Aharon Razon:

The structure of symmetric tensor powers of a symbol algebra of prime degree

Let A be a symbol algebra of prime degree p over a field F of characteristic 0. Consider a positive integer n. The symmetric group S_n acts faithfully on $A^{\otimes n}$. We show there is an embedding $\theta\colon F[S_n] \to A^{\otimes n}$ such that $\sigma(a) = \theta(\sigma) \cdot a \cdot \theta(\sigma)^{-1}$ for each $\sigma \in S_n$ and each $a \in A^{\otimes n}$. Let $R^{(n)}$ be the fixed ring of $A^{\otimes n}$ under the action of S_n, let $z^{(n)} \in F[S_n]$ be the sum of all transpositions, and let $w^{(n)} = \frac{1}{n!}\sum_{\sigma \in S_n} \sigma$. Then $A^{\otimes n} = A^{\otimes n} \cdot \theta(z^{(n)} - \binom{n}{2}) \oplus A^{\otimes n} \cdot \theta(w^{(n)})$. Moreover, $R^{(n)} \cdot \theta(w^{(n)})$ is a central simple algebra of degree $\binom{n+p-1}{n}$ over F. Let $\mathcal{L}$ be the set of all $e \in A$ such that $[F[e] : F] = p$ and $e^p \in F$. For each $e \in \mathcal{L}$ let $\mathfrak{a}_e^{(n)}$ be the left ideal of $A^{\otimes n}$ generated by the $n-1$ elements: $e \otimes 1 \otimes \cdots \otimes 1 - 1 \otimes e \otimes 1 \otimes \cdots \otimes 1, \ldots, e \otimes 1 \otimes \cdots \otimes 1 - 1 \otimes \cdots \otimes 1 \otimes e$. It is shown that $R^{(n)} \cap \bigcap_{e \in \mathcal{L}} \mathfrak{a}_e^{(n)} = R^{(n)} \cdot \theta(z^{(n)} - \binom{n}{2})$.