Direct and Indirect Network Effects are Equivalent: A Comment on “Direct and Indirect Network Effects: Are They Equivalent?”

by

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Abstract
Clements (2004) makes the following two claims: (i) unlike direct network effects, increases in the size of the market do not, in the case of indirect network effects, make standardization more likely, but (ii) indirect network effects are associated with excessive standardization. We show in Clements’ framework that neither of these results are correct: standardization is more likely as the number of software firms increases and when the type of market equilibrium is unique—there are only multiple networks or only standardization—there is never excessive standardization, but there could be insufficient standardization, just as is the case with direct network effects.

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1.0 Introduction

Clements (2004) suggests that there is an important difference between direct and indirect network effects. Clements finds that under direct network effects—a telephone network for example—a greater mass of consumers makes standardization more likely, but under indirect network effects—hardware/software networks—the opposite is true. The greater the size of the market, the less likely standardization. While direct networks are prone to tipping this is likely efficient, but the tendency in indirect networks is inefficient overprovision of standardization. Clements concludes that “a model of direct network effects is inadequate in analyzing a market in which network effects are in fact indirect” because of the difference in results.

Clements’ results are also the exact opposite of Church and Gandal (1992) (CG). CG observe that there are two (opposing) effects on profits from a software firm opting to join an existing network and support a hardware technology: a network or demand effect (demand for hardware rises and hence software sales go up) and a competitive effect (more competition as the number of software firms increases). In CG when the value of additional software is small, hardware differentiated, or there are few software varieties, the unique result is non-standardization. The only equilibria entail standardization when the value of additional software is large, hardware not differentiated, or there are lots of software varieties.

The main welfare result of CG is that when consumers place a high enough value on software variety, software companies, to avoid competition, support both hardware technologies even though consumers would be better off under standardization. Insufficient standardization results when there is sufficient hardware differentiation, relative to network effects, that a hardware technology is viable with a single software firm, but only for strong network effects and not too much differentiation. Insufficient standardization also occurs in settings with direct network effects (Farrell and Saloner 1986).

In CG network effects are driven by a love of variety. Consumers assemble systems comprised of one unit of hardware and one unit of software. Their valuation of the hardware rises as the
number of compatible software varieties increase because they can consume more systems.¹ Unlike CG, the Clements’ model is a matching model and consumers do not have a love for variety. Rather in Clements consumers have a single ideal system consisting of one unit of hardware and one unit of software that they consume (as in the components approach of Matutes and Regibeau (1988)). When an additional software firm in the Clements’ model supports a hardware technology, the advantage to consumers is a lower price and reduced mismatch costs. As in CG there is a demand effect and a competitive effect when a firm switches from one hardware technology to another. Indeed the competitive effect is larger, since rather than just a reduction in market share (as in CG), there is also a reduction in price.²

In the Clements’ model it is for small numbers of firms, limited hardware differentiation, or large software differentiation that the only equilibria are standardization. For large numbers of software firms, small software differentiation, or large hardware differentiation the unique equilibrium involves multiple networks. The results with respect to hardware and software differentiation (or benefit) are the same as CG. The difference is with respect to the number of firms. Clements gets standardization with a relatively small number of firms, CG multiple networks. Clements gets multiple networks with a large number of firms, CG standardization.

One of Clements’ main results is that unlike direct network effects, increases in the size of the market (or reduction in the fixed cost of software) do not, in the case of indirect network effects, make standardization more likely (Clements Result 1).³ Clements concludes (Clements 2004, p. 639 footnotes omitted):

- “standardization is less likely for a greater number of software firms. This is the opposite of Church and Gandal’s (1992) result.”
- “Under direct effects, a larger mass of consumers encourages standardization; the opposite is true under indirect effects.”
- “Under indirect effects, the mass of consumers determines the total number of software firms, and a large number of firms hinders standardization.”

¹ See Church and Gandal (2005) and Church, Gandal, and Krause (2008).
³ See Clements’ Corollary 2 (iii) and (iv) at p. 638.
Moreover, his welfare conclusions are also the exact opposite of CG. Rather than excessive variety, Clements finds that indirect network effects are associated with excessive standardization (Clements Result II).4

The two Clements’ results discussed above are counter-intuitive and, if correct, remarkable. We don’t believe they are correct. The reason the first is remarkable is that it does not appear to be consistent with network effects. For standardization to be an equilibrium, deviation by a software firm to another network cannot be profitable. The profitability of such a deviation presumably should be decreasing in the extent of network benefits enjoyed by consumers and these will be greater the larger the number of software firms. Indeed, in this comment, we show that neither Clements’ Result I nor Clements’ Result II are correct. Using the Clements’ model,5 we find, consistent with CG, that (I) standardization is more likely as the number of software firms increases and (II) when the market equilibrium is either multiple networks or standardization, there is never excessive standardization, but there could be insufficient standardization. There is a region of the parameter space where consumers would be better off if there was standardization but the unique equilibrium involves multiple networks. Thus, both models of direct network and indirect network effects have the same important welfare result: when the only market equilibria entail standardization, it is efficient. The inefficiency in both models of direct and indirect network effects is the tendency for there to be too little standardization when the unique market equilibrium involves multiple networks.

2.0 The Clements’ Model

To recap the assumptions in the Clements’ model: hardware is horizontally differentiated as per Hotelling with $X$ at the left end of the unit line and $Y$ at the right end. Consumers’ preferences are distributed uniformly. The density of consumers is $A$. The mismatch cost for hardware is $t_h$.

Consumers also buy a single variety of software. Consumers’ preferences for software are distributed uniformly on the unit circle. We assume that each hardware firm supplies a sole

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4 See Clements Corollary 5 at p. 641.
5 We use a variant of Clements’ model that insures when we consider the incentive for deviation by an independent software firm from a standard that there is competition on the alternative network. The Nash equilibrium software price that Clements uses is only valid if there is competition. We insure competition by assuming that hardware firms provide a single variety of software. See footnote 7.
variety of software. We denote the number of independently supplied software varieties for network $j$ as $N_j$. So the total number of varieties for network $j$ will be $N_j + 1$. We assume that software varieties are distributed at equal intervals around the unit circle. The mismatch cost for software is $t_s$. The anticipated mismatch costs of software for a consumer, on the basis of which they make their adoption decision are $t_s / (4(N_j + 1))$, where $N_j$ is the expected number of independent software varieties for hardware $i$, $i = X, Y$. This follows immediately from observing (i) that minimum mismatch costs are 0, (ii) that maximum mismatch costs are $t_s / (2(N + 1))$, and (iii) that the distribution of consumer preferences is uniform.

2.1 Consumers

The utility of a consumer that purchases hardware $X$ is,

$$U_X = U^0 - p^X - a t_h - p^{sx} - t_s / (4N_X + 4)$$  \hspace{1cm} (1)

and for hardware $Y$,

$$U_Y = U^0 - p^Y - (1-a) t_h - p^{sy} - t_s / (4N_Y + 4)$$ \hspace{1cm} (2)

where $a$ is the location of the consumer as measured from hardware $X$, $p^j$ the price of hardware $j$, and $p^{sj}$ the price of software for hardware $j$. The Nash equilibrium software price for hardware $j$ is

$$p^{sj} = c_s + t_s / (N_j + 1)$$  \hspace{1cm} (3)

where $j = X, Y$. Without loss of generality assume that the unit cost of software ($c_s$) equals zero. Denote the fixed cost of development of an independent software variety as $f$.

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6 All other assumptions are as in Clements except the assumption of a single variety of software provided by the hardware firm. See footnote 7 for why this is required.

7 As per Salop (1979). In Clements and in an Appendix available from the authors (3) is used for the software price even if there is a single variety of software. This would not be the profit maximizing price if there was a monopolist in software, i.e., a single variety. It is for this reason that we have deviated from Clements’ original formulation to allow for a “default” provision of a variety by the sponsor of a standard. Hence (3) will apply whenever there is a single independent source of software. The Appendix shows that our critique holds even if (3) is used for monopoly software provision and the software variety supplied by a hardware firm is zero, as per Clements’ analysis. In this Comment proper we adopt a specification for which (3) is in fact the Nash equilibrium price in software. Our critique of Clements does not depend on whether there is a default software variety or not.
By assumption, hardware is priced competitively and the marginal cost of the two technologies is the same, so the marginal consumer is defined, from (1) through (3) as,

\[ a = (1/2) + (5/8) \left( \frac{t_s}{t_h} \right) \left( 1/(N_y + 1) - 1/(N_x + 1) \right). \]  

(4)

### 2.2 Free-Entry Number of Software Firms

There is free-entry of software and two relevant cases. In the first case all independent software firms support only one of the two hardware technologies \( \left( N_j = N, N_i = 0 \right) \). \( U^0 \) is assumed to be sufficiently large that only the technology supported will be adopted and that it will be universally adopted. Its market share will be one and it is a de facto standard.\(^8\) The gross profits of a software provider that supports the hardware standard are

\[ \pi^j(N_j = N) = \frac{At_s}{(N+1)^2}. \]  

(5)

The free-entry number of independent software varieties/firms under standardization therefore is

\[ N = \sqrt{\frac{At_s}{f}} - 1. \]  

(6)

In the second case, there is a symmetric duopoly equilibrium in the hardware market. In this equilibrium the two hardware technologies have an equal number of complementary software varieties and equal market shares. The profits of a software provider are

\[ \pi^i(N_j = \hat{N}, N_i = \hat{N}) = \frac{At_s}{2(\hat{N}+1)^2} \]  

(7)

and the free-entry number of independent software varieties/firms (for each hardware technology) in a symmetric, multi-network, equilibrium is

\[ \hat{N} = \sqrt{\frac{At_s}{2f}} - 1. \]  

(8)

Using (6) and (8) we have \( \sqrt{2}(\hat{N} + 1) = N + 1 \) or \( N = \sqrt{2} \hat{N} + \sqrt{2} - 1 \).

\(^8\) We verify that in a standardization equilibrium the market share of the non-supported hardware is zero.
3.0 Equilibrium

Without loss of generality assume the standard supported is hardware X. The profits of a software firm on this standard are given by (5) with N defined by (6). The profits from deviating and being the sole independent software provider for hardware Y are:

\[
\pi^Y(N_Y = 1, N_X = N - 1) = (1/4)At_s(1/2 - (5/8)(t_s / t_h)(N - 2)(2N)^{-1}) . \tag{9}
\]

For standardization to be an equilibrium (5) must be greater than (9), so the condition for existence of the standardization equilibrium is

\[
(5/4)(t_s / t_h) \geq 2\left(\frac{N}{N - 2}\right)(1 - 8(N + 1)^{-2}) . \tag{10}
\]

The comparison in (10) is only valid if \( N > 2 \). If \( N > 2 \) then \( \hat{N} > (3/\sqrt{2}) - 1 > 1 \).

Notice that the right-hand side of (10) is declining in \( N \) (for \( N > 2 \)) and its limit as \( N \to \infty \) is 2. Hence standardization is more likely as the number of software firms increases (ceteris paribus), just as in Church and Gandal. Clements ‘Result I’ claims just the opposite (standardization is less likely and it is more likely that the unique equilibrium involves multiple networks as the number of software firms increases).

Standardization is an equilibrium if when all independent software firms support hardware X, a single independent firm would not benefit from deviating and switching its support from X to Y. In hardware-software models (and in Clements), the more software provided the more valuable the hardware. At some point, the more software on X, therefore, the less profitable it will be for a single software firm to switch to Y. As the software on X becomes larger, the demand effect from switching to Y becomes smaller and smaller.\(^{10}\) Indeed in the Church and Gandal model when the

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\(^9\) We have considered the case when \( N = 2 \) and \( \hat{N} = 1 \). See comments in Section 4.

\(^{10}\) Contrary to Clements’ intuition in footnote 15 at p. 639.
only equilibrium are standardization, a switch by a single software firm from the standard (X) is not enough to induce consumers to switch to Y, even those for whom Y is the best match.

A firm on hardware network $i$ would have an incentive to jump to hardware network $j$ at an interior equilibrium if its profits would increase. Clements uses a local condition on the profit function to assess the profitability of switching: for standardization to be the only equilibria the profit function of a independent software firm at $a=1/2$ and $N_y = N_x = \hat{N}$ must be increasing in the number of software firms. If this is the case, then deviating from the symmetric division between the two hardware technologies by a single software firm to the other network would increase its profits. The condition required for the profit function of a software firm to be increasing in independent software variety at $a=1/2$ when $N_y = N_x = \hat{N}$ (so total software support is $\hat{N} + 1$) is

$$
(5/4)(t_s / t_h) > \hat{N} + 1.
$$

So, following Clements, for standardization to be the only equilibria requires that both (10) and (11) hold. However, since $N = \sqrt{2}\hat{N} + \sqrt{2} - 1$, if (11) is satisfied so is (10) provided $\hat{N} \geq 1.9339$ and $N > 3.1492$. Henceforth we assume these parameter restrictions, except to illustrate the issues involved for smaller numbers of firm, we consider below $N = 2$ and $\hat{N} = 1$.

Figure 1 partitions the parameter space $(\hat{N}, (5/4)(t_s / t_h))$ using (10) and (11) for $\hat{N} > 1.9339$. For values of $(5/4)(t_s / t_h)$ above (11) the only equilibria involve standardization.$^{11}$ For values of $(5/4)(t_s / t_h)$ below (10) the unique equilibrium is multiple symmetric networks. For values of $(5/4)(t_s / t_h)$ between (10) and (11) both types of equilibria exist.

3.1 Summary of the Equilibrium

Our analysis indicates the following:

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$^{11}$ There are two possible standardization equilibrium: either all software firms support hardware X or they all support hardware Y.
Clements interprets the results as indicating that standardization is less likely as the number of firms grows. This is not true. As the number of firms grows, the range of parameter spaces for which standardization is an equilibrium becomes larger. Consequently, the results in Clements’ Corollary 2 (iii) and (iv) are incorrect. The factors that result in an increase in the number of firms—increases in the population and decreases in fixed costs—increase the propensity for standardization.

(ii) Further the claim (in footnote 14) by Clements that the positive results of this model are different than CG because of price competition in software is incorrect. There are no differences in the positive results. While it is true that price competition increases the competitive effect, this reinforces the results of CG since it makes it less likely that standardization is an equilibrium. The addition of price competition reduces the profits of standardization and increases the incentive to deviate.

4.0 Welfare

The optimality of the market equilibria is assessed by comparing the outcome with the choice a social planner would make between multiple networks and standardization. Since hardware is assumed to be competitive and the software sector characterized by free-entry, the choice is between which of the two equilibria is best, in aggregate, for consumers.

Aggregate consumer welfare under standardization is

\[ CS = A \left( U^0 - c_h - \frac{t_h}{2} - \frac{5t_s}{4(N + 1)} \right), \]  \hspace{1cm} (12)

while aggregate consumer welfare with multiple networks is

\[ \hat{CS} = A \left( U^0 - c_h - \frac{t_h}{4} - \frac{5t_s}{4(N + 1)} \right). \]  \hspace{1cm} (13)

Comparing (12) to (13) shows the trade off: under standardization the number of software varieties is larger, but there are increased mismatch costs attributable to a reduction in the variety in hardware. From (12) and (13) consumers prefer standardization if
\[
\frac{5s}{4h} > \frac{(N+1)(N+1)}{4(N - \hat{N})}.
\]

(14)

The welfare results are also shown in Figure 1 where we use the relationship \( N = \sqrt{2N + \sqrt{2} - 1} \) to rewrite (14) in terms of a relationship between \((5/4)(s/\hat{h})\) and \(\hat{N}\). For values of \((5/4)(s/\hat{h})\) above (14) the social optimum is standardization, while for values of \((5/4)(s/\hat{h})\) less than (14) the social optimum is multiple symmetric networks. For sufficiently high mismatch costs for software and low mismatch costs for hardware the reduction in software mismatch costs exceeds the increase in hardware mismatch costs from standardization.

4.1 Efficiency of the Market Equilibrium

4.1.1 Excessive Standardization

It is easy to verify that there is not socially excessive standardization when the only equilibria involve standardization. Figure 1 shows that Clements Result II does not hold (see his Corollary 5). As in Church and Gandal and Farrell and Saloner (direct network effects) excess standardization can only occur if there are multiple equilibria (below (14) and above (10)). This only occurs for \(\hat{N} > N^*\), where \(N^*\) equates (10) and (14).

4.1.2 Insufficient Standardization

For \(\hat{N} < N^*\) there is a region of the parameter space for which variety is the unique equilibrium, yet the social optimum is standardization (the shaded area below (10) but above (14)).

4.2 What happens when we restrict the number of firms to be an integer and N=2?

We have also considered the case when \(N = 2\) and \(\hat{N} = 1\).\(^{12}\) The interesting feature of this case is that only looking at the slope of the profit function to assess whether the symmetric interior configuration is a Nash equilibrium is insufficient. It ignores the potential for the profit function (as a function of the number of software varieties) to become concave and slope downwards if a

\(^{12}\)Fixing the number of firms is not a restriction, since there are parameter values \((A,t_s,t_h,f)\) for which in free-entry, \(N = 2\) with standardization and \(\hat{N} = 1\) under variety.
switch of a single firm from an equal allocation of software firms to each network is sufficient for standardization. If the extent of hardware differentiation is sufficiently low, then once a critical differential in the number of software firms is reached, the market standardizes on the hardware with the greater number of software varieties. The only effect of another software firm after the critical differential is reached is competitive, the demand effect is zero, and hence profits will decline as the number of software firms on the de facto standard increases.

For the case of \( N = 2 \) and \( \tilde{N} = 1 \), a switch by a single independent software firm from the symmetric interior equilibrium would result in standardization if, \((5/4)(t_s/t_h) > 3/2\), but it is not profitable. As a result the interior equilibrium is unique for all values of \((5/4)(t_s/t_h)\).

Notice that the deviation by a firm from standardization restores the market share of the hardware without an independent software from zero to half. The demand effect is very large and the competitive effect is small: software is very differentiated, but hardware is not. It is easy to show, using (14), that the social optimum also involves standardization when \((5/4)(t_s/t_h) > 3/2\). But in this case the unique equilibrium involves multiple networks, indicating insufficient standardization again when \((5/4)(t_s/t_h) > 3/2\).

### 5.0 Summary

As in Church and Gandal and Farrell and Saloner, the inefficiency in this setting is excessive variety or insufficient standardization. The intuition is the same as in Church and Gandal. When there are a small number of firms and standardization, a switch to support the excluded hardware technology has a large demand effect and increases the profits for the deviating software firm. As we observed, the suboptimal amount of standardization arises from the incentives for a single or small number of software firms to jump to an unsupported network. There is little or no competition on the unsupported network and there is a large demand effect. It is socially inefficient since the reduction in hardware mismatch costs is less than the increase in software mismatch costs.

Unlike Church and Gandal, the potential for insufficient standardization in Clements is limited to the case where there are a limited number of firms. In Church and Gandal the potential for
insufficient standardization is increasing in the value of variety, since this increases the extent of
the demand effect and hence the potential for an increase in profit from jumping to an
unsupported hardware technology. This creates a larger wedge between the social benefit of
variety and the private benefit to a software firm of switching to an unsupported network. In the
Clements’ model where network effects arise by reducing expected mismatch costs the potential
divergence between the social and private costs of switching to an unsupported hardware
technology are much more limited. We conjecture that this points to an important difference, not
between direct and indirect network effects, but between the source of indirect network effects.

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Figure 1
Red = (10). Market Multiple Networks Unique Below
Black = (11). Standardization Unique Above
Blue = (14). Standardization Socially Optimal Above