זרימה ברשתות / רפי חסין
סמסטר I – תשס״ו

1. (a) Define a totally unimodular matrix and explain its meaning in the context of network flows.

   (b) Given a network with lower and upper bounds (l_{ij} and u_{ij}) on the flows, a source s and a sink t. Formulate a theorem on the value of the *minimum* $s-t$ flow (as in Hoffman's theorem, this theorem involves sums of bounds over cuts). Illustrate the theorem by showing a simple example where the minimum flow is ~~smaller than the maximum flow~~ affected by the upper bounds.

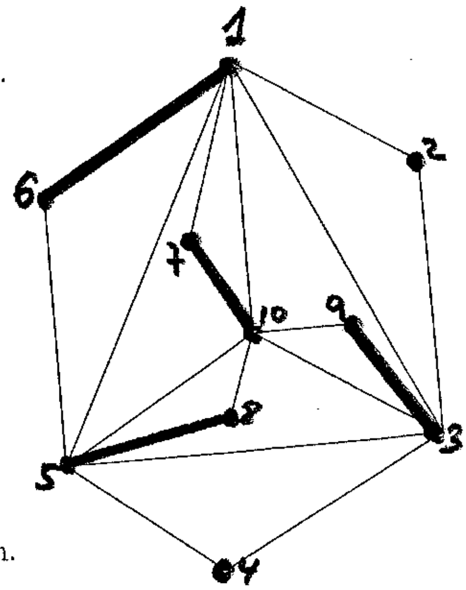
2. The problem is to compute the maximum possible reduction in the length of the shortest $s-t$ path when it is allowed to choose any set of K arcs and replace them by zero length arcs.

   (a) Is the following idea correct: Compute a shortest path and replace the longest K arcs in it by zero length arcs.

   (b) Design a dynamic program to solve the problem. For simplicity, assume first that the graph is acyclic. What is its complexity? What change is required if the graph has directed cycles?

3. (a) Describe Dantzig's algorithm for computing the shortest paths from node 1 and its complexity.

   (b) Describe Dial's algorithm and its scaling implementation.



4.

   (a) Prove that the matching in the figure is a maximum matching, or find a greater matching, by applying the algorithm for maximum (unweighted) matching.

   (b) Prove that the matching you got (the given one or the better one) is optimal by using the Tutte-Berge theorem.

   (c) Let M be a matching such that there are no augmenting paths with less than l edges of M (and $2l+1$ edges in total). Let M^* a maximum cardinality matching. Prove a lower bound on $|M|/|M^*|$. Hint: Consider the symmetric difference of M and M^*.