CHILDREN'S CONCEPT LEARNING IN SOLVING LOGIC PROBLEMS

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Abstract

In everyday life, new concepts are often built by combining existing concepts and by using classification of instances, objects, events, etc. Cognitive psychologists have paid particular attention to concepts that identify kinds of things, i.e. to those that classify or categorize objects. In our study, we gave to 6-7 year-old children logical set-completion tasks using cards from the "Set game"® (www.setgame.com). We examine the relation between the cognitive complexity of the tasks and the complexity of Boolean functions corresponding to them. The children were provided with a sequence of three cards having varying characteristics (such as a number of contours, their size, and fill), and were asked to select a fourth card that would fit to the three cards as a set. Our experiment aims to study how children recognize regularity within the shapes' characteristics and, consequently, how this recognition supports children in solving logic problems. The results of the study comprise both a set of the children's answers and a set of values of the function of regularity. Comparison between the above two sets of data serves the basis to analyze the correlation between the children's success in problem solving and the complexity of a certain logic task. Our paper provides results of the analysis. The research indicates that children recognize the regularity of certain characteristics more successfully than the regularity of others. This result is of a great importance for developing children's logical thinking.

Keywords: Cognitive complexity, logic problem (pattern recognition task), concept learning, mental model.

1 INTRODUCTION

One of the challenges inherent to the solution of logic problems is that of their "complexity". A problem's complexity, however, may mean different things, which do not necessarily overlap. It may refer, for instance, to the "objective" complexity of the problem's content, or it may refer to the problem's *cognitive* complexity – to how difficult its solution is *for the student to perceive*. In the study presented here, we focus on the latter aspect, attempting to quantify the complexity of different logic problems *for students* by using a mathematical model to determine which logical connections are easier for them to make.

Many new concepts used in everyday life are acquired through the act of connecting them to older concepts, and of correctly classifying new individual instances under preexisting, more general categories [1]. Cognitive psychologists have paid particular attention to concepts that identify kinds of things - those that classify or categorize objects. The ability to categorize objects by identifying a regularity of certain features is a critical one in learning and solving complex logical tasks. Humans instinctively recognize and identify regularities in their environment or in tasks they perform. This recognition process is flexible, and is guided by individual learners' goals [2].

We began our study with the assumption that learning and understanding a certain concept is strongly connected with identification of patterns and regularities in their characteristics. This understanding, as we propose, can be measured using an analytical function that applies a series of vectors to students' solutions of a logic task involving the mentioned patterns on the cards. Goodwin & Johnson-Laird [3] claim that given a set of properties and relations, humans are able to construct complex concepts by establishing relations between properties. One way to formalize relations between components of a mental model, to assess complexity of logical connections and to identify regularities is through Boolean operators. Feldman [4] proposed measuring the complexity of a logic problem by the number of literals in the minimal expression of a corresponding Boolean concept.

In our work, we study the children's ability to solve logic tasks formulated by a set of cards. Each of the cards represents from one to three red contours. The cards differ from one another by one or two characteristics from the following three: size, fill, and number of the contours.

An important point of our study is the understanding that each game card is in one-to-one correspondence with nodes of a specific algebraic structure called a Boolean Cube. The Boolean cube as an algebraic structure widely applied to various fields of knowledge. An Isomorphic Boolean cube successfully describes different concepts by using the same structure that can be visually expressed by a so-called Hasse diagram. Fig. 1 shows two Hasse diagrams. The left diagram corresponds to a set of subsets of the three-elements set $\{x, y, z\}$. The right diagram corresponds to our set of cards. Nodes of the first diagram correspond to 8 specific subsets, while nodes of the second diagram correspond to 8 different cards. Axes of the left cube are elements of the initial set (x, y and z). Axes of the right diagram are three characteristics of cards: feel, size and number.

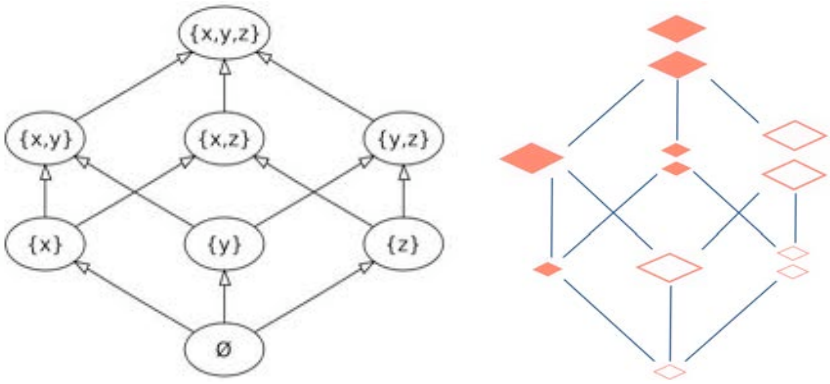

Figure 1: Isomorphic Boolean cube for: a) the set of subsets, and b) the Set of cards.

The Boolean cube (Fig.1b) presents relations between cards. Cards are distinguished from one another by size, number or fill. The Hasse diagram of the Boolean cube (Fig. 1a) is used to represent a finite partially ordered set. Based on the Boolean cube, we study a connection/correlation between specific regularities in subsets of cards (Fig. 1b) on the one hand, and students' success in solving logic tasks defined by the subsets (Fig. 1a) on the other hand. We also study the cognitive complexity of each logic task by 1) a formal method of measuring the complexity using a specific analytical function of regularity, and 2) an empirical method, by analysing the corresponding students' answers.

2 RESEARCH METHODOLOGY

This study was conducted with twenty-five elementary-school students aged 6-7, all from a small private school in central Israel. The meetings took place at school and lasted for ten to fifteen minutes. We met the first 10 students twice and recorded their verbal explanation of how they solved the task (see Fig. 1). The multiple meetings were designed to avoid taking their first impression as their final answer (children tend to change their answers). We therefore asked for explanations and checked these at the second meeting for consistency.

Our methodology is designed to assess children's method of solving so-called "sequence tasks" represented as a logical set with a missing piece. To examine the question of which characteristics are more important to determine the complexity of a problem, we created tasks for solving logic problems, based on the Set game®. The children were given twenty-one sets of cards. For each set, the children were asked to complete one of two tasks, each with its own set of instructions. The two tasks were as follows: the children needed to either a) find the card that completes the set in the most logical way; b) find the card that is least likely to complete the set.

In the example below, the students were provided with a set of cards upon which diamonds were drawn, the diamonds varied in their size, number and fill. The students were asked to choose the most suitable card (from the set of candidates, options A, B and C) to complete it.

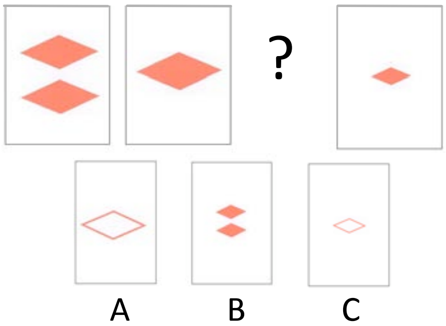

Figure 2: Set completion task. Initial set (top), subset of candidates (bottom).

Choosing a certain card from the candidates' set can be seen in light of a specific reason or a number of reasons corresponding to different regularities taking place in the initial set, for example:

A. Choosing the card with a big empty contour we consider as a priority to accomplish the task with regularity difference in the fill characteristics.

B. Choosing the card with two small red contours we consider as a priority to accomplish the task with a symmetry regularity in the number characteristics, and homogeneity regularity in the fill characteristics.

C. Choosing the card with the empty small contour we consider as priority for different regularity in the fill characteristic.

We compare the correctness of the above answers, by the computational distance of the mode in our example. Specifically, the highest grade corresponds to answer B since it includes two regularities: “Symmetry in number” and “Homogeneity in fill not empty”.

The above answers can be measured by the value of a newly introduced analytical function (see below). How the students choose to complete the set is based on their ability to identify the main *characteristics* of the figures on the cards (in our experiment these are Size, Fill & Number) and the manner in which the figures form regularity as a sequence (homogeneity, differentiation, monotonic character or symmetry).

3 DATA ANALYSIS

We propose to estimate the correctness of the answer for the task of the above type, by a *function of regularity* R. The function ***R*** measures the level of regularity of the sequence of cards in cases when a specific card is chosen from a set of cards-candidates. Obviously, function R has a certain value for each of the cards-candidates.

Let us describe the essence of the function R. Tab. 1 is used for this purpose.

Rows of Tab. 1 correspond to characteristics of the cards: size, number and fill. Columns of the table correspond to four regularities: Homogeneity, Difference, Monotony and Symmetry. If certain regularity j takes place for a certain characteristic i, than we put 1 in the intersection of row i and column j; we put 0 if that certain regularity doesn’t take place. The Tab. 1 is filled out according to the task described above for the card B.

Table1: Table for calculating the value of Function of Regularity.

	Homogeneity	Difference	Monotony	Symmetry
Size (S)				
Number (N)				1
Fill (F)	1			

The regularity of a specific characteristic of a card is a vector comprising all possible regularities. We measure the function of regularity R_s of the card characteristic “Size” by the length of the vector as follows:

$$R_S = \sqrt{w_h r_{Sh}^2 + w_d r_{Sd}^2 + w_m r_{Sm}^2 + w_s r_{Ss}^2} \ ,$$

where: w_h, w_d, w_m, w_s - weights of regularities, that has to be included to take into account cognitive features of different regularities. The values of the weights are one of the subjects of our study.

It is understood that, for each card, three functions of regularity, respectively corresponding to three characteristics of the cards have to be calculated: R_S, R_N and R_F. We propose to estimate the total regularity of a specific card by the length of the vector of the card characteristics:

$$R = \sqrt{w_S R_S^2 + w_N R_N^2 + w_F R_F^2} \ .$$

We use the term "weights" to emphasize which characteristics are stronger among the three (size, fill or number), and which regularities are more influential among the following four (homogeneity, differentiation, monotonic or symmetry). The weights were added to the answers as a "factor" after the first meeting with the students, in which they explained the reasoning behind their choices. The factors were given at the decoding phase: to the characteristics of size & fill (2 points) after we found that the students tend to explain their choices first of all by the number of the shapes; later, to the most popular regularity, Difference (2 points), and then to the less popular Homogeneity (3 points).

In our example, according to the regularity function, the act of choosing A - the card with the big empty contour - indicates a tendency to complete the set using the regularity "difference", with an emphasis on the "fill" characteristic. The act of choosing B - the card with two small red contours - indicates a tendency to accomplish the task with the ”symmetry" regularity in the "number" characteristic, and ”homogeneity" regularity in the "fill" characteristic. The fact of choosing C – the empty small contour – also indicates a tendency towards the ”difference" regularity in the "fill" characteristic.

According to the function of regularity, the highest score, in this case, would therefore go to answer B, since it accounts for two regularities of the characteristics: Symmetry in number (two) & Homogeneity in fill (not empty).

4 RESULTS

Our findings relied primarily on the observation of the way our students solved the logic problems we placed before them. Their explanations were used to comprehend what guided their choices of how to complete the sets. The students' explanations of their choices, when asked to recognize the pattern of a set, suggested that they were looking for difference between the objects rather than similarity. The most common choice of an object to complete the set was the one that stood out most prominently. This finding justifies our addition of "weights" to the regularity "homogeneity", which was usually disregarded by the students. We also learned that, out of the three characteristics presented to the students, the one most commonly noticed by the children was size. That explains why we decided to add weight to the two relatively disregarded characteristics "fill" and "number".

Table 2: Meanings of marks in 6 questions' Categories (N=25) with weights to Homogeneity.

Fill	Number	Size	
2.06 2.43*	2.82 3.17*	2.30 2.92*	Homogeneity
2.75 3.78*	3.22 2.90*	4.52 3.00*	Difference

With weights to Size and Fill*

Tab. 2 shows the scores in each of the questions' categories. Analysis of the marks by comparing meanings of T-Test revealed significant differences between the categories. The highest score (4.52) was achieved in the category of different size (T = 142, df=24, p<0.001) with weights to Homogeneity, and the lowest mark (2.06) was achieved in the category of Homogeneous fill (T = 44.38, df=24, p<0.001).

Table 3: Marks (means) in 4 questions' Categories.

	Homogeneity	Difference	Monotony	Symmetry
Mean (Std.)	3.33 (.09)	2.82 (.16)	3.81 (.41)	2.92 (.14)

Tab. 3 shows the score in each question's regularity category (H, D, M & S). The difference between the four categories was significant. The highest score was achieved in questions belonging to the Monotonic category (T=48.79, df=27, p<.001), and the lowest in questions belonging to the Difference category.

5 DISCUSSION AND CONCLUSIONS

In this study, we attempted to use the pattern/ way students solve logic problems for quantitatively assessing the sources of these problems' cognitive complexity. In doing so, we tested our assumption that complexity comes from the way children perceive the regularity and the characteristics of the objects represented in the series.

The findings of previous studies have shown that learners tend to be biased toward concepts with simpler representation [5]. This supports Piaget's theory of cognitive levels, which showed that children at the age of 6-7 have not reached the formal operation stage, in which they gain the ability to think in an abstract manner and the ability to combine and classify items in a more sophisticated way [6]. At the age of 6-7, children have not yet attained the thinking level that enables them to take in all three characteristics (i.e. number, size, fill) at once.

We therefore originally assumed that our students would be able to perceive the concept easily when asked to complete sets based on common characteristics. We assumed that they would look for similarity (Homogeneity) rather than difference in the characteristics of the missing object and would notice the fill as a main regularity of the objects. Our results did not reflect these assumptions.

While we found that some categories of problems were simpler for them to solve than others, the simpler ones were those that relied on size as a main characteristic, and those with solutions reliant on finding a different object to accomplish the set, rather than finding a similar one. Logic problems with Homogeneity in their solution seem to be more complex to solve than problems that required solvers to look for a different object as the one suitable to accomplish the task. Our experiment showed that the students preferred a solution emphasizing a maximum difference to one emphasizing similarity, to accomplish the series represented in a more complex way, which is inconsistent with our assumption that learners are biased to prefer similarity. For example, we found that learners notice size as a solution to the logic problem more often when it is different, and notice fill more often when it is the same.

We were also surprised to find that, while according to the function of regularity, the "Monotony" is a more complex concept than "Homogeneity" and "Difference" (the score is higher at that category), our population of young students looked for monotonic regularity in the way objects connect into series, easily completing sets in which the object monotonically gets bigger. This means that there are differences in the way students conceive logic problems of the kind we presented to them, and that these differences can be identified and measured by our model of regularity.

Our findings show differences in the way young learners identify the relevant characteristic when they are asked to complete a set. We strongly agree with previous studies about the high value of looking for a "code" in concept learning [4], [7] and we suggest that our "function of regularity" model can be used for such a purpose. For a future research, it would be interesting to use the model to analyse new series constructed spontaneously by the children themselves, comparing it to their work when they merely complete existing ones.

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