Horn–Schmid Duality

In 1962 K. Igi[1] pointed out that Regge asymptotic behavior for reaction amplitudes implies relations connecting integrals over cross sections to Regge-pole trajectories and residues. He employed one of these relations to secure information about the P' trajectory (now believed to contain the f_0 meson) from experiments on low-energy πN scattering. The Igi idea then lay almost dormant until 1966 when it was rediscovered by Alfaro, Fubini, Furlan, and Rossetti[2] in a special situation involving amplitudes whose highest Regge trajectories lie below a certain critical J-value. The integral conditions here became known as "superconvergence relations". In 1967 several groups, including D. Horn and C. Schmid,[3] K. Igi and S. Matsuda,[4] as well as Logunov, Soloviev, and Tavkhelidze,[5] gave a more unified discussion, and the term "finite-energy sum rule" or "FESR" was introduced. This terminology characterizes an approximate relation between an integral up to a finite upper limit and the leading Regge poles for the amplitude appearing under the integral. Igi's original work, as well as the superconvergence relations, can be regarded as special examples of FESR.

Horn and Schmid[3] made an analysis of FESR and found the accuracy thereof to be determined by the accuracy with which the included Regge poles approximate the amplitude at the upper limit of the integral. The precise height of the first neglected trajectory is unimportant so long as it makes only a small contribution to the amplitude at this particular energy. Further, Schmid[6] has pointed out how the FESR implies that extrapolation of the Regge formula to low energy must give a semilocal average of the amplitude even in the resonance region. Since the Regge formula, like any peripheral representation, contains no energy-poles, this approximate description of resonances through peripheralism constitutes a novel and profound aspect of the hadronic S matrix. We refer to it as "Horn–Schmid duality".

Although the essential ideas underlying FESR and the attendant duality have been discussed since 1962, the recent concrete identification of this concept has catalyzed a remarkable outpouring of developments. First have come tests of the concept in situations where both high- and low-energy experimental data are available. The current literature overflows with such tests, some carried out with more care than others. Where data are adequate and the number of relevant Regge poles small, the FESR has been confirmed.[7] Encouraged, theorists have proceeded to make deductions about Regge trajectories and residues from low-energy resonance data in situations where the relevant high-energy experimental information is still lacking. An especially interesting facet of the preliminary deductions has been the support they give to conjectures about residue-function zeros.[7] More generally it appears that a major new source of experimental information about Regge parameters has been tapped. In fact the paradox is conceivable that data taken below 3–4 GeV may become more decisive in determining trajectories and residues than those taken at high energies!

A second exploitation of duality has focussed on the bootstrap concept.[8] Duality implies that Regge poles in "crossed" reactions determine resonances in "direct" reactions and the converse is of course also true; if at the same time all resonances belong to trajectories, bootstrap conditions follow. To date only the simplest aspects of this self-consistency problem have been investigated, but it is already apparent that duality provides a more reliable bootstrap tool than do the traditional N/D or Bethe–Salpeter equations. Duality involves no channel truncation in the unitarity condition and accommodates indefinitely-rising trajectories. The only essential approximation in bootstrap studies based on duality has been that resonances are narrow—a condition well satisfied in nature. Correspondingly, a deficiency in such studies has been their inability to explain the ratio of resonance width to resonance mass. Only ratios of widths and ratios of masses appear in the narrow-width approximation. The old-fashioned Bethe–Salpeter or N/D approach superficially seems capable of explaining absolute widths, but in practice the neglected high-energy channels play a controlling role. Nothing has been lost, therefore, through the duality approach; all successes of earlier bootstrap models have been maintained.

The bootstrap aspect of Horn–Schmid duality throws new light on the peculiar dynamical role of the Pomeranchuk trajectory, the Regge singularity which dominates the very high energy behavior of all elastic and some inelastic amplitudes, even those which lack prominent low-energy resonances. Schmid[6] has emphasized the consistency of the latter circumstance with the absence of low-mass resonances on the Pomeranchuk trajectory itself, as well as with the small slope thereof. Precisely these features allow the Pomeranchuk trajectory to stand apart from the interplay of first-order bootstrap constraints. Harari[9] has suggested associating the Pomeranchuk contribution with "background" at low energy, whether or not prominent resonances there appear. Such an association might explain why total cross sections systematically approach their high-energy limits from above.

Clarified by Horn–Schmid duality are a variety of "double counting" puzzles. The first of these arose in two-particle reactions at intermediate energies. Should one here add Breit–Wigner resonances to the Regge peripheral formula? The Horn–Schmid–Harari answer is that at most the Pomeranchuk singularity can be added to Breit–Wigner without double counting. The "Deck effect" has been a second puzzle. Deck[10] evaluated a doubly-peripheral model for the reaction $\pi N \to \pi \rho N$ and found a spectrum in the final $\pi\rho$ mass that peaked near the A_1 resonance even though his model lacked any pole in this variable. This phenomenon was demoralizing to particle spectroscopists, who saw it as undermining their ability to identify new resonances. But Horn and Schmid tell us we can have it both ways! The process *is* peripheral, but resonances are present just the same.[11]

An extension of the above reasoning promises great simplification in the analysis of multiple-production processes. The experimenter, for example, need no longer sweat over whether to characterize three pions as an A_1 meson, a ρ meson plus a pion, or simply as three pions. The theorists now can interpret a single formula so as to fit any of these alternative descriptions.

Considering the brief time interval thus far devoted to analysis of Horn–Schmid duality, the flow of physical deductions has surely not yet exhausted the content. The topics touched above ultimately may be judged a small sample.

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