Suspensions and polymer solutions

Solution of Exercise 6

1. The total charge of the chain is $Q = N\varphi e$. The Flory free energy is, therefore,

$$F \sim \frac{k_\mathrm{B}T}{Na^2}R^2 + \frac{(N\varphi e)^2}{\varepsilon}\frac{1}{R}. \tag{1}$$

2. Minimizing F with respect to R, we get

$$R \sim (a^2 l_\mathrm{B})^{1/3}\varphi^{2/3}N, \tag{2}$$

where $l_\mathrm{B} = e^2/(\varepsilon k_\mathrm{B}T)$ is the Bjerrum length encountered earlier in the course (about 7 Å in water at room temperature). Note the unusual dependence on temperature ($R \sim T^{-1/3}$) and on the charge density ($R \sim \varphi^{2/3}$).

3. We see that the swelling exponent is $\nu = 1$, i.e., the dimensionality is $D = 1$. This means that the polyelectrolyte (in the absence of screening) is stretched like a rod, for any value of φ. This is a consequence of the long range of the Coulomb repulsion between monomers.