Outsourcing in a Global Economy*

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Abstract
We study the determinants of the location of sub-contracted activity in a general equilibrium model of outsourcing and trade. We model outsourcing as an activity that requires search for a partner and relationship-specific investments that are governed by incomplete contracts. The extent of international outsourcing depends \textit{inter alia} on the thickness of the domestic and foreign market for input suppliers, the relative cost of searching in each market, the relative cost of customizing inputs, and the nature of the contracting environment in each country.

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“Subcontracting as many non-core activities as possible is a central element of the new economy.” — Financial Times, July 31, 2001, p.10.

1 Introduction

We live in an age of outsourcing. Firms seem to be subcontracting an ever expanding set of activities, ranging from product design to assembly, from research and development to marketing, distribution, and after-sales service. Some firms have gone so far as to become “virtual” manufacturers, owning designs for many products but making almost nothing themselves.¹

Vertical disintegration is especially evident in international trade. A recent annual report of the World Trade Organization (1998) details, for example, the production of a particular “American” car:

Thirty percent of the car’s value goes to Korea for assembly, 17.5 percent to Japan for components and advanced technology, 7.5 percent to Germany for design, 4 percent to Taiwan and Singapore for minor parts, 2.5 percent to the United Kingdom for advertising and marketing services, and 1.5 percent to Ireland and Barbados for data processing. This means that only 37 percent of the production value ... is generated in the United States. (p.36)

Feenstra (1998), citing Tempest (1996), describes similarly the production of a Barbie doll. According to Feenstra, Mattel procures raw materials (plastic and hair) from Taiwan and Japan, conducts assembly in Indonesia and Malaysia, buys the molds in the United States, the doll clothing in China, and the paints used in decorating the dolls in the United States. Indeed, when many observers use the term “globalization,” they have in mind a manufacturing process similar to what Feenstra and the WTO have described.

To us, outsourcing means more than just the purchase of raw materials and standardized intermediate goods. It means finding a partner with which a firm can establish a bilateral relationship and having the partner undertake relationship-specific investments so that it becomes able to produce goods or services that fit the firm’s particular needs. Often, but not always, the bilateral relationship is governed by a contract, but even in those cases the legal document does not ensure that the partners will conduct the promised activities with the same care that the firm would use itself if it were to perform the tasks.\footnote{Marsh (2001, p.10) notes some of the pitfalls in outsourcing: “Outsourcers depend on others caring as much about the product as they do. If you ask someone else to make a vital component, you may lose control over the way it evolves.”}

Because outsourcing involves more than just the purchase of a particular type of good or service, it has been difficult to measure the growth in international outsourcing. Audet (1996), Campa and Goldberg (1997), Hummels et al. (2001) and Yeats (2001) have used trade in intermediate inputs or in parts and components to proxy for what they have variously termed ‘vertical specialization’, ‘intra-product specialization’ and ‘global production sharing’. While these are all imperfect measures of outsourcing as we would define it, the authors do show that there has been rapid expansion in international specialization for a varied group of industries that includes textiles, apparel, footwear, industrial machinery, electrical equipment, transportation equipment, and chemicals and allied products. It seems safe to tentatively conclude that the outsourcing of intermediate goods and business services is one of the most rapidly growing components of international trade.

In this paper, we develop a framework that can be used to study firms’ decisions about where to outsource. We consider a general equilibrium model of production and trade in which firms in one industry must outsource a particular activity. These firms can seek partners in the technologically and legally advanced North, or they can look in the low-wage South. Our model of a firm’s decision incorporates what we consider to be the three essential features of a modern outsourcing strategy. First, firms must search for partners with the expertise that allows them to perform the
particular activities that are required. Second, they must convince the potential suppliers to customize products for their own specific needs. Finally, they must induce the necessary relationship-specific investments in an environment with incomplete contracting.

Using the framework developed in Sections 2 and 3, we are able to examine in Sections 4 and 5 several possible determinants of the location of outsourcing. First, the size of a country can affect the ‘thickness’ of its markets. All else equal, a firm prefers to search in a thicker market, because it is more likely to be able to find a partner there with the appropriate skills that would make it able and willing to tailor a component or service for the final producer’s needs. Second, the technology for search affects the cost and likelihood of finding a suitable partner. Search will be less costly and more likely successful in a country with good infrastructure for communication and transportation. Third, the technology for specializing components determines the willingness of a partner to undertake the needed investment in a prototype. Finally, differences in contracting environments can impinge on a firm’s ability to induce a partner to invest in the relationship. We study the contracting environment by introducing a parameter that represents the extent to which relationship-specific investments are verifiable by an outside party.

While our model is rich in its description of the outsourcing relationship, we focus here only on the location of outsourcing activity, without allowing firms a choice of whether to produce components themselves as an alternative to outsourcing. Our analysis thus complements that in Grossman and Helpman (2002a), where we studied the make-or-buy decision but did not allow firms any choice of where to produce or source their components.\(^3\) The next step in our progression would be for us to construct a model in which firms have a four-way choice of whether to undertake an activity in-house or to subcontract, and whether at home or abroad. Such a model

\(^3\)There are other important differences between this model and that in Grossman and Helpman (2002a). There we allowed for variable search and assumed imperfect contracts governing the production and sale of customized components. Here we introduce partial contracting over investments so that we can study the implications of differences in the legal environments across countries. Also, our other paper featured a closed economy, whereas here we incorporate international trade.
would come closer to describing the central decisions facing modern, multinational firms. But the current paper takes an important intermediate step, because it highlights considerations that are bound to be important in a more complete analysis.

2 The Model

Consider a world economy with two countries, North and South, and two industries. Firms in either country can produce a homogeneous consumer good $z$ with one unit of local labor per unit of output. Firms in the North also can design and assemble varieties of a differentiated consumer good $y$. The South lacks the know-how needed to perform these activities. Both countries are able to produce intermediate goods, which we henceforth call “components” but might also represent business services. The components are vital inputs into the production of good $y$.

The varieties of good $y$ are differentiated in two respects. First, as is usual in models of intra-industry trade, consumers regard the different products as imperfect substitutes. Second, the varieties require different components in their production. We capture product differentiation in the eyes of consumers with the now-familiar formulation of a CES sub-utility function. On the supply side, we associate each final good with a point on the circumference of a unit circle, so that the “location” of a good represents the specifications of the input needed for its assembly.

Consumers in both countries share identical preferences. The typical consumer seeks to maximize

$$u = z^{1-\beta} \left[ \int_0^1 \int_0^{\hat{n}(l)} y(j, l)^{\alpha} dj dl \right]^{\frac{1}{\beta}}, \quad 0 < \alpha, \beta < 1,$$

(1)

where $z$ is consumption of the homogeneous final good and $y(j, l)$ is consumption of the $j^{th}$ variety located at point $l$ on the unit circle (relative to some arbitrary zero point). We assume that there is a continuum of goods located at each point on the circle, but (1) implies that consumers regard the various goods at the same location on the circle as differentiated. In the limit to the integral, $\hat{n}(l)$ is the measure of varieties available to consumers that require an intermediate input at location $l$.  

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Note that, as usual, $\beta$ gives the spending share that consumers optimally devote to the homogeneous good and $\varepsilon = 1/(1 - \alpha)$ is the elasticity of substitution between any pair of varieties of good $y$.

Production of any variety of good $y$ requires a fixed investment in product design plus one unit of the customized input per unit of output. Potential final producers enter in the North by devoting $f_n$ units of Northern labor to product development.\footnote{Since there is a continuum of differentiated final goods, the fixed cost of designing a single product is infinitesimally small. Of course, the total resources used in designing a positive measure of such goods is finite.} The location of the requisite intermediate input on the unit circle is a random element in the design process, with all locations being equally likely. The designers-cum-final-producers cannot manufacture the intermediate inputs themselves; rather they must 	extit{outsource} this activity to specialized suppliers in one country or the other. If a final producer finds a partner who is willing and able to manufacture the requisite components, the firm can assemble final goods without additional inputs.\footnote{This is an inessential simplification. We could as well assume that production of final goods requires labor and components in fixed proportions.} If a final producer fails to identify a suitable supplier, the firm must exit the industry.

Component suppliers may enter in either market. Such entry requires an investment in expertise and equipment, the cost of which is $w^i f_m^i$ in country $i$, where $w^i$ is the wage rate and $f_m^i$ is the (fixed) labor requirement, for $i = S, N$. A supplier’s expertise is represented by a point on the unit circle. The investment in developing expertise is large relative to the cost of designing a single final product, so there are relatively few suppliers of components in each market and each supplier serves multiple final producers in equilibrium. The suppliers who enter a given market space themselves equally around the circle. For simplicity, we neglect the integer “problem,” and treat the finite number of input suppliers in country $i$, $m^i$, as a continuous variable.

After the entry stage, the Northern firms that have developed product designs seek suppliers for their specialized inputs. The search process is specific to a geographic region, so each firm must decide whether to hunt for a supplier in the North or in
the South. The search and associated research activities require \( f_s \) units of Northern labor at a cost of \( w^N f_s \). We assume that by bearing this cost, a firm can ascertain the expertise of all suppliers active in the selected country and, in particular, identify the one whose expertise is closest to its own needs.\(^6\) For reasons that will become clear, this closest supplier is the one with whom the final producer enters into any subsequent discussions.

At the time when a final producer must choose where to conduct its search, it does not know the expertise of all of the various potential suppliers in the two markets (i.e., their locations on the circle), but only the total numbers of such suppliers and the fact that the suppliers in each market are equally spaced around the circle. A final producer regards all equi-spaced configurations of suppliers in a market as equally likely. Accordingly, when choosing to search in a market with \( m^i \) suppliers, a final producer knows that the nearest supplier will be at a random distance \( x \), where \( x \) is a draw from a uniform distribution with range from 0 to \( 1/2m^i \). We will refer to the number of suppliers in a country as the “thickness” of the market and will find that market thickness plays an important role in the search decision.

Any supplier must develop a prototype before it can produce the customized inputs needed by a particular final producer. The cost of this investment varies directly with the distance between the location of the supplier’s expertise and that of the final producer’s input requirements. In particular, if a supplier in country \( i \) wishes to provide components to a final producer whose location in input space is at a distance \( x \) from its own expertise, then it must pay a fixed cost of \( w^i \mu^i x \) to develop the prototype. Thereafter, it can produce customized components for its partner at constant marginal cost, with one unit of local labor needed per unit of output.

To summarize, the production of varieties of good \( y \) entails a number of fixed and variable costs. A final producer of any variety must bear a fixed cost of product

\(^6\)In our working paper, Grossman and Helpman (2002a), we treat the case in which the final producers have variable search costs and choose the intensity of their search effort. There we assume that final producers are not guaranteed to find all suppliers in a given market, unless their search efforts are sufficiently intense. The specification that we have employed here (with only a fixed cost of search) substantially simplifies the analysis without sacrificing essential insights.
design ($w^N f_n$) and a fixed cost of searching for a component supplier ($w^N f_s$). Such a firm needs one unit of a specialized input per unit of output. A component supplier in country \( i \), in turn, bears a fixed cost of investment in expertise and equipment ($w^j f^i_m$) and a fixed cost customizing a component for each of its customers ($w^j \mu^i x$) that depends on the distance between its own expertise and the customer’s needs. Component producers employ one unit of local labor per unit of output.

### 2.1 Bargaining and Contracting

Once a final producer has identified the supplier whose expertise is most suitable to its needs, the two firms can begin to explore a bilateral relationship in the light of the local legal environment. For such a relationship to develop, the supplier must be willing to invest in a prototype that is specific to the particular differentiated product.\(^7\) And whereas the supplier’s investment (or its result) can be perfectly observed by the final producer, it is not fully verifiable to outside parties. The lack of verifiability constrains the contracting possibilities, as is familiar from the work of Williamson (1985), Hart and Moore (1990), and others. Later, we shall assume that some aspects of the investment are verifiable, while others are not. This permits partial (imperfect) contracts. But for now we take the extreme view that none of the up-front investment is contractible. The supplier must be willing to undertake the investment itself in anticipation of an order contract that will be negotiated and fulfilled only after a suitable prototype has been built.\(^8\)

Let $S^i$ denote the profits that the parties will share if the supplier develops a com-

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\(^7\)In other words, we assume that a firm’s input requirements are unique, and in particular different from those of other firms located at the same point on the unit circle. Also, final producers may not use components that nearly fit but not precisely so. These assumptions simplify the analysis without significantly affecting the nature of the hold-up problem.

\(^8\)Segal (1999) and Hart and Moore (1999) have developed detailed models that provide the microeconomic underpinnings of contractual incompleteness. Applying their reasoning to our setting, it will be impossible for the parties to negotiate an order contract before the prototype exists if some details about the components are revealed to the supplier only after the prototype has been built and if the parties cannot commit to refrain from renegotiating any initial order contract.
ponent that fits the buyer’s needs and if the two parties subsequently reach agreement on an order contract. The parties anticipate that if they reach a stage where a suitable prototype exists, their negotiation will lead to an equal sharing of $S^i$. So the supplier expects to earn $S^i/2$ if it chooses to invest $w^i\mu^i x$ in the prototype, where $x$ is the distance between the final producer’s needs and the (closest) supplier’s expertise. The supplier willingly undertakes the investment if and only if its share of the prospective profits exceeds the investment cost; that is, if and only if $w^i\mu^i x \leq S^i/2$.

A final producer that finds a (nearest) supplier in the chosen market at a distance greater than $S^i/2w^i\mu^i$ cannot acquire components and thus has no choice but to exit the industry. One that finds a supplier at a closer distance than this can expect the investment to be made and the relationship to proceed. We define $r^i$ as the greatest distance in input space between any producer that remains active after having searched for a partner in country $i$ and its supplier. Considering that the suppliers in market $i$ are separated by a distance $1/m^i$, it follows that

$$r^i = \min \left\{ \frac{S^i}{2w^i\mu^i}, \frac{1}{2m^i} \right\}.$$  \hfill (2)

Once the input supplier has invested in the prototype, the partners have coincident interests concerning the production and marketing of the final good. We assume that they reach an efficient agreement to govern the manufacture of components. The preferences in (1) imply that the producer of the $j^{th}$ variety of good $y$ at location $l$ faces a demand given by

$$y(j,l) = Ap(j,l)^{-\varepsilon},$$  \hfill (3)

when it charges the price $p(j,l)$, where

$$A = \frac{\beta \sum_i E^i}{\int_0^1 \int_0^{r^i(l)} p(j,l)^{1-\varepsilon} dj dl}$$  \hfill (4)

and $E^i$ denotes spending on consumer goods in country $i$, for $i = N, S$. This is a constant-elasticity demand function, which means that profits are maximized by fixed mark-up pricing. Any partnership in which the supplier resides in country $i$ faces a marginal cost of output of $w^i$. Thus, joint profits are maximized by a price $p^i = w^i/\alpha$. 

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of final output. Maximal joint profits are

$$S^i = (1 - \alpha)A \left( \frac{w^i}{\alpha} \right)^{1-\varepsilon},$$

which are independent of the distance between the supplier’s expertise and the final producer’s input type. The order contract that generates the maximal joint profits dictates a quantity of inputs

$$y^i = A \left( \frac{w^i}{\alpha} \right)^{-\varepsilon}$$

and a total payment by the final producer to the input supplier of

$$\frac{1 + \alpha}{2} A \left( \frac{w^i}{\alpha} \right)^{1-\varepsilon}.$$ 

### 2.2 Search

We consider now the search problem facing a typical final producer. The firm must decide whether to search for a supplier in the North or in the South. Suppose the firm searches in country $i$. Recall that the firm finds a partner at a random distance $x$, where $x$ is uniformly distributed on the interval $[0, 1/2m^i]$. If the firm finds a nearest potential supplier at distance greater than $r^i$, where $r^i$ is given by (2), then the supplier will be unwilling to undertake the investment in customizing the intermediates. In the event, both the final producer and the supplier firm derive

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9The payment is such that the input supplier’s reward net of manufacturing costs is half of the joint profits. Thus, the payment is $S^i/2 + w^iy^i$, which, with (5) and (6), implies the expression in the text.

10We assume that final producers search for an outsourcing partner in only one country. This can be justified by assuming that the search cost $f_s$ is large enough. Note that the equilibria described below with outsourcing in both countries would remain equilibria even if we were to allow firms to search in both markets. In these equilibria, some firms break even by searching only in the North and others by searching only in the South, so a firm that searched in both places would suffer an expected loss. However, if firms were free to search in both markets, there might be additional equilibria in which all firms search in both countries and firms choose ex post where to outsource. This choice would be based on the distance between their input requirement and the expertise of the two potential partners and on the profit opportunities that would ensue from production of intermediates in the alternative locations.
no profits from the relationship. If, however, the final producer finds a supplier at a distance closer than \( r^i \), the supplier will be prepared to invest in customization and the final producer earns \( S^i / 2 \) from the relationship. It follows that by searching in market \( i \) the final producer expects to earn operating profits of \( S^i / 2 \) with probability \( 2 r^i m^i \) and operating profits of zero with probability \( 1 - 2 r^i m^i \). Its expected profits from searching in country \( i \) are

\[
\pi^i_n = r^i m^i S^i. \tag{7}
\]

Now we can identify the market or markets in which the Northern firms will choose to conduct their searches. If \( \pi^N_n > \pi^S_n \), all search is conducted in the North and all outsourcing takes place there. Similarly, if \( \pi^S_n > \pi^N_n \), all search focuses on the South and there is no domestic outsourcing. Mostly, we will study equilibria in which outsourcing occurs in both regions. This requires \( \pi^S_n = \pi^N_n \).

### 2.3 Free Entry and Market Clearing

The remaining equilibrium conditions comprise a set of free-entry conditions for producers of components and final goods and a pair of market-clearing conditions for the two labor markets.

Final-good producers must enter in positive numbers, since consumers spend a positive fraction of their income on differentiated products. All entrants earn zero expected profits in equilibrium. The expected operating profits for a typical firm that enters industry \( y \) is \( \pi_n = \max \{ \pi^N_n, \pi^S_n \} \), and the free-entry condition is

\[
\pi_n = w^N f, \tag{8}
\]

where \( f = f_n + f_s \) represents the sum of the fixed entry and search costs.

Positive numbers of component producers may enter in one or both countries.\(^{11}\) A firm that enters in country \( i \) will serve a measure \( 2n^i r^i \) of final-good producers,\(^{11}\)

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\(^{11}\)The intermediate producers also choose their expertise (i.e., location). We assume that this choice is made with rational expectations about the choices of others. It is a dominant strategy for each firm to locate at a point mid-way between the expected locations of the two most-distantly-spaced adjacent producers of intermediates. This strategy gives rise to a symmetric equilibrium with equi-spaced input producers.
where \( n_i \) is the total measure of final-good producers that searches in country \( i \).

A firm’s customers are spread uniformly at distances ranging from 0 to \( r^i \) in each direction from the point representing the firm’s expertise. A component producer earns profits of \( S^i/2 - w^i \mu^i x \) from its relationship with a final-good producer whose input requirement is at a distance \( x \) from its own expertise. Thus, operating profits for an input producer that enters in country \( i \) are

\[
\pi^i_m = 2n^i \int_0^{r^i} \left( \frac{1}{2} S^i - w^i \mu^i x \right) dx = r^i n^i \left( S^i - w^i \mu^i r^i \right).
\]

We assume that the number of entrants is sufficiently large so that, in making its entry decision, each firm ignores the small effect of its own choice on \( r^i \) and \( S^i \). Then free-entry implies

\[
\pi^i_m \leq w^i f^i_m \quad \text{and} \quad (\pi^i_m - w^i f^i_m) m^i = 0 \quad \text{for} \quad i = N, S.
\]

We turn next to the labor-market clearing condition in the South. We examine equilibria in which the wage rate in the North is higher than that in the South, so that \( \omega \equiv w^N/w^S > 1 \). In such equilibria, the entire world output of the homogeneous good \( z \) is produced in the South. Since aggregate profits are zero in both countries, all income is labor income. Aggregate spending equals aggregate income in country \( i \), which implies \( E^i = w^i L^i \), where \( L^i \) is the labor supply there. A fraction \( 1 - \beta \) of spending is devoted to homogeneous goods, which carry a price of \( w^S \). This means that in equilibrium the South employs \((1 - \beta)(\omega L^N + L^S)\) units of labor in the production of good \( z \).

The South also devotes labor to entry by input producers, to investment in customization, and to the manufacture of components. Entry absorbs \( m^S f^S_m \) units of labor. Customization requires \( \mu^S x \) units of labor for a final-good producer whose needs are a distance \( x \) from the expertise of the input producer. Each of the \( m^S \) producers of intermediates undertakes such an investment for all final-good producers that search in the South and that are located within \( r^S \) to its right or to its left. Since a constant density \( n^S \) of final-good producers searches in the South, the Southern labor needed for developing prototypes is

\[
2\mu^S m^S n^S \int_0^{r^S} x dx = \mu^S m^S n^S (r^S)^2.
\]
the density $n^S$ of Northern firms searching in the South results in a measure $2m^S r^S n^S$ of viable bilateral relationships. Each such partnership generates a demand for $y^S$ units of Southern labor to manufacture components. Therefore, manufacturing absorbs $2m^S r^S n^S y^S$ units of Southern labor. Summing the components of labor demand, and equating this to the fixed labor supply, we have

$$(1 - \beta)(\omega L^N + L^S) + m^S f_m^S + \mu^S m^S n^S (r^S)^2 + 2m^S r^S n^S y^S = L^S. \quad (11)$$

In the North, labor is used in the design of final goods, in searching for outsourcing partners, and in entry, investment and manufacturing by producers of intermediate goods. Entry and search by final-good producers requires $f(n^N + n^S)$ units of labor. The components of labor demand by intermediate-good producers in the North are analogous to those in the South. Therefore, the labor-market clearing condition in the North is given by

$$f \sum_i n^i + m^N f_m^N + \mu^N m^N n^N (r^N)^2 + 2m^N r^N n^N y^N = L^N. \quad (12)$$

This completes the description of the model.

\section{3 Outsourcing with Unverifiable Investment}

To gain an understanding of the workings of the model, we focus in this section on the key general equilibrium interactions. It is possible that the market for components will be sufficiently thick in country $i$ that all final-good producers who search there are able to find willing suppliers. This will be true if $m^i \geq w^i \mu^i / S^i$, which may hold in equilibrium for $i = S$, $i = N$, or both. Also, there may exist equilibria in which suppliers enter in only one country, so that $m^S = 0$ or $m^N = 0$. We discuss all of these possibilities in some detail in our working paper, Grossman and Helpman (2002b).\footnote{There we discuss a more general case in which final producers choose also the intensity of their search. In this more general setting, either market may fall into one of three regimes, depending upon whether the intensity of search is limited by the marginal cost of search, by the limitations on investment contracts, or by each firm being assured of finding a suitable partner.} Here, we focus on one type of equilibrium, namely that which arises when
outsourcing takes place in both countries and some final-good producers who search in each market are unable to find suppliers with expertise sufficiently close to their needs for a supply relationship to be consummated. We refer to this regime as one with a “binding investment constraint” in both countries. In such a setting, the limitations on contracting have real effects.

In the case of interest, (2) implies that the greatest distance between an active final producer who outsources in country $i$ and its supplier is given by

$$r^i = \frac{S^i}{2w^i \mu^i} \quad \text{for } i = N, S. \quad (13)$$

Assuming that outsourcing takes place in both countries, the free-entry conditions (10) together with (9) imply

$$r^i n^i (S^i - w^i \mu^i r^i) = w^i f_m^i \quad \text{for } i = N, S. \quad (14)$$

Substituting (13) and (14) into the South’s labor-market clearing condition (11) gives

$$(1 - \beta) \left( \omega L^N + L^S \right) + 2 \frac{1 + \alpha}{1 - \alpha} m^S f_m^S = L^S. \quad (15)$$

In (15), the first term on the left-hand side represents the labor used in the South in producing the homogeneous good while the second term reflects that used in all activities by component suppliers.

Next, we use (7) and (8), together with the fact that $\pi^N_n = \pi^S_n$ when outsourcing takes place in both countries, to write

$$r^i m^i S^i = w^i N f \quad \text{for } i = N, S. \quad (16)$$

\[\text{13 We also use } y^i = \alpha S^i/(1 - \alpha)w^i, \text{ which follows from (5) and (6).}\]
Using this equation together with (4) and (5), we derive\textsuperscript{14}

\[
f \sum_i n^i = \frac{1}{2} (1 - \alpha) \beta \left( L^N + \frac{1}{\omega} L^S \right);
\]

that is, the value of labor used by final-good producers for product design and search amounts to a fraction \((1 - \alpha)\beta/2\) of world income.\textsuperscript{15} Finally, we substitute this equation together with (13) and (14) into the North’s labor-market clearing condition (12) to derive

\[
\frac{1}{2} (1 - \alpha) \beta \left( L^N + \frac{1}{\omega} L^S \right) + 2\frac{1 + \alpha}{1 - \alpha} m^N f^N_m = L^N .
\]

(17)

The second term on the left-hand side of (17) represents the labor used in all activities by component suppliers in the North.

The two equations, (15) and (17), involve \(m^S\), \(m^N\), and the relative wage, \(\omega\). But the relative wage can be solved as a function of \(m^S\) and \(m^N\) using the requirement that, if \(m^S\) and \(m^N\) are both positive, search for input suppliers must be equally profitable in the two countries. Substituting (13) into (16) and noting that (5) implies \(S^N = \omega^{1-\varepsilon} S^S\), we obtain another statement of the equal-profit condition,

\[
\omega = \left( \frac{\mu^S m^N}{\mu^N m^S} \right)^{\frac{1-\alpha}{1+\alpha}} .
\]

(18)

This equation indicates that for search to be equally profitable in the two countries, the relative wage must be aligned with the relative costs of customization and the

\textsuperscript{14}The derivation uses the fact that \(p^i = w^i/\alpha\) for all differentiated products assembled using intermediate inputs from country \(i\), and that the number of varieties of good \(y\) that are actually produced using intermediate inputs from country \(i\) is \(2m^i r^i n^i\). Together, these considerations and (4) imply

\[
A = \frac{\beta \sum_i w^i L^i}{\sum_i 2m^i r^i n^i \left( \frac{w^i}{\alpha} \right)^{1-\varepsilon}} .
\]

\textsuperscript{15}The reason for this result is as follows. A fraction \(\beta\) of world income is spent on differentiated products while the total surplus (operating profits) from sales of these products is a fraction \(1 - \alpha\) of revenue. Therefore total surplus equals the fraction \((1 - \alpha)\beta\) of world income. Half of this surplus is earned by final-good producers. Since they break even on average, the fraction \((1 - \alpha)\beta/2\) of world income has to equal their entry and search costs.
relative numbers of suppliers. The relatively more costly it is to customize components in the South, the more profitable it will be to search for a supplier in the North. To offset this advantage, the relative wage must be higher in the North. On the other hand, the "thicker" is the market for components in the South relative to that in the North, the more profitable it will be to search for a supplier in the South, and therefore the lower must be the relative wage of the North if search in either country is to be equally profitable.

Combining (15) with (18) yields the reduced-form $SS$ curve,

$$m^N = \frac{\mu^N}{\mu^S}m^S \left[ \frac{\beta L^S - 2^{1+\alpha} m^S f^S_m}{(1 - \beta) L^N} \right]^{\frac{1+\alpha}{1-\alpha}},$$

(19)

which gives combinations of $m^N$ and $m^S$ that are consistent with labor-market clearing in the South and equal profitability of search in the two countries. Similarly, combining (17) with (18) yields the reduced-form $NN$ curve,

$$m^S = \frac{\mu^S}{\mu^N}m^N \left\{ \left[ 1 - \frac{1}{2}(1 - \alpha) \beta \right] L^N - 2^{1+\alpha} m^N f^N_m \right\}^{\frac{1+\alpha}{1-\alpha}} \frac{1}{\frac{1}{2}(1 - \alpha) \beta L^S},$$

(20)

which has a similar interpretation in regard to the Northern labor market. Typical $SS$ and $NN$ curves are depicted in Figure 1.16

16The curves in this figure and in Figure 2 were drawn with the following parameters: $\alpha = 0.5$, ...
Consider the $SS$ curve. It is evident from (19) that the origin lies on the curve, as does the point $(m_{max}^S, 0)$, where $m_{max}^S = \beta L^S (1 - \alpha) / 2 (1 + \alpha) f_m^S$. The right-hand side of (19) is rising in $m^S$ when the number of component suppliers in the South is small and declining in $m^S$ when this number is close to $m_{max}^S$. It follows that the $SS$ curve reaches a peak somewhere between $m^S = 0$ and $m^S = m_{max}^S$. It also can be shown that the right-hand side of (19) is a concave function of $m^S$ to the left of this peak and that the slope of the curve approaches zero as $m^S$ approaches $m_{max}^S$.

To understand the economics behind the shape of the $SS$ curve, note from (15) that as the number $m^S$ of component suppliers in the South rises, so too does their demand for labor at a given relative wage, and thus $\omega$ must fall in order to preserve equilibrium in the Southern labor market. From (18), a decline in the relative wage requires a decline in the relative number of component producers in the North; i.e., $m^N / m^S$ must fall to preserve the equal profitability of search in each market. If $m^S$ is small, the indicated decline in $m^N / m^S$ is achieved by a rise in $m^N$ that is proportionately smaller than the rise in $m^S$; thus, the $SS$ curves slopes upward for $m^S$ small. If $m^S$ is large, on the other hand, the indicated drop in $m^N / m^S$ requires an absolute fall in the number of Northern component producers; thus $SS$ slopes downward for $m^S$ close to $m_{max}^S$ and $m^N$ close to zero.

Observe from (15) that the relative wage of the North consistent with labor-market clearing in the South declines as the number of component producers in the South rises. Thus, along the $SS$ curve, $\omega$ attains its maximum value of $\beta L^S / (1 - \beta) L^N$ for $m^S = 0$. But recall that the existence of an equilibrium with production of homogeneous goods concentrated in the South requires $\omega > 1$. It follows that $\beta L^S / (1 - \beta) L^N > 1$ is a necessary condition for the existence of an equilibrium of the sort we are describing.\[17\]

\[\beta = 0.8, \mu^S = \mu^N, f_m^S = f_m^N = 1, L^S = 1000 \text{ and } L^N = 1500. \text{ Note that these curves do not depend on } f, \text{ i.e., the fixed costs of entry and search for final-good producers.} \]

\[17\] The condition $\beta L^S / (1 - \beta) L^N > 1$ is required for an equilibrium with $\omega > 1$ no matter what final producers decide about their search for components inasmuch as $\omega > 1$ implies that the entire demand for the homogeneous good must be satisfied by producers the South. In the event, the demand for Southern labor by the $z$ sector equals $(1 - \beta) (\omega L^N + L^S)$. This exceeds the South’s
Figure 2: Equilibria with binding investment constraints in North and South

Analogous arguments explain the shape of the $NN$ curve in Figure 1. We omit the details for the sake of brevity. Note only that the relative wage $\omega$ rises along the $NN$ curve as we move away from the origin. As the $NN$ curve approaches the vertical axis, the relative wage $\omega$ tends to infinity.

Now consider Figure 2, where we have depicted portions of the $SS$ and $NN$ curves from Figure 1 on a common scale. The ray through the origin represents combinations of $m^S$ and $m^N$ for which $\omega = 1$. Only points above and to the left of this line (which have $\omega > 1$) are of interest to us. The figure shows two equilibria, labelled $E_1$ and $E_2$, each characterized by active outsourcing in both countries. Recall that the $SS$ and $NN$ curves were constructed under the assumption that the equilibrium number of supplier firms is not so large as to allow every final producer to find a partner willing to incur the relationship-specific investment. It can be shown that this is indeed the case when the fixed cost $f$ for final-good producers is sufficiently small. We do not draw the boundaries of the region in which this condition is satisfied, but do assume labor supply when $\omega > 1$ and $\beta L^S / (1 - \beta) L^N < 1$. 

17
that $f$ is small enough for $E_1$ and $E_2$ to fall in the relevant region.\footnote{According to (2), the investment constraint binds in the North when $m^N < w^N \mu^N/S^N$. But (13) and (16) imply that this inequality holds if and only if
\[
m^N < \frac{\mu^N}{2f}.
\] Similarly, the investment constraint binds in the South when $m^S < w^S \mu^S/S^S$. Equations (13) and (16) imply, however, that this is satisfied if and only if $m^S < \mu^S/2\omega f$. Together with the equal profit condition (18) this inequality becomes
\[
(m^N)^{1-\alpha} (m^S)^{1-\frac{1-\alpha}{2}} < \left(\frac{\mu^N}{2f}\right)^{1-\alpha} \left(\mu^S\right)^{1-\frac{1-\alpha}{2}}.
\]
The equilibrium points $E_1$ and $E_2$ satisfy these conditions when $f$ is small enough. In Grossman and Helpman (2002b) we describe equilibria for world economies that do not satisfy these conditions.}

The possibility of multiple equilibria — as illustrated in the figure — reflects an important positive feedback mechanism that is present in our model. The greater is the number of input suppliers active in a country, the more profitable it is for a final producer to search there for a partner. With more component producers, the suppliers are more closely packed in input space, and a final producer is more likely to find a partner willing to undertake the necessary investment in customization. At the same time, the greater is the number of final producers that search for partners in a given country, the more profitable it is for an input producer to operate there.\footnote{McLaren (2000) was the first to study the thick-market externality in international trade. He pointed out that this externality can give rise to multiple equilibria when firms can choose between outsourcing and in-house production.}

The positive feedback associated with the thick-market externality is, however, limited by a wage response. As more intermediate producers enter in a country, their demand for labor bids up the country’s relative wage. This tends to dampen the incentive for final producers to search there. In our model, the general-equilibrium wage response creates the possibility of multiple equilibria with production of components in both countries and different patterns of outsourcing.

When several equilibria exist, it is natural to ask which ones are stable. We have performed a stability analysis and report the results in an appendix available.
at the journal’s web site. In the analysis, we take the numbers of each type of firm (final producers, component producers in the North, and component producers in the South) as the state variables and assume that entry and exit respond to profit opportunities. When profits net of entry costs are positive for a typical firm of a given type, more firms of that type enter. When profits are negative, firms exit. For simplicity, we assume that small final-good producers react to profit opportunities more quickly than the larger, component producers. With this adjustment process, stability requires that the $NN$ and $SS$ curves both be downward sloping and that the $SS$ curve be the steeper of the two at the point of intersection.

To understand these stability conditions, consider a point on the $SS$ curve in Figure 1. Now suppose that the number of Northern component producers $m^N_N$ were to fall slightly. As is evident from equation (15), a decline in the number of component suppliers in the North has no direct effect on labor-market conditions in the South. That is, as long as the number of component producers in the South and the relative wage do not change, the Southern labor market will continue to clear. But note that the fall in the number of suppliers in the North reduces the relative profitability of search in that country (see (18)). As a result, final-good producers will switch their search from North to South, thereby raising the net profits of suppliers in the South above zero. We indicate the profit opportunity, which induces entry by component producers in the South, by a rightward horizontal arrow for points below the $SS$ curve in Figure 1. A similar argument establishes that net profits of component producers in the South are negative above the $SS$ curve. This triggers exit and explains the leftward arrows we have drawn for such points.

By similar reasoning, the number of Northern component producers will grow in response to positive profit opportunities for points to the left of the $NN$ curve and will shrink in response to losses for points to the right of this curve. The arrows in Figure 1 describe these adjustments in the number of such producers.

Using the entry and exit dynamics described by the arrows in Figure 1, we obtain

\footnote{In Grossman and Helpman (2002b) we provide a stability analysis for the more general case in which search costs vary with the intensity of search. The findings are similar to those reported here.}
the combined dynamics depicted by the arrows in Figure 2. Thus, the equilibrium labelled $E_1$ is stable, whereas that labelled $E_2$ is unstable. Note that for a stable equilibrium such as $E_1$ to exist, we need two conditions: the $SS$ curve must be downward sloping at its intersection with the ray $\omega = 1$; and the $NN$ curve must intersect the ray $\omega = 1$ to the right of its intersection with $SS$. These two conditions can be represented by

$$\frac{\beta L^S}{(1-\beta) L^N} > \frac{2}{1+\alpha}$$

and

$$\frac{[1 - \frac{1}{2}(1-\alpha)\beta L^N] - \frac{1}{2}(1-\alpha)\beta L^S}{\beta L^S - (1-\beta) L^N} > \frac{\mu^N f_m^N}{\mu^S f_m^S},$$

respectively. They are satisfied by the parameters that we used to construct Figures 1 and 2. In what follows, we focus on economies that satisfy these conditions.

There also may be equilibria with outsourcing concentrated in one country. For example, an equilibrium with all outsourcing in the North (and $\omega > 1$) always exists when $\beta L^S > (1-\beta)L^N$. In such an equilibrium, $\omega = \beta L^S/(1-\beta)L^N$, and the fact that there are no input suppliers in the South ($m^S = 0$) discourages final producers from searching there. Given that no final producers search for partners in the South, no input suppliers have an incentive to enter there. Point $E_3$ in Figure 2 represents such an equilibrium. When an equilibrium exists with all outsourcing activity concentrated in the North, that equilibrium always is stable. We do not consider such equilibria further in this paper.

Note that the stable equilibrium $E_1$ in Figure 2 will not exist when the fixed costs of developing expertise for producing intermediate goods are very low in the South or very high in the North (see the inequality in (22)). A decline in $f_m^S$ shifts the $SS$ curve upward (see (19)), moving the intersection point $E_1$ to the right along the $NN$ curve. The intersection point does not lie above the equal wage ray when $f_m^S$ is small. Similarly, a rise in $f_m^N$ shifts the $NN$ curve to the left (see (20)), moving the intersection point $E_1$ to the right along the $SS$ curve. Again, this intersection point falls outside the region with $\omega > 1$ when $f_m^N$ is large enough. In both cases, there remains an unstable equilibrium such as $E_2$ with outsourcing in both countries and production of the homogeneous good in the South and a stable equilibrium such $E_3$. 
with all components produced in the North and all homogeneous goods produced in the South.

4 Comparative Statics

In this section, we study how the pattern of outsourcing and world trade are affected by the sizes of the two countries and by the technologies for customization. We begin with country size, because this allows us to illustrate some important properties of the model.

4.1 Country Size

Consider growth in the resource endowment of the South, as would be reflected in an increase in $L^S$. An initial stable equilibrium with outsourcing in both countries is depicted by point $E$ in Figure 3. An increase in $L^S$ shifts the $SS$ curve upward, because for given $m^S$ the added labor in the South more than suffices to serve the country’s increased demand for homogeneous goods. The relative wage $\omega$ must rise in order to eliminate the incipient excess supply of labor. But a higher relative wage in the North makes search in the South relatively more profitable, so $m^N$ must rise to restore equal profitability. The new $SS$ curve is represented by the broken curve in the figure.

In the North, the growth in Southern income means additional demand for differentiated products, and thus a greater demand for labor by final-good producers. This generates a leftward shift of the $NN$ curve, as can be seen from (20). This is because, for given $m^N$, the relative wage $\omega$ must rise to curtail the excess demand for Northern labor that results from the income growth in the South. As a result, search becomes more profitable in the South, and the number of suppliers must decline there to restore equal profitability. The leftward shift in the $NN$ curve is represented in the figure by a broken curve. The new equilibrium is at point $E'$.

As the figure shows, an expansion of resources in the South induces entry by local producers of components and exit by such producers in the North. This has immediate
implications for the composition of world outsourcing activity. We define the volume of outsourcing as $v^i = 2m^i r^i n^i y^i$; that is, the number of units of intermediate goods manufactured by input suppliers in country $i$. In a regime with a binding investment constraint, (5), (6), (13) and (14) imply that

$$v^i = \frac{4\alpha}{1 - \alpha} m^i f_m^i.$$  

(23)

In this setting, the volume of outsourcing in a country is proportional to the number of component producers active there. Evidently, an increase in $L^S$ boosts outsourcing activity in the South while curtailing such activity in the North.

It is interesting to note the effect on the relative wage. Figure 3 shows the combinations of $m^N$ and $m^S$ that imply the same relative wage as at point $E$. These points satisfy the equal-profit condition (18) for $\omega = \omega_E$, where $\omega_E$ is the relative wage at $E$. Points above the ray correspond to a higher relative wage in the North than $\omega_E$, while points below it correspond to a higher relative wage in the South. We see that, as long as outsourcing continues to take place in both countries, an increase in $L^S$ must boost the relative wage of the South. The direct effect of an increase in $L^S$ is to generate excess supply for labor in the South and excess demand in the North. But the shift in outsourcing activity has the opposite effects. Moreover, the thick-market
externality implies that outsourcing is an increasing returns activity at the industry level. Only when the wage of the North falls relative to that of the South will the final producers find it to be equally profitable to search in either region in view of the now thinner market in the North and the thicker market in the South.\footnote{That the rise in the supply of an input can lead to a rise in its relative reward has been pointed out in other circumstances as well. For example, Grossman and Helpman (1991, chapter 11) show that in a world in which the North innovates and the South imitates an increase in the size of the South raises its relative wage, and Acemoglu (1998) shows that an increase in the supply of skilled workers can induce skill-biased technical change that leads to an increase in their relative wage.}

An increase in $L^S$ also increases the value of world trade, the share of trade in world income, and the fraction of world trade that is intra-industry trade. The value of world trade is the sum of the value of Northern imports of homogeneous goods, the value of Southern imports of final goods, and the value of Northern imports of components. But trade balance implies that the total value of Southern imports, $\beta w^S L^S$, equals the value of its exports of homogeneous goods and of components. Therefore, the value of world trade is

$$T = 2\beta w^S L^S, \quad (24)$$

which rises with $L^S$ when measured either in terms of the numeraire good (so that $w^S = 1$) or in terms of Northern labor. The ratio of trade volume to world income is

$$\frac{T}{GDP} = \frac{2\beta L^S}{\omega L^N + L^S} \quad (25)$$

while the fraction of trade that is intra-industry trade is\footnote{The volume of intra-industry trade is defined as twice the smaller of the North’s exports of differentiated final goods and the South’s exports of intermediates. In this case, the latter quantity is smaller. Since the volume of these exports equals $\beta L^S - (1 - \beta)\omega L^N$ (the difference between the South’s imports of differentiated goods and the South’s exports of homogeneous goods), the expression in (26) follows from (24).}

$$\frac{T_{\text{intra}}}{T} = 1 - \frac{1 - \beta}{\beta} \frac{\omega L^N}{L^S}. \quad (26)$$

It is clear that both of these ratios rise with $L^S$, because the direct effect and the indirect effect that derives from the change in the relative wage both point in the same direction.
We will not repeat the analysis for the case of an increase in $L^N$. The reader may confirm that the qualitative effects on the number of component producers in each country, the location of outsourcing activity, the relative wage, the ratio of trade to world income, and the share of intra-industry trade are just the opposite of those for an increase in $L^S$.

4.2 Outsourcing Technology

The technology for outsourcing is reflected in the parameters that describe the cost of customizing a prototype for a particular producer of final goods ($\mu^i$). Arguably, changes in production methods associated with computer-aided design have reduced the cost of customizing components. We investigate how improvements in the investment technologies affect the location of outsourcing activity.\(^{23}\)

First, consider equi-proportionate improvements in the investment technologies; i.e., $\mu^N$ and $\mu^S$ fall by similar percentage amounts. From the equal-profit condition (18), we see that such technological change has no effect on the relative profitability of searching in the North versus the South. Moreover, the investment parameters have no direct effect on labor demand in neither the North nor the South (see (15) and (17)). As a result, only the ratio of these parameters appears in the reduced-form $SS$ and $NN$ equations (see (19) and (20)). It follows that a uniform improvement in investment technologies leaves the $SS$ and $NN$ curves in their initial locations. There is no effect on the number of component producers in either country, on the relative wage, on the levels of outsourcing activity, or on the level and composition of international trade.

Now we consider improvements in the technology for customizing components in the South alone. When only $\mu^S$ falls, it becomes more profitable for final producers to search for partners in the South at the initial relative wage. The relative wage of the North must fall to restore equal profitability of search (see (18)). But then the $SS$ curve shifts up and the $NN$ curve shifts to the left, as illustrated in Figure 4.

\(^{23}\)In Grossman and Helpman (2002b), we analyze as well the effects of reductions in the marginal cost of search due, for example, to improvements in transportation and communications technologies.
The equilibrium moves from $E$ to $E'$, implying an increase in the number of input suppliers in the South and a fall in their number in the North. Outsourcing activity shifts from North to South (see (23)).

The fall in $\mu^S$ implies, by (18), that the relative wage $\omega$ can be realized with a smaller number of input suppliers in the South (given $m^N$) than was true before the technological change. Thus, the $\omega = \omega_E$ ray rotates as drawn. At $E'$, the relative wage of the North is lower than $\omega_E$. It follows, from (25) and (26), that an improvement in the investment technology in the South results in an increased ratio of trade to world income and an increased share of intra-industry trade.

To summarize, a rise in international outsourcing with concomitant growth in the importance of trade and of intra-industry trade can be explained by improvements in the technologies for customization, but only if these improvements have occurred to a disproportionate extent in the South. It is certainly plausible that such technological catch-up has occurred in recent years.  

\footnote{In Grossman and Helpman (2002b) we show that similar results obtain when the search technology improves disproportionately in favor of the South, when search costs are a function of distance.}
5 Contracting with Partial Verifiability

We would like to examine how differences in the contracting environment across countries affect equilibrium patterns of outsourcing. To this end, we proceed now to extend our model to incorporate intermediate cases between the familiar extremes of “no contracts” and “perfect contracts.” Specifically, we assume that, in country $i$, an outside party can verify a fraction $\gamma_i < 1/2$ of the investment in customization undertaken by an input supplier for a potential downstream customer.\(^{25}\) The parameter $\gamma_i$ may be given a literal interpretation: the development of a prototype requires a number of stages or sub-investments, some of which are verifiable and others are not. More figuratively, we imagine that $\gamma_i$ captures the quality of the legal system in country $i$; the greater is $\gamma_i$, the more complete are the contracts that can be written there. When investments are partially verifiable, potential business partners are able to write (limited) contracts governing the supplier’s investment in customization.

We assume now that bargaining occurs in two stages. When a final producer approaches a potential supplier in a given market, the two firms first negotiate over the extent of the supplier’s investment in customization and the amount of compensation that the customer will pay for the prototype. Later, the parties negotiate over the quantity and price of components. We will refer to the contract that governs the supplier’s investment in the prototype as an investment contract, to distinguish it from the subsequent order contract.

Consider the negotiation of an investment contract between an input producer in country $i$ and a final-good producer whose required component is at a distance $x$ from the supplier’s expertise. The contract can require a level of investment up to but not exceeding $\gamma_i w_i \mu_i x$, because only verifiable investments can be covered by contract. The contract also can specify a payment $P_i$ for which the final-good producer will be liable if the supplier carries out the stipulated investment. We

\(^{25}\)Since the component producer garners one-half of the returns to any investment in the Nash bargain, full efficiency can be achieved whenever it is feasible for the parties to write a contract that calls for an equal sharing of costs. Accordingly, our assumption that $\gamma_i < 1/2$ corresponds to an assumption that full efficiency is not attainable.
assume Nash bargaining wherein the parties share equally in the surplus that accrues from any such contract. The parties anticipate that if they reach a stage where a suitable prototype exists, their negotiation will lead to an equal sharing of $S_i$ under an efficient order contract. So each party expects to earn $S_i/2$ if a first-stage bargain is reached and if the supplier chooses to invest the full amount $w^i \mu^i x$ needed for the development of a suitable prototype.

Suppose first that the distance $x$ between the supplier and potential downstream customer is such that $S_i/2 < (1 - \gamma^i)w^i \mu^i x$. Then the supplier’s share of the prospective profits is not large enough to cover the cost of the discretionary investment (i.e., the part of the investment that cannot be governed by any contract). In the event, the supplier will not make the full investment in customization. Foreseeing this outcome, the final producer will not be willing to pay anything for a partial investment of $\gamma^i w^i \mu^i x$, and the producer will not make any relationship-specific investment. In short, there is no investment contract and no investment under these conditions.

Next suppose that $S_i/2 \geq w^i \mu^i x$. In such circumstances, the supplier’s share of the prospective profits covers the full cost of the requisite investment in the prototype. Then the supplier is willing to proceed with the full investment even if there is no contract requiring a partial investment and no initial payment whatsoever. In this case, an investment contract creates no surplus relative to the joint profits that would result without it. It follows that the investment contract can be a null contract, or (what amounts to the same) it can require an investment of $\gamma^i w^i \mu^i x$ with a payment by the final-good producer of $P^i = 0$.

Finally, suppose that $(1 - \gamma^i)w^i \mu^i x \leq S_i/2 < w^i \mu^i x$. In this situation, the component producer would not be willing to bear the full cost of customizing the component absent an investment contract, but it would be willing to undertake the marginal investment of $(1 - \gamma^i)w^i \mu^i x$ if an investment of $\gamma^i w^i \mu^i x$ were stipulated by an enforceable contract and justified by a sufficiently large payment $P^i$. Therefore, the parties can share a positive surplus if they manage to agree on an investment contract in the first stage of negotiations. The Nash bargain calls for an equal sharing of the potential surplus relative to their outside options of zero, which means that an initial payment.
by the final-good producer must equalize the net rewards to the two parties. The final producer’s reward net of the payment is $S_i^i/2 - P^i$, where the input producer’s reward including the payment but net of the investment cost is $S_i^i/2 + P^i - w^i\mu^i x$. Equating the two, we have $P^i = w^i\mu^i x/2$. In other words, when $(1 - \gamma^i)w^i\mu^i x \leq S_i^i/2 < w^i\mu^i x$, the two sides share the investment cost equally.

To summarize, the investment contract and the induced investment behavior depend upon the contracting environment in the supplier’s home country, on the distance between the supplier’s expertise and the final producer’s input requirement, and on the size of the potential profits that would be generated by an efficient order contract. Let $P^i(x)$ be the payment that is dictated by an investment contract between a final producer in the North and an input supplier in country $i$ when the supplier’s expertise differs from the buyer’s input needs by $x$, and let $I^i(x)$ be the induced investment level. Then

$$P^i(x) = \begin{cases} \frac{1}{2}w^i\mu^i x & \text{for } \frac{S_i^i}{2w^i\mu^i} < x \leq \frac{S_i^i}{2w^i\mu^i(1-\gamma^i)} \\ 0 & \text{otherwise} \end{cases}$$

and

$$I^i(x) = \begin{cases} w^i\mu^i x & \text{for } x \leq \frac{S_i^i}{2w^i\mu^i(1-\gamma^i)} \\ 0 & \text{otherwise} \end{cases}$$

Now recall that the distance in input space between a final-good producer who searches in country $i$ and its nearest supplier is a random variable uniformly distributed on $[0, 1/2m^i]$. Together with the investment equation (28), this means that the greatest distance between an active final-good producer and its supplier of compo-

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26 We have also examined what happens when the bargaining shares are unequal at one or both stages. The greater is the bargaining power of the component supplier at the first stage, the larger is the payment $P^i$ that this firm collects from the customer in situations where $P^i > 0$. But the division of surplus in the first stage does not affect the range of distances between supplier and customer for which the investment takes place. In contrast, the greater is the bargaining share of the component supplier in the second stage, the larger will be the maximum distance between supplier and customer for which $I^i > 0$, unless it is anyway the case that all final producers are served.
nents in country $i$ is given by
\[ r^i = \min \left\{ \frac{S^i}{2w^i \mu^i (1 - \gamma^i)}, \frac{1}{2m^i} \right\}. \] (29)

Once a component supplier has borne the cost of investment in the prototype, the partners share similar interests regarding the production and marketing of the final good. As in the case with $\gamma^i = 0$, they write an efficient order contract to govern the manufacture and sale of the intermediate inputs, sharing equally the surplus $S^i$ given by (5).

A final-good producer who searches for a supplier in country $i$ finds a partner willing to invest in customization with probability $2r^im^i$. The prospective supplier may be at any distance between zero and $r^i$ with equal probability. It follows that the expected profits of a final-good producer who searches in country $i$ are
\[ \pi^i_n = 2m^i \int_0^{r^i} \left[ \frac{S^i}{2} - P^i(x) \right] dx. \] (30)

As before, final-good producers search in the country offering the higher expected profits. Therefore the expected operating profits of a final-good producer are $\pi_n = \max \{ \pi^N_n, \pi^S_n \}$, and the free-entry condition remains (8).

A component producer that enters in country $i$ will serve a measure $2n^ir^i$ of buyers. The firm’s customers will be spread uniformly at distances ranging from zero to $r^i$. A supplier earns profits of $P^i(x) + S^i/2 - w^i \mu^ix$ from its relationship with a final-good producer whose input requirement is at a distance $x$ from its own expertise. Thus, potential operating profits for an input producer that enters in country $i$ are
\[ \pi^i_m = 2n^i \int_0^{r^i} \left[ P^i(x) + \frac{1}{2}S^i - w^i \mu^ix \right] dx. \] (31)

Changes in the contracting environment, as measured by changes in the $\gamma^i$’s, can affect the outsourcing equilibrium only when they alter either the probability that a typical firm will find a willing partner or the payments made by final producers to their suppliers. But every final producer is sure to be supplied with components when $r^i = 1/2m^i$ and $P^i$ does not depend on $\gamma^i$. Therefore, a change in $\gamma^i$ does not affect the equilibrium unless the investment constraint binds in country $i$. We henceforth focus on equilibria in which the investment constraint binds in both countries.
When the investment constraints bind, (29) implies that
\[ r^i = \frac{S^i}{2w^i\mu^i(1 - \gamma^i)} \quad \text{for } i = N, S. \] (32)

Expected operating profits for a final producer searching in country \( i \) can be calculated using (27) and (30). Substituting these expected profits into the free entry condition (8) yields
\[ r^i \left[ m^i S^i - \frac{1}{2} m^i w^i \mu^i r^i \gamma^i (2 - \gamma^i) \right] = w^N f \quad \text{for } i = N, S. \] (33)
The difference between this equation and (16) — which applies when \( \gamma^i = 0 \) — is the second term in the square brackets. When multiplied by \( r^i \), this term reflects the expected first-stage payment by a final producer to its prospective parts supplier.

Similarly, we can use (27) and (31) to calculate the operating profits for a component producer in country \( i \). Equating these to the fixed cost of entry (and thereby assuming that \( m^i > 0 \) for \( i = N, S \)), we have
\[ r^i n^i \left[ S^i + \frac{1}{2} w^i \mu^i r^i \gamma^i (2 - \gamma^i) - w^i \mu^i r^i \right] = w^i f_m^i \quad \text{for } i = N, S. \] (34)
The product of the second term in the square brackets and \( r^i n^i \) is the total amount of up-front payments received by the typical input supplier from its various customers.

We now are ready to derive the reduced-form labor-market clearing conditions that apply when \( \gamma^i > 0 \) and the investment constraint binds in both countries. Substituting (5), (6), (32) and (34) into (11), we find that
\[ (1 - \beta) \left( \omega L^N + L^S \right) + \left[ \frac{2^{1+\alpha} - \frac{1+\beta \alpha}{1-\alpha} \gamma^S - \frac{1}{2} (\gamma^S)^2}{1-\gamma^S - \frac{1}{2} (\gamma^S)^2} \right] m^S f_m^S = L^S. \] (35)
The first term on the left-hand side represents labor demand by producers of the homogeneous product, while the second term represents labor demand by Southern producers of intermediate inputs for entry, investment and production.

Similarly, we use (4), (5), (6), (32), (33) and (34) to substitute for the terms in
This yields
\[
\frac{1}{2} (1 - \alpha) \beta \left( L^N + \frac{1}{\omega} L^S \right) - \left[ \frac{\gamma^N (1 - \frac{1}{2} \gamma^N)}{1 - \gamma^N - \frac{1}{2} (\gamma^N)^2} \right] m^N f_m^N
\]
\[
- \left[ \frac{\gamma^S (1 - \frac{1}{2} \gamma^S)}{1 - \gamma^S - \frac{1}{2} (\gamma^S)^2} \right] \frac{1}{\omega} m^S f_m^S + \left[ \frac{2^{1+\alpha} - \frac{1+3\alpha}{1-\alpha} \gamma^N - \frac{1}{2} (\gamma^N)^2}{1 - \gamma^N - \frac{1}{2} (\gamma^N)^2} \right] m^N f_m^N = L^N.
\]

The first three terms on the left-hand side represent the total demand for labor by final-good producers for entry and search, while the last term represents the labor used by component producers in the North for entry, investment, and production.

To complete the construction of the reduced-form SS and NN curves, we need an equal-profit condition. We substitute (5) and (32) into the free-entry condition for final producers (33), and equate the expected operating profits from search in either country, to derive
\[
\omega = \left( \frac{\mu^S m^N}{\mu^N m^S} \right)^{\frac{1}{1+\alpha}} \left[ \frac{(1 - \gamma^S)^2}{(1 - \gamma^N)^2} \right] \frac{2 - 3\gamma^N + \frac{1}{2} (\gamma^N)^2}{2 - 3\gamma^S + \frac{1}{2} (\gamma^S)^2} \right]^{\frac{1}{1+\alpha}}.
\]

As before, a relatively thicker market for components in the North raises the profitability of search in the North relative to search in the South. To offset this imbalance with \(m^N\) and \(m^S\) fixed, the relative wage of the North must rise. Similarly, an improvement in the technology for customization in a country mandates a rise in the country’s relative wage if equal profitability is to be preserved. These relationships are the same as before. Now, in addition, the degree of contract incompleteness affects the profitability of search in a given country. When \(\gamma^N < 1/2\), an increase \(\gamma^N\) raises the expected profits from searching in the North relative to the South. To restore equal profitability with the same number of firms, the relative wage of the North must rise. Similarly, for \(\gamma^S < 1/2\), a rise \(\gamma^S\) requires a fall in \(\omega\) for equal profitability at the initial \(m^N\) and \(m^S\). In short, an improvement in the contracting environment in a country raises the probability that a given match will result in a viable bilateral relationship and so enhances the attractiveness of searching for a supplier in that
country. In equilibrium, ω, m^N or m^S must adjust if outsourcing is to persist in both locations.

The new SS curve is obtained by substituting the relative wage from the equal-profit condition (37) into (35), while the new NN curve is obtained by substituting this relative wage into (36). When γ^S = γ^N = 0, the formulas for these curves collapse to (19) and (20), respectively.

5.1 Improvements in Contracting in the North

We begin by examining improvements in the contracting environment in the North. Suppose that, initially, γ^N = γ^S = 0. Figure 5 depicts the initial equilibrium point at E.

Now consider a marginal increase in γ^N. As we have just noted, this raises the relative profitability of search in the North. The relative wage ω must rise at given m^S and m^N to equalize the expected profits from search in either market. As a result, the SS curve shifts downward.\(^{27}\) This shift reflects the greater amount of Southern labor needed to produce homogeneous goods for the now better-paid Northern consumers.

In the Northern labor market, there are several effects that must be taken into account. First, the fall in the relative wage of the South spells a reduction in Southern demand for differentiated products, which tends to reduce employment by final producers. The demand for labor by final producers at given m^N also falls for another reason: the improvement in the contracting environment means that viable outsourcing relationships will develop between final producers and suppliers who are not able to consummate such a relationship without any investment contracts. Since each final producer ultimately has a better chance of finding a suitable partner, there are more final goods produced for any given number of entrants. But the implied intensifica-

\(^{27}\)For γ^S = 0, (37) and (35) yield the following formulae for the SS curve

\[
m^N = \frac{\mu^N}{\mu^S}m^S \left[ \frac{\beta L^S \cdot 2^{1+\gamma^S}m^S f^S}{(1-\beta)L^N} \right]^{\frac{1+\gamma^S}{1+\gamma^N}} \frac{(1 - \gamma^N)^2}{2 - 3\gamma^N + \frac{1}{\gamma^N} (\gamma^N)^2}.
\]

The right hand side of this equation declines in γ^N for γ^N < 1/2. It follows that an increase in γ^N shifts the SS curve downward.
tion of competition in the market for differentiated products means that fewer such producers are willing to bear the fixed cost of entry. This effect is reflected in the second term on the left-hand side of (36), which is zero when \( \gamma^N = 0 \) but becomes negative when \( \gamma^N \) turns positive.

The fall in labor demand by final-good producers is offset by an increase in demand by component producers. Component producers need more labor to make additional investments in customization and to serve their greater numbers of customers. The demand for labor by component producers is captured by the fourth term on the left-hand side of (36), and it unambiguously grows as \( \gamma^N \) increases. It is easy to verify that the fall in labor demand by final producers at given \( m^N \) and \( m^S \) exactly offsets the increase in labor demand by final producers when \( \gamma^N \) increases slightly from zero. This leaves only the effect of the rise in \( \omega \) that is needed to maintain equal expected profits from searching in either country. It follows that the \( NN \) curve shifts to the right for a small increase in \( \gamma^N \), as illustrated in the figure.28

The net result is an increase in the number of component producers in the North, a decline in the number of such producers in the South, and a hike in the North’s relative wage. This can be seen in Figure 5, which shows the new equilibrium at \( E' \), above and to the left of \( E \). Note that this equilibrium lies above the broken ray, which shows the combinations of \( m^S \) and \( m^N \) that give the same relative wage as at \( E \) in view of the new, higher value of \( \gamma^N \). It is also easy to show that the volume

\[ m^S = \frac{\mu^S}{\mu^N} m^N \left\{ \frac{[1 - \frac{1}{2} (1 - \alpha) \beta] L^N - 2 \frac{1+\alpha}{1-\alpha} \frac{1-\gamma^N}{1-2\gamma^N} m^N f_m}{\frac{1}{2} (1 - \alpha) \beta L^S} \right\} \frac{1+\alpha}{2} \frac{2 - 3\gamma^N + \frac{1}{2} (\gamma^N)^2}{(1 - \gamma^N)^2}. \]

The right-hand side of this equation is rising in \( \gamma^N \) when \( \gamma^N \) is small. Therefore, an increase in \( \gamma^N \) from \( \gamma^N = 0 \) shifts the \( NN \) curve to the right.

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28 We obtain from (37) and (36) the following equation for the \( NN \) curve when \( \gamma^S = 0 \):
of domestic outsourcing rises while the volume of outsourcing in the South falls. It follows further from equations (25) and (26) that the ratio of trade to world income and the share of intra-industry trade in total trade both fall.

While an initial improvement in contracting conditions in the North causes outsourcing to relocate from South to North, further improvements in the contract environment need not have this effect. In fact, once \( \gamma^N \) is positive, the boost in labor demand by component producers at given \( \omega \) and \( m^N \) induced by further growth in \( \gamma^N \) outweighs the fall in such demand by final-good producers (i.e., the fourth term in (36) grows by more than the second term shrinks). Still, there is an additional

\[ dv^N / d\gamma^N > 0 \] at \( \gamma^N = \gamma^S = 0 \), because there are an increased number of Northern component producers and each produces a larger volume of components. In the South, a fall in outsourcing results from the exit of Southern component suppliers.
fall in demand by final-good producers owing to the decline in Southern income (and reflected in the shift in \( \omega \) from (37)). On net, the \( NN \) curve may shift in either direction. It is easy to find situations in which an increase in \( \gamma^N \) from an initially high level causes exit by component producers in the North, entry by component producers in the South, and an expansion in international outsourcing and trade.\(^{30}\)

We have solved the model numerically for a wide range of parameter values. Holding \( \gamma^S = 0 \), we varied \( \gamma^N \) gradually from zero to 0.4 and found a recurring pattern. Namely, the volume of outsourcing in the North rises then falls as \( \gamma^N \) increases, but always remains above the level for \( \gamma^N = 0 \). Meanwhile, the volume of outsourcing in the South falls and then rises, while remaining below the level for \( \gamma^N = 0 \). The relative wage of the North rises, then falls, which implies that the ratio of world trade to world income and the share of intra-industry trade in total trade do just the opposite.\(^{31}\)

### 5.2 Improvements in Contracting Worldwide

Before we turn to the contracting environment of the South, it is helpful to discuss the effects of worldwide gains in contracting possibilities. We again take an initial situation in which all investment is unverifiable in both countries (\( \gamma^N = \gamma^S = \gamma = 0 \)), but now consider a change in the legal environment that makes some investment tasks contractible in both countries (\( d \gamma > 0 \)). We will show that, perhaps surprisingly, such a development would not be neutral with respect to the siting of outsourcing activity.

\(^{30}\)This occurs when \( \gamma^N > \bar{\gamma}^N \), where \( \bar{\gamma}^N < 1/2 \) is the unique solution to

\[
\frac{\bar{\gamma}^N (1 - \frac{1}{2} \bar{\gamma}^N)}{1 - \bar{\gamma}^N - \frac{1}{2} (\bar{\gamma}^N)^2} = \frac{1 - 2 \bar{\gamma}^N}{2 - 3 \bar{\gamma}^N + \frac{1}{2} (\bar{\gamma}^N)^2}.
\]

The value of \( \bar{\gamma}^N \) has been calculated so that the downward shift in \( SS \) at the initial \( m^S \) exactly matches the downward shift in \( NN \). With this initial value of \( \gamma^N \), an improvement in the contracting environment in the North results in a fall in \( m^N \) and no change in \( m^S \) or the relative wage. For still larger initial values of \( \gamma^N \) than \( \bar{\gamma}^N \), the \( NN \) curve shifts down by more than the \( SS \) curve, so \( m^S \) rises and \( m^N \) falls.

\(^{31}\)Such a pattern obtains, for example, when \( \mu^N = \mu^S = 50 \), \( \alpha = 0.5 \), \( \beta = 0.75 \), \( f^N_m = f^S_m = 0.01 \), \( L^N = 40 \) and \( L^S = 32 \).
From the equal-profit condition (37) we see that as long as \( \gamma^S = \gamma^N = \gamma \), the degree of contract incompleteness has no direct effect on the relative profitability of search in the two countries. As can be seen from (35), an increase in \( \gamma^S \) increases the demand for labor (at given \( m^S \) and \( \omega \)) by Southern component producers who need more labor, because each undertakes a greater number of investments and serves a larger number of customers. The demand for the homogeneous product must decline worldwide in order for the Southern labor market to clear at the initial \( m^S \), which means that \( \omega \) would have to be lower than before. But a decline in the relative wage makes search more profitable in the North, which requires a decline in \( m^N \) in order to restore the equal-profit condition. Therefore, an increase in \( \gamma \) from an initial situation with \( \gamma = 0 \) causes the SS curve to shift downward, as depicted in Figure 6.

The NN curve, in contrast, shifts to the right. While it is true that Northern component producers demand more labor (at given \( m^N \) and \( \omega \)) for much the same reason as their Southern counterparts, this is more than offset by a decline in employment by final producers. As we noted previously, at \( \gamma^N = 0 \), the second term on the left-hand side of (36) decreases with \( \gamma^N \) by the same amount as the fourth term increases. But now we also have a decline in the third term of (36) due to the growth in \( \gamma^S \). The additional bilateral relationships that are consummated by final producers with input suppliers in the South are an added source of intensified competition in the product market. In response, final producers exit in even greater number than they do when contracting improves only in the North. The result is an overall decline in labor demand in the North at given wages and given numbers of component producers. To restore labor-market equilibrium in the North, the relative wage in the North must decline, given \( m^N \). Such a decline in the relative wage would make search in the North more profitable. To restore equal profitability, more component producers would have to enter in the South. That is, \( m^S \) must increase for given \( m^N \) in order for the Northern labor-market to clear after \( \gamma \) rises.

As the figure shows, a worldwide improvement in contracting possibilities is not neutral with respect to the location of outsourcing; the equilibrium point shifts from \( E \) to \( E' \). In the new equilibrium, there are more component producers in the North and
fewer such producers in the South. The improvement in the legal environment induces a shift in outsourcing activity from South to North.\footnote{Recall that the volume of outsourcing from country $i$ is given by
$$v_i = \frac{4\alpha}{1 - \alpha} \frac{1 - \gamma^i}{1 - \gamma^i - \frac{1}{2} (\gamma^i)^2} m_i f_m.$$}

The asymmetric effects of the change in $\gamma$ come about, because the improved prospects for investment by input suppliers mitigates the need for entry by final-good producers. With the resources freed from the activity of designing differentiated products, the North can expand its input supply activities. Meanwhile, the improvements in contracting possibilities raise world income (evaluated in terms of the numeraire good), and with it the demand for homogeneous goods. More labor must be devoted by the South to producing these goods, which means that less is available for serving the needs of final producers.\footnote{It can readily be shown that an increase in $\gamma$ has no effect on the volume of outsourcing in a}
5.3 Improvements in Contracting in the South

We are now ready to explain why improvements in the contracting environment in the South, even if achieved from a very low initial level, need not result in an expansion of outsourcing activity there. We take an initial situation with $\gamma^N > \gamma^S = 0$ and consider a marginal increase in $\gamma^S$.

For reasons that are familiar by now, an increase in $\gamma^S$ raises (at given $\omega$, $m^S$ and $m^N$) the relative profitability of search in the South. To restore the equal-profit relationship, the relative wage $\omega$ must decline. The movement in the relative wage (or the terms of trade) expands the demand for differentiated goods by the South and reduces the demand for homogeneous goods by the North. Thus, the shift in $\omega$ exerts upward pressure on the SS curve and leftward pressure on the NN curve, both of which tend to generate an expansion of outsourcing activity in the South and a contraction of such activity in the North.

But the effects of the change in relative profitability are offset by impacts on labor demand at the initial pattern of search activity. In the South, component producers are able to serve more customers, and so their demand for labor grows for both investment and production purposes. This alone would shift the SS curve down. At the same time, the intensified competition in the product market that results from the broader search efforts of firms seeking partners in the South spells the exit of some final producers in the North. This alone reduces labor demand, tending to push the NN curve to the right. On net, the SS curve can shift in either direction, as can the NN curve.

Again, we resort to numerical computations to explore possible outcomes. Holding $\gamma^N$ fixed at $\gamma^N = 0.4$, we varied $\gamma^S$ from 0 to 0.4 for a wide range of values of the remaining parameters. Repeatedly, we find that the volume of outsourcing in the North rises monotonically with $\gamma^S$, while the volume of outsourcing in the South rises at first, but then falls to a level below that for $\gamma^S = 0$. So too does the ratio of world trade to world income, the share of intra-industry trade in total trade, and closed economy. The aggregate effects described here reflect the general equilibrium interactions between two asymmetric economies with segmented markets for components.
the relative wage of the South (i.e., $1/\omega$). In other words, the volume of international outsourcing, the share of world trade in world income, and the relative wage of the South typically are largest when the legal environment allows somewhat less complete contracts in the South than in the North.$^{34}$

6 Conclusions

We have developed a model that can be used to study outsourcing decisions in a global economy. In our model, producers of differentiated final goods must go outside the firm for an essential service or component. Search is costly and specific to a market. Each final producer decides where to conduct its search for a supplier. If a firm finds a potential partner with suitable expertise, the supplier can customize an input for the final producer’s use. Such relationship-specific investments are governed by incomplete contracts, and the contracting environment may differ in the two countries.

Our model features a thick-market externality: search in a market is more profitable the more suppliers are present there, while input producers fare best when they have many customers to serve. This externality creates the possibility for multiple equilibria, some of which may involve a concentration of outsourcing activity in one location. But stable equilibria need not involve complete specialization of input production in a single country. In the paper, we focused our attention on stable equilibria in which some firms outsource at home while others do so abroad.

First, we studied how labor supplies and the investment technologies affect the equilibrium location of outsourcing activity. As the South expands, its share of world outsourcing grows, as does the ratio of trade to world income and the share of intra-industry trade in total world trade. Uniform worldwide improvements in the investment technologies, as might result from advances in computer-aided design, have no effect on the volume of outsourcing or its international composition. But disproportionate improvements in the technology for customization in the South generate shifts

$^{34}$These patterns obtain, for example, when $\mu^N = \mu^S = 50$, $\alpha = 0.5$, $\beta = 0.75$, $f^N_m = f^S_m = 0.01$, $L^N = 40$ and $L^S = 32$. 
in outsourcing activity from North to South.

Next, we investigated the role of the contracting environment. We characterized the legal setting in a country by the fraction of a relationship-specific investment that is verifiable to a third party. An improvement in the contracting possibilities in a country raises the relative profitability of outsourcing there, given the numbers of component producers in each country and the relative wage. But changes in the contracting environment also affect the demand for labor by component producers and final-good producers at a given wage. A global increase in the fraction of contractible investment tends to favor outsourcing in the North, whereas an improvement in the legal environment of the South can raise or lower the volume of outsourcing there while raising outsourcing from the North.
References


