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Optimal allocation of quotas

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Abstract

Consider a government which fixed the supply of a good (e.g. by an import quota). Suppose the market for the good can be partitioned (e.g. sales in different years or in different areas of the country can be considered sales in different markets). We show that maximization of consumer surplus in the domestic economy may require segmentation of the domestic market into two, but no more, parts. © 1998 Elsevier Science S.A.

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1. Introduction

Government often allocates a good among different segments of the population or in different markets: an airport allocates landing slots to different airlines serving different routes; the Federal Communications Commission allocates the radio spectrum for different uses; dams deliver water and electricity to different geographic areas. Since different allocations will generate different market prices, different profits, and different values of consumer surplus, understanding the effects of allocations is necessary for understanding government policy.

This paper focuses on one aspect of the effects, consumer surplus, which is well recognized as critical in political decision making.[1] We agree that other effects are important. But for some policies consumer surplus may be primary in determining allocations. The clearest case where government may care little about profits arises with import quotas. How should it allocate application the goods across different markets? For example, should a quota be set at one million cars a year, or instead at half a million cars each 6 months? Should that quota be on all vehicles, or subdivided between vehicles made by different manufacturers? Could the importing country increase consumer welfare by allowing imports into only one area of the country (for example by constraining ports of entry), or

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[1]For the seminal work on an agency's objective function, see Peltzman (1976). For an application to quotas, related to our example below, see Fabella (1991).

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should it allow sales everywhere? U.S. trade policy has used both types of restrictions. Quotas on footwear in force from 1977 to 1981 allowed the owners of import rights to determine the composition of imports among men's, women's, children's, and athletic footwear, though some distinctions were made by types of material and country of origin. In contrast, quotas on steel, textile, and apparel include detailed restrictions on specific products within the categories.[2]

Though most of the following focuses on import quotas, as shown in the concluding section, similar issues arise in other areas.

We shall give conditions calling for subquotas, showing that maximization of consumer surplus requires segmenting the market into no more than two parts, and that some segmentation may increase consumer welfare. To accentuate the results, we shall consider homogeneous rather than heterogeneous products: it is not surprising that if demand differs for different goods, then differential quotas can increase consumer welfare compared to uniform quotas. Because we consider homogeneous goods, we must determine the welfare-maximizing number of subquotas, and the number of persons eligible to buy the goods in each market, rather than taking as exogenous the number and composition of markets in which a good can be sold.

Our analysis of quotas, however, is incomplete. It does not ask whether, or by how much, tariffs are superior to quotas. We do not ask why government may want to impose a quota. We instead take a quota as given, and ask how it should be allocated to maximize consumer surplus in the importing country.

The analysis must consider two opposing effects. First, in a unified market units of the good will be purchased by the consumers in the population who most highly value the good. Second, however, in a unified market the price will be higher than in segmented markets. The higher price, though it benefits the seller, harms consumers. Thus, central to our analysis are the effects of subquotas on the terms of trade.[3]

To see the intuition for the benefits of segmenting a market, suppose the population consists of 500 persons who value the good at \$100, and 500 people who value the good at \$25. Suppose 300 units of the good can be sold. Then when the market consists of everyone, the market price will be \$100, and consumer surplus will be zero. Suppose instead that the population is divided into two identical submarkets, and that 300 units of the good are supplied to one of the submarkets. In this market only 250 people value the good at \$100, and the rest value it at \$25. The market price will therefore be \$25, and consumer surplus will be positive. As shown below, these qualitative results hold far more generally.

2. Assumptions

Consider a fixed quota, Q, on a good. For our purposes two items are considered the same good if they have the same production costs (both marginal and average). Consumers, however, may differ, placing different valuations on the same good. Moreover, we shall suppose that the distribution of

[2] See Berg and Bond (1991).

[3] Changes in the terms of trade can arise even for a small country. As Berg and Bond (1991) point out, under quotas a good is sold at the domestic market-clearing price, allowing even a small country to change the domestic terms of trade. Anderson (1985) considers subquotas for differentiated goods when the terms of trade are fixed.

consumers in each submarket is the same. Thus, we do not merely apply standard analyses of price discrimination or of product variety. We instead show that even under stringent conditions partition of markets can be beneficial.

Price (or marginal valuation) is given by the demand function $p(q)$. If a global quota is set, the price in the importing country is $p(Q)$. Consumer surplus is $\int_0^Q p(q)\,dq - p(Q)Q$.

Alternatively, let the quantities Q be divided among several markets. Consumers' preferences have the same distribution in all markets, but the number of consumers in each, and the quantity of goods sold in each, can differ. For example, goods sold in July may serve different consumers from goods sold in June, but with the demand curve in each month the same. The size of the population and of each market are continuous variables.

Suppose the quota rights are given to the exporters, so that the amount paid by domestic consumers is a social cost to the domestic economy. Let the volume of imports be sufficiently small to make a partial equilibrium analysis reasonably approximate the general equilibrium effects. Our assumptions thus make social welfare in the importing country equal the welfare of consumers who purchase the good.

3. Optimal number of markets

A solution which maximizes consumer welfare must consider the value of the goods to the consumers who obtain them, and the prices the consumers pay.

Theorem 1. *The partition that maximizes consumer surplus need have at most* 2 *markets.*

Proof: Call *per capita* demand at a price of 0 *potential demand.* Since the distribution of consumers types is the same in all markets, potential demand is the same in all markets. Define the *allocation* to a given market as the ratio of the *per capita* quota allocated it to its potential demand; denote the allocation by α. For example, if $\alpha = 1/3$ the supply allocated to that market equals 1/3 of potential demand in that market. Our aim is to allocate units of the good across markets to maximize aggregate consumer surplus. We can view the problem as considering a finite (but possibly extremely large) number of possible markets, each characterized by its allocation α_j. Without loss of generality, each market is supposed to get a different allocation. If several markets get the same allocation the same welfare is obtained by combining these markets into our larger market. Let the *per capita* consumer surplus in such a market be c_j; that value is determined by the value of α.

Since in our model government decides on the partition of the population into markets, we must also define the number of persons in each market. We do so with the choice variable X_j: market j (which has allocation α_j) has size X_j if a fraction X_j of the total population lies in this market. If for example, the total population is 1 million, then the number of goods allocated to market j is 1 000 000 $X_j\alpha_j$ times potential demand. If the optimal solution has $X_j = 0$, then no person consumes in a market with allocation α_j; if $X_j = 1$, then all persons consume in a market with allocation α_j.

The choice variables are therefore the values of X_j. One constraint to the problem is that the aggregate allocation not exceed the total supply. Supply of the good is specified by β, the proportion of potential demand for the good that can be satisfied. The other constraints to the problem require the size of each market to be non-negative, and the total quantity allocated to be no greater than the size of the population.

The problem is thus one of linear programming, where we need to determine the X values:

$$\max \sum_j c_j X_j$$

subject to:

$$\sum_j \alpha_j X_j \leq \beta$$

$$\sum_j X_j \leq 1$$

$X_j \geq 0$ for all possible allocations α_j.

A fundamental property of linear programming assures the existence of an optimal basic solution to the problem. Since the problem has two constraints (excluding non-negativity constraints), such a solution has at most two strictly positive variables.[4] □

3.1. *Examples for optimality of submarkets*

To see when quotas should be divided into one or into two markets we give some examples. Recall that an allocation in a market is described by a single number, α, with $0 \leq \alpha \leq 1$ expressing the proportion of potential demand allocated to it.

Example 1. Consider a population with three types of consumers, where each type constitutes one third of the population. In each market one third of the members are therefore of each type. The value of the good to the consumers is 0, 1, or 2, depending on the type of consumer. Consider a market supplied with a fraction α of its potential demand. Then, if $0 \leq \alpha < 1/3$, the price in that market is 2; consumer surplus there is zero. If $1/3 \leq \alpha < 2/3$ then the price is 1; consumer surplus per person is $1/3$. If $2/3 \leq \alpha \leq 1$ then the price is zero; consumer surplus per person is 1.

Since the surplus is constant for values of α within each interval, it suffices to consider two variables. Let Y_1 be the size of the market (as a fraction of the total population) to which $1/3$ of the supply is allocated. Let Y_2 be the size of the market to which $2/3$ of the supply is allocated. In a market allocated $1/3$ of the supply, the surplus per consumer is $1/3$; in a market allocated $2/3$ of the supply the surplus per consumer is 1. As before, let β represent supply – the fraction of potential demand that can be satisfied. Thus, the problem is to

$$\max (1/3)Y_1 + (1)Y_2$$

subject to:

$$(1/3)Y_1 + (2/3)Y_2 \leq \beta$$

$$Y_1 + Y_2 \leq 1$$

[4] The same argument shows, more generally, that when n goods are to be allocated, maximization of consumer surplus requires segmentation into at most $n+1$ markets.

$Y_1, Y_2 \geq 0.$

For $\beta \leq 1/3$ the optimal solution allocates nothing to one market, and allocates all the supply to a single market of size 3β of the population. One third of this market's potential demand is thus satisfied, making the price in that market 1. Thus, if few units of a good are to be allocated, optimization requires allocation to one small market, rather than to the entire population.

The optimum solution when $\beta \geq 2/3$ allocates aggregate supply to one market consisting of the entire population.[5] For $1/3 \leq \beta \leq 2/3$ two markets should share the supply. One market has size $(2-3\beta)$; $1/3$ of its potential demand is satisfied. The other market consists of the rest of the population; $2/3$ of its potential demand is satisfied.

In the following, the mean consumer surplus in a market with allocation α is called $c(\alpha)$.

Theorem 2. *Let $n=1$. Let $c(\alpha)/\alpha$ be monotone increasing in α. Then maximizing aggregate consumer surplus requires making sales in only one market.*

Proof: Theorem 1 states that an optimal solution exists with at most two markets. Let $\alpha_1 > \alpha_2$ be the fractions of the potential demands satisfied in the two markets. Let the proportion of the population in market 1 be X_1; the proportion of the population in market 2 is X_2. Let consumer surplus *per* consumer in these markets be $c_i \equiv c(\alpha_i)$, for $i=1,2$. The X values are computed by

$$\max c_1 X_1 + c_2 X_2$$

subject to

$$\alpha_1 X_1 + \alpha_2 X_2 \leq \beta$$

$$X_1 + X_2 \leq 1$$

$$X_1, X_2 \geq 0.$$

Fig. 1 depicts the problem and its solution. The first two constraints are shown by the solid lines. Values of X_1 and X_2 which yield constant values for the objective function are lines parallel to the dashed line. By assumption the dashed line is steeper than the line representing $\alpha_1 X_1 + \alpha_2 X_2 = \beta$. The optimal solution thus has $X_2 = 0$. □

Example 2. Consider a linear demand curve, $p(\alpha)=1-\alpha$. Then $c(\alpha)=\alpha^2/2$ and the condition of Theorem 2 holds. Maximizing aggregate consumer surplus therefore requires making sales in only one market. It can also be shown that total consumer surplus is double that obtainable when the goods are divided equally among two markets, or double that when the goods are allocated to a market consisting of the whole population.

Theorem 3. *Let $n=1$, let $c(\alpha)/\alpha$ be monotone decreasing in α, and let $c(\alpha)$ be concave. Then*

[5] The solution of the linear program calls for allocating only 2/3 of the potential demand. But such a solution is equivalent to satisfying all potential demand: the price declines to zero and the excess supply does not increase welfare.

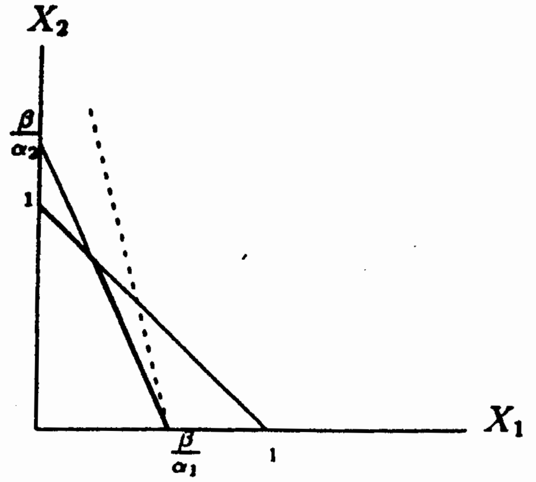

Fig. 1. The linear programming problem of allocating quotas.

maximizing aggregate consumer surplus requires distributing all the goods to the whole population (that is, the optimal solution has a single market).

Proof: We start as in the previous proof. But this time the dashed line is steeper than $\alpha_1 X_1 + \alpha_2 X_2 = \beta$ (and flatter than $X_1 + X_2 = 1$). The optimal solution lies at the intersection of the lines: $X_1 = (\beta - \alpha_2)/(\alpha_1 - \alpha_2)$ and $X_2 = (\alpha_1 - \beta)/(\alpha_1 - \alpha_2)$.

The objective function is then a convex combination of the two c values,

$$c_1 \frac{\beta - \alpha_2}{\alpha_1 - \alpha_2} + c_2 \frac{\alpha_1 - \beta}{\alpha_1 - \alpha_2}.$$

Since c is assumed to be concave, the objective is maximized at $\alpha_1 = \alpha_2$. This optimum can also be attained by limiting sales to a single market. □

Example 3. Consider the demand function $p = (1-q)/(1+q) = 2/(1+q) - 1$. Then $c(\alpha) = \int_0^{\alpha}(2/1 + q - 1)\mathrm{d}q = 2\ \ln(1+\alpha) - \alpha$ and the conditions of Theorem 3 can be shown to hold. With this demand function, partition of the market yields no benefits.

4. Applications

The Introduction claimed that import quotas are not the only problem to which the model can be applied. Instead, the results are relevant whenever supply is quantitatively restricted. A city which runs an airport and which must ration landings may care about the welfare of its citizens, and not about the profits of airlines; it must therefore consider whether to uniformly restrict flights to all destinations, or instead to provide good service to some destinations, and poor service to others. Similarly, a state under political pressure to restrict entry to some profession must decide whether to impose the restrictions more stringently on some specialities than on others.

Perhaps most importantly, an environmental agency considering limits on industrial pollution may want to impose severe limits on some industries and weak limits on others; the motivation would be not to maximize social welfare, but to limit the decline in consumer surplus caused by the regulations which reduce output and increase price. To make the analogy to quotas clearer, suppose each unit of output generates one unit of emissions, and therefore that emissions can be restricted only by restricting output. Then the question arises of how to allocate the output reductions across different industries. When the demand and cost functions are similar for the different goods, the standard analysis, which considers the maximization of social welfare, states that production should be restricted by about the same amount across all industries. Our analysis, which considers only consumer surplus, suggests that even under such conditions of homogeneity, some industries should be subject to much more stringent regulation than others. Such discriminatory regulation will be especially attractive when locally consumed goods are produced by foreign-owned firms, for then the local regulator may care about consumer surplus, but not about profits which enter into social welfare. With an increase in sales by multinational corporations, the considerations we emphasize may become increasingly important in explaining local and national regulatory policies.

Of course segmentation of a market may arise not because government attempts to maximize welfare, but as a result of lobbying by producers to protect their segment of the industry. Nevertheless, the results we consider arise even if consumer welfare is only one of several factors government considers when allocating a restricted supply.

5. Notation

α_j	Fraction of potential demand satisfied in market j
β	Fraction of aggregate potential demand for the good that can be satisfied
c_j	*Per capita* consumer surplus in market j
$p(Q)$	Demand function
Q	Aggregate allocation
X_j	Size of market j as a proportion of the total population, which has allocation α_j

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