Chemical physics of polymer solutions

Exercise 2

3 November 2003

Show that the propagator of a one-dimensional Gaussian chain composed of N segments of length a under an external potential $V(x)$,

$$G(x_0, 0; x, N) = \int \mathcal{D}x(n) \mathrm{e}^{-\int_0^N \mathrm{d}n \left[\frac{1}{2a^2}\left(\frac{\mathrm{d}x}{\mathrm{d}n}\right)^2 + \frac{V(x)}{k_\mathrm{B}T}\right]},$$

satisfies the Schrödinger-like equation

$$\partial_N G = \frac{a^2}{2}\partial_{xx} G - \frac{V}{k_\mathrm{B}T} G.$$

You may find the following points helpful:

- In the absence of potential ($V = 0$) we have already proved this result (cf. the diffusion equation). Let us call the propagator of this case G_0.
- Consider the addition of a small number δN of monomers to the chain under potential. Write the propagator of the new chain and decompose it into two parts (a long chain connected to a very short chain). Note that

$$G(x_0, 0; x, M) = \int \mathrm{d}x' G(x_0, 0; x', N) G(x', N; x, M), \quad M > N.$$

- Express the G of the short part in terms of G_0 and V recalling that δN is very small.
- Calculate the required integral noting that $G_0(x', N; x, N + \delta N)$ has a sharp peak at $x = x'$.