Quarks in the Bootstrap Era

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The quark model emerged from the Gell-Mann–Ne'eman flavor SU(3) symmetry. Its development, in the context of strong interactions, took place in a heuristic theoretical framework, referred to as the Bootstrap Era. Setting the background for the dominant ideas in strong interaction of the early 1960s, we outline some aspects of the constituent quark model. An independent theoretical development was the emergence of hadron duality in 1967, leading to a realization of the Bootstrap idea by relating hadron resonances (in the s-channel) with Regge pole trajectories (in t- and u-channels). The synthesis of duality with the quark-model has been achieved by duality diagrams, serving as a conceptual framework for discussing many aspects of hadron dynamics toward the end of the 1960s.

1. Introduction

In this short memoir I wish to present a personal perspective concerning the early history of the quark model. During the early 60s I was a graduate student of Yuval Ne'eman, spending several formative years at the Weizmann Institute, and in the late 60s I have been a postdoc at Caltech, witnessing the bustling research activity influenced by Murray Gell-Mann. The theoretical framework, within which Yuval and Murray and all their students worked at that time, was based on fundamental notions and principles which can be derived from Quantum Field Theory (QFT). Particle interactions were divided into four categories: strong, electromagnetic, weak and gravitational interactions. It seemed clear that, given the strength of the strong interactions, and lacking any analog of the electromagnetic fine structure constant α, there existed no hope of formulating a fundamental field theory of strong interactions.

Studying strong interactions one tried to get the utmost out of basic principles, like analyticity and unitarity of the S-matrix, and PCT symmetry principles, all of which did have their origin in QFT. Other than that, one tried to come up with principles that seemed to fit the observed experimental phenomena. I refer to this period as the bootstrap era, whose basic leading ideas will be described in short in the next paragraph. It is within this background that we, as young practitioners of novel particle symmetry ideas, tried to make critical observations and work out the consequences of the ideas that we have promoted or criticized.

2. The Bootstrap Era

S-matrix models of two-to-two particle-scattering were described in terms of the s, t, u Mandelstam variables[1] $s = (p_1 + p_2)^2 = (p_3 + p_4)^2$, $t = (p_1 - p_3)^2 = (p_2 - p_4)^2$ and $u = (p_1 - p_4)^2 = (p_3 - p_2)^2$, obeying the over-all constraint $s + t + u = \sum_i m_i^2$. The physical region for the process $1 + 2 \rightarrow 3 + 4$ was characterized by positive s and negative t. Physical regions in the crossed channels described processes involving the anti-particles, according to conventional associations in Feynman diagrams. This is exemplified in Fig. 1, which is taken from Mandelstam's paper, describing πN scattering.

Analyticity of the S-matrix implied the appearance of poles below the scattering threshold of the s-channel, and cuts above the threshold. The same types of structures appear in physical regions of the S-matrix in the t-channel and in the u-channel, where t or u obtain positive values. Analyticity was expressed in terms of dispersion relations, relating real and imaginary parts of the scattering amplitude to each other. These analytic structures, whose analogs can be defined for general m to n particle scattering amplitudes, were further supplemented by unitarity constraints. The simplest of these relationships is the optical theorem, relating the imaginary part of the elastic scattering amplitude $a + b \rightarrow a + b$ to the total cross-section $\sigma_T(ab)$. The rich set of analytic and unitarity constraints has led Chew and Frautschi[2] to propose the Nuclear Bootstrap idea, stating that these constraints may suffice to determine a unique set of poles (i.e. particles and resonances) in all channels, thus providing the basis of a theory of the strong interactions. An example of a detailed summary of all these ideas is the set of lectures delivered by Chew in the 1965 Les Houches Summer School.[3]

Another important element of the dynamics of strong interactions was the use of Regge poles.[4,5] The asymptotic behavior of a scattering amplitude at high energies (large s and negative t) has been composed of a set of terms of the type $s^{\alpha(t)}$ where $\alpha(t)$, for negative t values, was a (linear) extrapolation of angular momenta of particles (or strong interaction resonances) of masses m observed for a given set of quantum numbers in the crossed channel, i.e. for positive $t = m^2$ values.[6] Their presumed linear behavior is depicted in

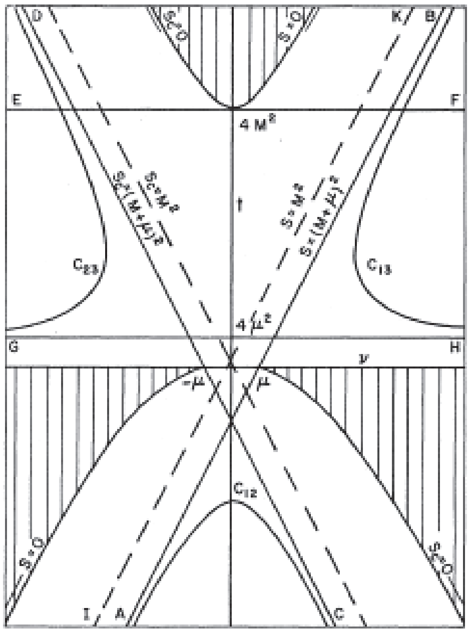

Fig. 1. The Mandelstam plot for πN scattering, taken from the original paper.[1] Masses of π and N are denoted by μ and M, respectively. The physical region of s is the shaded region on the bottom right.

the Chew–Frautschi plot reproduced in Fig. 2. The Froissart bound,[7] stating that σ_T cannot increase asymptotically faster than $\log^2(s)$, meant that all $\alpha(0)$ had to be smaller than 1. The Regge pole with $\alpha(0) = 1$ in Fig. 2 was called the Pomeron, leading to constant total cross-section and implementing the Pomeranchuk theorem, which stated that the asymptotic total cross-sections of particles and of their antiparticles were identical. The Pomeron was exceptional, because no particles were identified for $t > 0$ which were fit to lie on its trajectory. All other Regge poles were associated with known particles and resonances with relevant quantum numbers, accounting for many phenomenological observations at large s and negative t. Corrections to this picture were presumed to be due to lower-lying cuts in complex angular momentum. For positive t, one expected to find further resonances with ever-increasing angular momenta.

3. SU(3) and the Quark Model

The late 50s and early 60s have witnessed the uncovering of an increasing zoo of particles and resonances. Some of them can be seen in Fig. 2, where

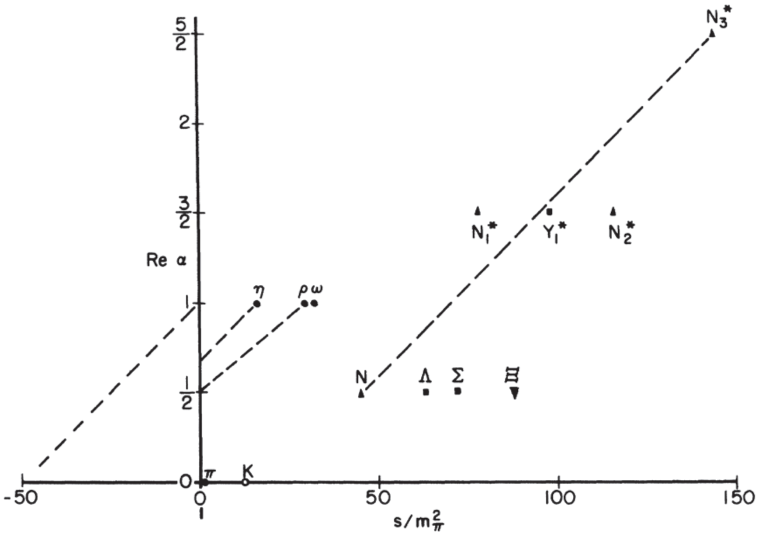

Fig. 2. Chew–Frautschi plot of Regge pole trajectories, taken from Ref. 6.

they served as the basis of deciding which Regge trajectories should influence high-energy scattering amplitudes. By that time there existed already many sets of particles with different spin (J) and parity (P) quantum numbers, and the hope was that a symmetry higher than the spin symmetry SU(2) will be found to account for ordering spectra of the same J^P, but different isospin and strangeness, into the same representation. The SU(3) symmetry (which today should be referred to as flavor-SU(3)) proposed by Gell-Mann[8] and Ne'eman[9] was one of the suggested symmetry models. It has accommodated pseudo-scalar and vector mesons in singlet and octet representations, and the lowest baryons of spin-$\frac{1}{2}$ were also nicely accounted for by an octet representation. There existed 9 spin-$\frac{3}{2}$ resonances which could fit into a decuplet, if a tenth strangeness -3 particle could be found.[10] With the experimental discovery of the stable Ω^- particle,[11] which served to complete the spin-$\frac{3}{2}$ decuplet, the Gell-Mann and Ne'eman SU(3) model has won overall recognition as the correct symmetry model of strong interactions. This symmetry model provided also a framework for mass formulas, as well as their electromagnetic corrections, and it also served as a basis for postulating the structure of weak interaction currents, etc. The symmetry considerations became so popular that, within a few years, various extensions of SU(3) into higher symmetries have been proposed, like SU(6) (including spin degrees of freedom) and more, but all of them looked quite speculative and the interest in them diminished throughout the years while flavor-SU(3) stood its ground. The early period of flavor-SU(3) has been summarized in "The Eightfold Way".[12]

In the mean time, Gell-Mann[13] has proposed the quark model, and Zweig[14] has independently proposed his ace model. The quark model built the isospin degrees of freedom from u and d quarks, and associated the strangeness quantum number with the s quark. Thus all mesons were accounted for by quark–antiquark combinations and all baryons could be viewed as three-quark structures. Most physicists have regarded this viewpoint as a mnemonic for SU(3) symmetry considerations, rather than viewing quarks as real physical objects. The strong belief that all true physical variables should be experimentally measurable was at the heart of this refusal to accept quarks as physical building blocks, because of their fractional electric charges and wrong spin-statistics relations. Nonetheless it won popularity because the quark model seemed to be the natural way to explain SU(3) representations, i.e. why representations other than 1, 8 and 10 have not been observed in hadron physics. Clearly, most of the dynamic consequences of quark-based descriptions like SU(3) breaking (mass differences) and electromagnetic mass-shifts, etc., could just as well be stated in terms of operators with specific SU(3) characteristics. There were however a few indications of experimental phenomena that required a quark-based rule. One example was the Zweig rule,[14] explaining the amazing dominance of the decay mode $\varphi \to K^+ + K^-$ in terms of the assumption that φ is a bound state of an s quark and anti-quark. Note that this quark structure meant a particular mixing of SU(3) singlet and octet states. The striking argument was that if strong decay can proceed only in terms of Feynman diagrams in which the quark–antiquark pair does not annihilate ("rule of ace conservation" in Zweig's language), it accounts for the dominance of this particular decay mode.

Zweig's approach was an early example of what became known as the constituent-quark point of view; regarding quarks within particles the same way one considers nucleons within nuclei, without trying to explain the unresolved conceptual problems. An early review of hadron physics as accounted for by this naïve quark approach was given by Dalitz.[15] A detailed account of the development of the quark model has been presented by Lipkin.[16]

4. Duality of the Strong Interactions

By the mid-sixties there existed then two important observations regarding hadron spectra. One was that their states fit well into quark model constructs, and the other was that towers of resonances with ever-increasing

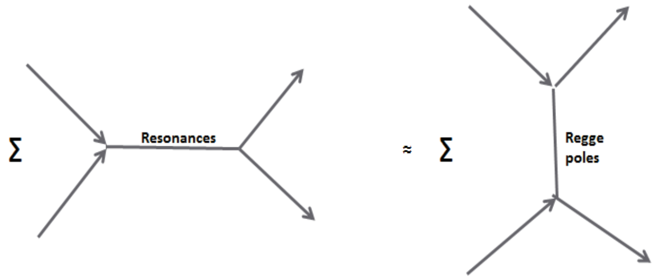

Fig. 3. Duality as implemented by the FESR.

masses and angular momenta are expected to exist in conjunction with the Regge model. The question still remained how all this can fit into the bootstrap approach. It turned out that what was missing was a formulation of the bootstrap idea that can unite the two.

Such a formulation has been proposed by the Finite Energy Sum Rules (FESR).[17–21] Employing analyticity of the S-matrix, the FESR approach incorporated experimental scattering amplitudes (which was dominated by resonances) as input, up to some finite energy, and Regge pole parametrization from that point on. This allowed relating these two contributions using a Cauchy integral approach. Applying it to inelastic scattering, such as $\pi^- p \to \pi^0 n$, one ended up with a relationship of the type expressed in Fig. 3, implying that the Regge pole amplitude, when continued to low-energies, can approximate on average the resonance contributions. Or, vice versa, the resonances can be used to account for Regge pole properties. Inelastic scattering channels were used in order to avoid the Pomeron issue: the elastic channel was assumed to be dominated by Pomeron exchange, with $\alpha(0) = 1$ (see Fig. 2), and its s-channel amplitude was evidently not dominated by resonances.

The FESR approach was quite revolutionary on two counts. First, it went against the then common trend to sum both types of contributions, resonances and Regge exchanges, to the scattering amplitude. This practice has followed the common experience from the use of Feynman diagrams, which the FESR have shown to involve double-counting. Second, it has led to a concrete realization of the bootstrap idea: the resonances in the s-channel have now successfully been related to Regge poles that are the analytic continuation of resonances in the t-channel. This has also become to be known as hadron duality.[22]

The dual relationship of Fig. 3 can also be captured within a mathematical model, as demonstrated by Veneziano.[23] Using linear Regge trajectories

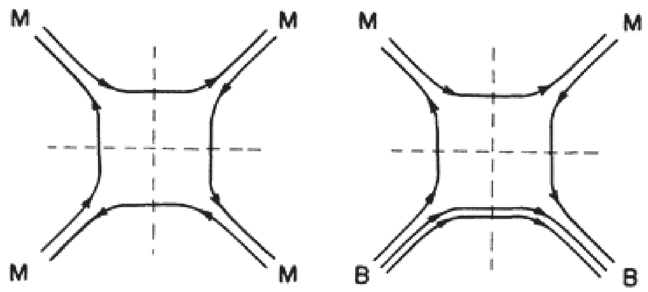

Fig. 4. Duality diagrams, taken from Harari,[24] representing meson-meson and meson-baryon scattering amplitudes. They may be realized as representing resonances in the s-channel and Regge exchanges in the t-channel, both obeying quark-model assignments.

he expressed his model in terms of the Euler Beta function. This mathematical realization has led to a flurry of theoretical activity studying dual resonance models in the following years.

5. Duality Diagrams

With hadrons appearing as quark-model resonances in both the s-channel and the t- and u-channels, it seemed natural to try and account for both in one diagrammatic description. A merger of the duality principle and the quark model came about through the duality diagrams which have been proposed by Harari[24] and by Rosner.[25] The diagrams, such as the examples shown in Fig. 4, display scattering amplitudes in terms of two- or three- quark components, as befitting mesons and baryons, but their s- and t-structure is that of a dual amplitude. Although they did not account for Pomeron exchanges, duality diagrams presented a theoretical framework which has described the understanding of strong interaction dynamics at that time. They have also extended the thinking underlying Zweig's rule into the realm of scattering amplitudes, thus encapsulating all allowed and forbidden resonances and Regge exchanges in their various channels.

One should however admit that, although duality diagrams served as an underlying conceptual framework, explaining allowed and forbidden reaction channels, they fell short of providing a model which can explain the observed dynamical features of scattering phenomena and multi-particle production. The discussion of these phenomena continued to make use of concepts derived from various field-theoretical or statistical models, which have dominated the relevant literature.[26] One important reason was that high-energy reactions consisted mostly of multi-pion production, in flagrant violation of what might be expected from a system which is symmetric under flavor-SU(3). Therefore one still needed theoretical tools which could deal with pion dominated phenomena.

6. Epilog

Now we know that the quark model is more fundamental than the flavor-SU(3) symmetry which gave birth to it in the 60s. After the discoveries of additional and heavier quarks it became clear that flavor SU(3) symmetry reflected the coincidence that the quarks u, d and s possess light masses. Now we also know that strong interactions can be formulated as a quantum field theory, QCD. Why should one then look back at the 1960s?

Other than just the simple answer, that it is nice to reminisce about the 1960s, these stories may serve as an account of a very active era, in which theoretical developments and experimental discoveries followed rapidly one another. It was a great achievement of the HEP community that, even in the absence of an underlying theory, it could proceed very far in establishing an adequate framework to provide consistent understanding of observed particles and their interactions.

Aficionados of field theory have criticized the bootstrap approach at the time, arguing that having everything emanate from self-consistency is too much to be asking for. It turned out that the underlying quark model was necessary in order to provide the duality framework, which was the manifestation of the bootstrap approach, with its particle-flavor foundation. This was sufficient to produce a conceptual framework for many aspects of hadron physics at the end of the 1960s.

References

1. S. Mandelstam, Determination of the pion-nucleon scattering amplitude from dispersion relations and unitarity, *Phys. Rev.* **112**(4), 1344 (1958).
2. G. F. Chew and S. C. Frautschi, Principle of equivalence for all strongly interacting particles within the S-matrix framework, *Phys. Rev. Lett.* **7**, 394 (1961).
3. G. F. Chew, The analytic S-matrix: A theory of strong interactions, pp. 189–250, in *High Energy Physics*, 1965 Les Houches Summer School, eds. C. DeWitt and M. Jacob (Gordon and Breach, New York, 1965).
4. T. Regge, Introduction to complex orbital momenta, *Nuovo Cim.* **14**, 951 (1959).
5. T. Regge, Bound states, shadow states and Mandelstam representation, *Nuovo Cim.* **18**, 947 (1960).
6. G. F. Chew and S. C. Frautschi, Regge trajectories and the principle of maximum strength for the strong interactions, *Phys. Rev. Lett.* **8**, 41 (1962).

7. M. Froissart, Asymptotic behavior and subtractions in the Mandelstam representation, *Phys. Rev.* **123**, 1053 (1961).
8. M. Gell-Mann, The Eightfold Way: A theory of strong interaction symmetry, Unpublished report CTSL-20, 1961, *Phys. Rev.* **125**, 1067 (1962).
9. Y. Ne'eman, Derivation of strong interactions from a gauge invariance, *Nucl. Phys.* **26**, 222 (1961).
10. M. Gell-Mann, in *Proc. Int. Conf. on High-Energy Nuclear Physics*, Geneva, 1962, p. 805.
11. V. E. Barnes *et al.*, Observation of a hyperon with strangenss minus three, *Phys. Rev. Lett.* **12**, 204 (1964).
12. M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (W.A. Benjamin, New York, 1964).
13. M. Gell-Mann, A schematic model of baryons and mesons, *Phys. Lett.* **8**, 214 (1964).
14. G. Zweig, An SU3 model for strong interaction symmetry and its breaking, CERN Report 8419/TH.401 (January 17, 1964). An SU3 model for strong interaction symmetry and its breaking II, CERN Report 8419/TH.412 (February 21, 1964).
15. R. H. Dalitz, Quark models for the "Elementary Particles", pp. 253–323, in *High Energy Physics*, 1965 Les Houches Summer School, eds. C. DeWitt and M. Jacob (Gordon and Breach, New York, 1965).
16. H. J. Lipkin, Quarks for pedestrians, *Phys. Reports* **8**, 173–268 (1973).
17. D. Horn and C. Schmid, Finite energy sum rules, Unpublished report CALT-68-127, April 1967.
18. A. Logunov, L. D. Soloviev and A. N. Tavkhelidze, Dispersion sum rules and high energy scattering, *Phys. Lett.* **24B**, 181 (1967).
19. K. Igi and S. Matsuda, New sum rules and singularities in the complex J plane, *Phys. Rev. Lett.* **18**, 625 (1967).
20. R. Dolen, D. Horn and C. Schmid, Prediction of Regge parameters of rho poles from low-energy pi-N scattering data, *Phys. Rev. Lett.* **19**, 402–407 (1967).
21. R. Dolen, D. Horn and C. Schmid, Finite-energy sum rules and their application to pi-N charge exchange, *Phys. Rev.* **166**, 1768–1781 (1968).
22. G. F. Chew, Horn-Schmid duality, *Comments on Nuclear and Particle Physics*, 74–76 (1968).
23. G. Veneziano, Construction of a crossing-symmetric, Regge behaved amplitude for linearly rising trajectories, *Nuovo Cimento A* **57**, 190 (1968).
24. H. Harari, Duality diagrams, *Phys. Rev. Lett.* **22**, 562 (1969).
25. J. L. Rosner, Graphical form of duality, *Phys. Rev. Lett.* **22**, 689 (1969).
26. D. Horn and F. Zachariasen, *Hadron Physics at Very High Energies* (W.A. Benjamin, Reading, MA, 1973).