Concept of Non-Exactness in Science Education

Ilya Levin, Osnat Keren, Hillel Rosensweig

What is **Non-Exactness**

- "..the brain is a device that has evolved in a less exact world than the pristine one of orbiting planets... Therefore, mathematical simulation has to be replaced by **abstraction**, which involves **discarding the irrelevant**" (D. Hoftstader, Metamagical Themas, pp. 650)

What is **Non-Exactness**

Basic Assumptions:

1. The world is an extremely complex place
2. Humans manage to survive in it despite it's complexity

Humans survived by simplifying their
view of the world around them

We will discuss **classification** of objects as a form of simplification, and describe non - exact classifications

What is a **Classification**

Important aspect of human understanding is the ability to classify objects (whether physical or abstract). Classification, in this context means:

- **Extrapolation**: finding underlying similarities of objects in a group (similar properties) and formulating descriptions of these similarities.
- **Interpolation**: concluding whether a previously unexamined object, belongs to a class due to its properties.

Classification

An object that shares all the similarities described in a classification ‘belongs’ to that class of objects - *i.e. objects that share those similarities*

A classification is a **simple** representation of a large group of objects:

- Instead of dealing with a large list of objects, use one constant classification!
- Only object *properties* relevant to the classification matter – a valuable source for scientific theories!

Non-Exact **Classification**

For a group of objects (physical, or abstract) Non Exact classification can mean:

1. Formulating **partial** description of group similarities: i.e. describing some but not all of the underlying similarities
2. Formulating **inaccurate** description of group similarities: i.e. describing properties that do not exist in all objects in the group

Boolean **Classification**

- For given Class $\boldsymbol{C}$, and object of a group $\boldsymbol{O}$, A Classification function returns 'True' if $\boldsymbol{O}$ belongs to class $\boldsymbol{C}$.
- Boolean functions are well suited to serve as classification functions for classes that
  - Allow only full 'belonging' to a class (Boolean 'belonging' - function returns only '1','0')
  - Class properties are themselves Boolean

Implication

- For a given object group, **several simplified classifications** may exist – each one neglecting a different property.
- If one **partial** description of group similarities is possible, probably partial description is possible – neglecting other group properties!
- What comes first – removing/neglecting data (properties)? Or basic pattern recognition?

Research Goals

- Classification: study how people classify groups of objects where these are objects are describable by Boolean models.
- Specifically:
  - Examine redundancy in human concept learning – the ability to give **partial** description of group similarities.

Conducted Experiment

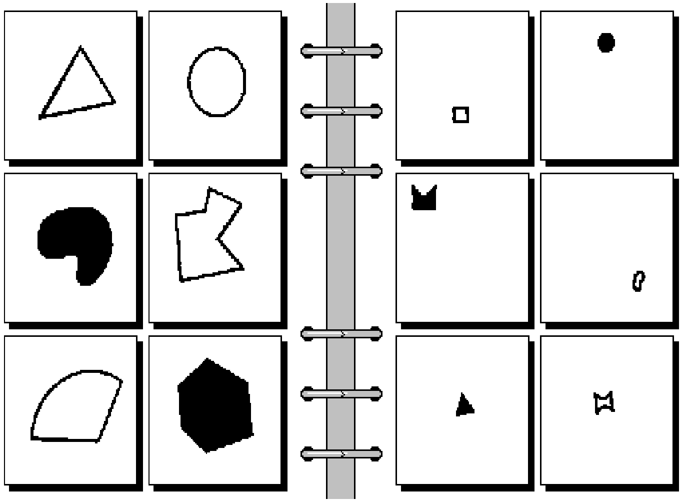
- We presented datasets to human subjects, and asked them to generalize each set into classification.

  (W -> R)

- Each dataset should not have an obvious classification.
- Each dataset needs to allow, on our part, an analysis of all its characteristics as well as **the process** by which a classification is derived – It has to have a certain degree of **compactness**.

Conducted Experiment

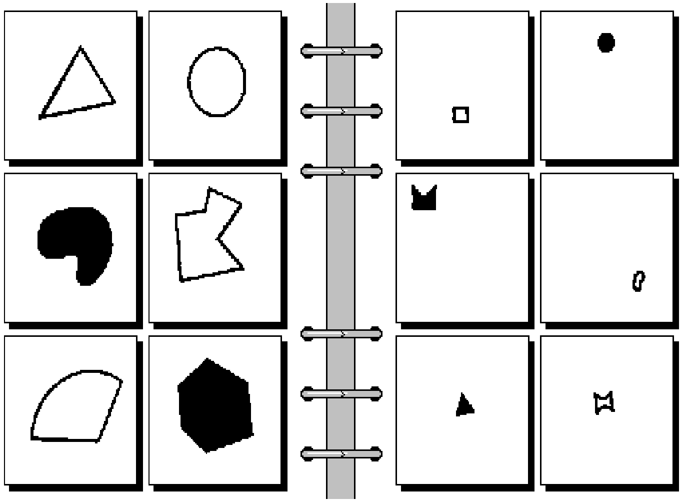
- Each subject is given 4 Bongard Puzzles. For instance:



- Subjects are asked to describe a common property for objects on both sides of the puzzle, and to describe other avenues attempted before reaching final conclusion

Case Study

- Bongard Problem #2:



- Solution (size): BP_{left}=Big; BP_{right}=Small
- One Subject Suggested interesting prior option: positioning (centered, not centered).

Analysis

- Boolean variables:
- a – position (0=non-centered, 1=centered)
- b – size (0=small, 1=large)

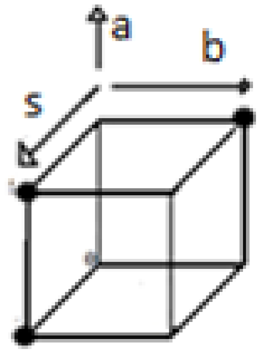
$$BP_{left}: a \cdot b$$

$$BP_{right}: a \cdot \bar{b} + \bar{a} \cdot \bar{b} = \bar{b}$$

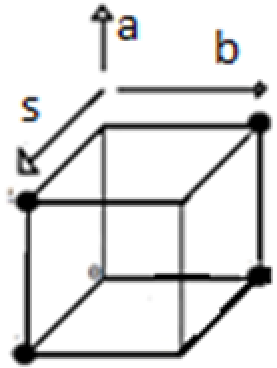
- The resulting expression (including variable '*s*' for side) is: $\bar{s} \cdot (a \cdot b) + s \cdot \bar{b}$

Analysis

- The Bongrad Problem as initially viewed by the subject:



- After we distinguish on basis of size alone we get:



Interpretation

- The final model ignores **positioning** –making it a ‘don’t care’ situation. We have inserted redundancy to the system – properties without consequence!
- Someone else could see a different solution – only positioning and not size, or size and positioning. There is a subjective element because of redundancy.
- Creating rules out of reality **changes** how we view reality.

Conclusions

- The proposed Boolean analysis provides a powerful methodology for study of processes of human concept learning in solving problems of recognition.
- The distinction between properties is clear in the case of Boolean cube representation.
- Redundancy in Boolean description opens a way for understanding non-exactness and subjectivity

Future Work

- This work focused on recognition problems, where the subject chose Boolean variables and formed the Classification
- Work should be done in parallel on cases where the variables are given to the subject, and only Classification forming is left to the subject.