .

With 2 subjects, a different distinction was first attempted: checking if curves are smooth or sharp. In Boolean terms, we have 4 evident variables :

a - smooth curves

b - sharp curves

c - convexity/concavity (0=convex, 1=concave(

s - side in Bongard Problem (0=left, 1=right(

Reviewing items on both sides in terms of a,b, we find the following distinction:

BP_{left}: - items on the left either sharp curves or smooth curves - but not both.

BP_{right}: =items on the right can be smooth, sharp or both.

In other words, there is a distinction between the BP_{left} and BP_{right} - the Boolean function . BP_{right} includes the function - its' items may contain smooth curves and sharp curves at the same time. If we include Convexity (c) and sides (s) we get the following expression :

In the hypercube domain, this function can be represented upon a 4-dimensional hypercube:

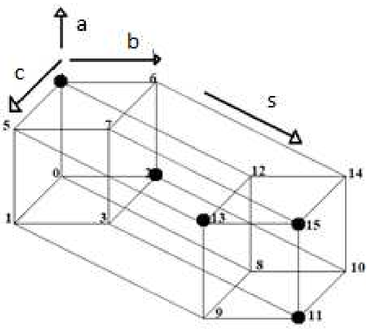


Fig. 5 Boolean hypercube representation of BP #4

In this representation, points 0-7 () represent BP_{left}. Points 8-15 () represent BP_{right}. The dataset in BP_{left} congregates on the back panel of the cube (). The dataset in BP_{right} congregates on the front panel (). This provides a simple method of distinction :

BP_{left} = ();BP_{right}.() =

As mentioned above, another point of distinction is the function . Still, distinction on the basis of convexity represents a simpler separation between sides, as is evident from the graph. However, it is important to note that choosing convexity as a distinction introduces redundancy to the model - there are points now included in BP_{right}, BP_{left} that in actuality do not appear in the BP. Convexity clearly differentiates between the BP_{left} and BP_{right}, however describes neither accurately. We have here a choice of simplicity over Non-redundancy.

3 .The common interpretation of the following problem (BP #12) is a distinction between elongated items on the left, and non-elongated items on the right - the distinction given by 3 subjects interviewed:

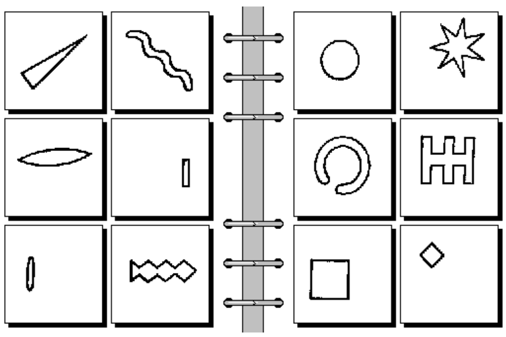
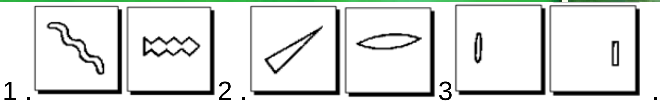


Fig. 6 - BP #12

One subject provided a more complex distinction: **The left side pairs shapes that are the same except for smoothing of the curves .**

Upon further discussion, the exact pairings were provided:



An analysis of this subjects' answer requires much care for detail, as it demands a format for representing characteristics for **pairs**. For our purposes, a smoothing pair includes 3 characteristics:

1 .an item with sharp curves

2 .an item with smooth curves

3 .correspondence between the two items: One item is a fully smoothed version of the other.

There is an obvious difficulty with this description - terms such as smoothing and correspondence are relative. We will initially ignore the relativity of the term. As far as we are concerned, a 'smoothed' item has no sharp curves at all, thus giving it a Boolean definition: an item is either fully smoothed or not. We will declare correspondence between items only if there is some basic resemblance, and one can be said to be a fully smoothed version of the other.

BF_{right} has only 2 smooth images with curves, and therefore by definition would not fall into these categories. We chose to describe each pairing according to the above mentioned list, using the following notation:

a - there exists an item in the pair with sharp curves

b - there exists an item in the pair with smooth curves

c – correspondence

Pairings 1,3 could be said to fulfill all 3 requirements: They both items with sharp curves, fully smoothed items, and some correspondence. Pairing 2 is somewhat problematic: The sharp curved item has 3 sharp curves, whereas the smoothed item is not fully smoothed - it contains 2 sharp edges. In this case we would say that a,b were fulfilled, but not c: BF_{left}.=

The complete function would now be . In other words, this definition must ignore one case for the sake of the other two. In graphical representation this would appear as follows:

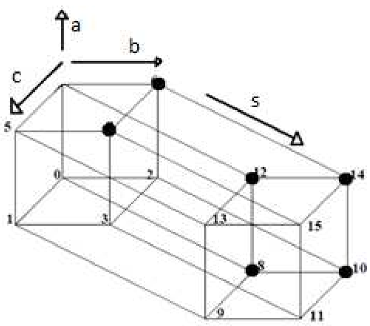


Fig. 7 - Hypercube representation of BP #12

This is a perfect example of Non-exactness – the given rule does not comply with all the data in our **World**. There is one aspect of this question that allows for ignoring the one pairing for the sake of the other two. One could say that there is a partial smoothing in pairing 2, as there are less sharp edges after some transformation. If we were to employ this fact, we would introduce fuzziness into the system - i.e. a pair doesn't have to be either smoothing or non-smoothing - there are degrees of smoothing. The hypercube representation now takes the following form:

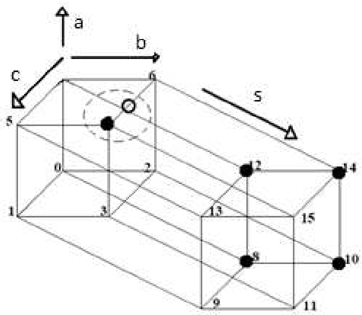


Fig 8. Fig. 7 - Hypercube representation with introduced fuzziness,

This representation manifests the ability to dim the details of the problem without ignoring them. In this case, we allow for partial membership of an item in the 'correspondence' parameter, and the graph expresses that all pairings in BPleft manifest this trait to some degree.

4 .Conclusions

The concept of non-exactness is studied formally in the case of Boolean recognition. The formal Boolean analysis provides important findings:

1 .The proposed Boolean analysis provides a powerful methodology for study of processes of human concept learning in solving problems of recognition.

2 .The distinction between properties becomes clearly in the case of Boolean cube representation.

3 .Introducing the concept of redundancy in the Boolean description opens a way for understanding non-exactness. Specifically, the possibility for redundancy inserts a potential level of subjectivity to any formulation of a unifying or distinguishing rule.

4 .Derived Rules that do not match the data entirely, may match partially if we allow for less rigid definitions. In other words, inserting fuzzy definitions may allow for certain underlying trends to arise, in essence providing us with a rule that is Non - Exact.

References

[1] Polanyi, M.(1966). The tacit dimension. London, Routledge

[2] Feldman, J. (2006). An algebra of human concept learning. Journal of Mathematical Psychology, 50, pp. 339–368**.**

[3] Bongard, M. M. (1970). Pattern Recognition. Rochelle Park, N.J.: Hayden Book Co., Spartan Books**.**

[4] Linhares, A. (2000). A glimpse at the metaphysics of Bongard problems. Artificial Intelligence, Volume 121(1-2), pp. 251–270**.**

[5] H. Fleisher et al. (1983)IBM J. Res. Develop. Vol. 27(4) July 1983

[6] Harry E. Foundalis. (2006). PHAEACO: A COGNITIVE ARCHITECTURE INSPIRED BY BONGARD’S PROBLEMS, 2006, University of Indiana