Suspensions and polymer solutions

Exercise 5

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DNA is a semiflexible polymer having a persistence length $l_\mathrm{p} \simeq 50$ nm. Consider the molecule as a three-dimensional chain containing N freely-jointed (i.e., uncorrelated) segments of length l_p each. Suppose that one end of the chain is fixed at the origin and the other end is pulled with force $\vec{f}$ in the $\hat{\mathbf{x}}$ direction. The chain is in contact with a thermal bath of temperature T.

1. Calculate the partition function of the chain. Hint: The work required to bring the other end from the origin to a point $\vec{R}$ is $-\vec{f} \cdot \vec{R}$; hence, this is the energy of a configuration with end-to-end vector $\vec{R}$. Note that the problem is analogous to that of N non-interacting dipoles under external field.

2. Find the free energy.

3. Calculate the mean extension of the chain in the direction of pulling, $\langle x \rangle$. What is the value of $\langle x \rangle$ for very small f? What is $\langle x \rangle$ for very large f?

4. What is the force required to get $\langle x \rangle$ which is one half the total length of the chain at room temperature?