Suspensions and polymer solutions

Exercise 4

13 April 2011

1. We arrived at the following expression for the Oseen tensor in Fourier space:

$$\tilde{G}_{ij}(\mathbf{q}) = \frac{1}{\eta q^2}\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right).$$

   Invert back to real space to obtain

$$G_{ij}(\mathbf{r}) = \frac{1}{8\pi\eta r}\left(\delta_{ij} + \frac{r_i r_j}{r^2}\right).$$

2. Two spherical particles of radius $a = 1\ \mu$m are suspended in water ($\eta = 0.01$ poise $= 0.001$ Pa s). At $t = 0$ particle 1 is at the origin and particle 2 at $r\hat{\mathbf{x}}$. Their displacements from these initial positions as a result of Brownian motion are denoted $\mathbf{r}^{(1)}(t)$ and $\mathbf{r}^{(2)}(t)$. The displacement of the center mass from its initial position is

$$\mathbf{r}_{\rm CM}(t) = [\mathbf{r}^{(1)}(t) + \mathbf{r}^{(2)}(t)]/2.$$

   We define a diffusion coefficient for the center of mass as

$$\langle r^2_{\rm CM}(t)\rangle = 6D_{\rm CM}t.$$

   (a) Calculate $D_{\rm CM}$ in the absence of hydrodynamic interactions. Is it larger or smaller than $D_{\rm s}$, the self-diffusion coefficient of a single particle?

   (b) Now add hydrodynamic interactions (assuming $r \gg a$). Do they enhance or suppress the diffusion of the center of mass?

   (c) At what inter-particle distance r will the contribution from hydrodynamic interactions to $D_{\rm CM}$ be 10% of the bare one of item (a)? Reach a conclusion regarding the validity of the uncorrelated description: At what particle concentration do you expect significant deviations (of order 10%) of the suspension dynamics from the uncorrelated description? Translate this concentration to particle volume fraction.