Network Flows: July 2017

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1. (a) Formulate the assignment problem as a min-cost flow problem with artificial source s and terminal t.

   (b) Use the min-cost flow algorithm for this formulation to solve the assignment problem with the following cost matrix: $\begin{bmatrix} 1 & 2 & \infty \\ 2 & 4 & 5 \\ \infty & 5 & 20 \end{bmatrix}$

2. Prove or disprove by a counterexample: The dual graph of a planar Eulerian graph is bipartite.

3. A bus company considers offering bus service between some pairs of cities. Before service can be offered between a pair of cities, a terminal must be constructed in each of them. The cost of constructing a terminal in City i is c_i. The profit received from offering service between i and j is p_{ij}. Solve the four cities problem with $c = (3, 7, 5, 5)$, $p_{12} = 2$, $p_{23} = 4$, $p_{13} = 8$, $p_{34} = 6$, and $p_{ij} = 0$ otherwise.

4. Design an efficient algorithm for checking the uniqueness of the minimum $s-t$ cut and justify its correctness.

5. Prove that there exists an optimal solution for the min-cost circulation problem such that the edges for which $l_{ij} < x_{ij} < u_{ij}$ do not contain a cycle (ignoring their directions). [The proof can be algorithmic or by using the theory of linear programming.]

6. Consider the following generalized b-matching problem. The input consists of

   (a) Graph $G = (V, E)$ with edge weights.

   (b) Partition $V_1, \dots, V_k$ of V into disjoint sets.

   (c) Integers $b_1, \dots, b_k$.

   The problem is to compute a minimum-weight subgraph such that for $i = 1, \dots, k$ the sum of degrees of the nodes $v \in V_i$ is exactly b_i. [In the common b-matching problem each V_i is a single node.] Transform the problem to a perfect-matching problem.

7. Given a directed acyclic graph (DAG), suppose that if flow x enters edge (i, j) from node i then flow $x + g_{ij}$ reaches node j. Note that g_{ij} can be positive, zero, or negative. Suggest a dynamic program that checks whether there exists a path from node 1 to node n such that if one unit of flow leaves node 1 and follows this path then the flow is positive everywhere along this path.