Karnaugh Maps

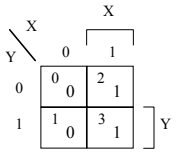
Karnaugh Maps

- Karnaugh Map : a representation of the truth table by a matrix of squares(cells) , where each square corresponds to a minterm (or a maxterm) of the logic function.
- For n-variable function, we need 2^n rows truth table and 2^n squares(cells).
- The square number is equivalent to the row number in the truth table
- To represent a logic function, the truth table values are copied into their corresponding cells .
- The arrangements of the squares help to identify the input variable redundancy (X.Y.Z+X.Y.Z'=X.Y)

Two-variable Karnaugh map

- Example : F = XY'+XY

ROW	X	Y	F
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1

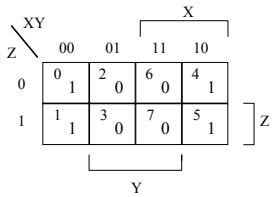


- Simplification : F = X(Y+Y') = X.1 = X

Three-variable Karnaugh map

- Example : F=X'.Y'.Z'+X'.Y'.Z+X.Y'.Z'+X.Y'.Z

Row	X	Y	Z	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0



- F = X'.Y'.(Z'+Z)+X.Y'.(Z'+Z)=X'.Y'+X.Y'=(X'+X).Y' = Y'

Four-variable Karnaugh map

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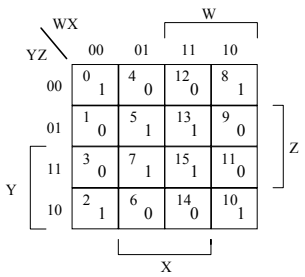
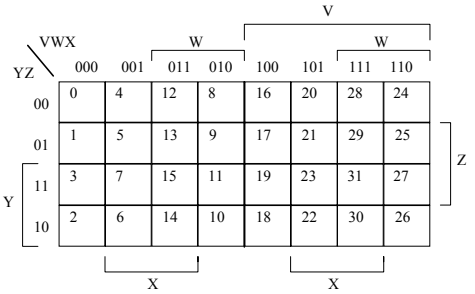
Row	W	X	Y	Z	F
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1



- F= W'X'Y'Z'+W'X'YZ'+WX'YZ'+W'XYZ+WX'Y'Z'+WX'YZ'+WXY'Z+WXYZ

Five-variable Karnaugh map

- Five variable K-map is formed using two connected four-variable maps:



Dont Care Conditions :

- In some applications, the Boolean function for certain combinations of the input variables is not specified. The corresponding minterms (maxterms) are called “dont care minterms(maxterms)”.
- In K-map , the “dont care minterms/maxterms” are represented by “d”.
- Since the output function for those minterms(maxterms) is not specified, those minterms(maxterms) could be combined with the adjacent 1 cells(0-cells) to get a more simplified sum-of-products (product-of-sums) expression.

Example

- Build a logic circuit that determines if a decimal digit is >= 5
- Solution:
- - The decimal digits(0,1,2,...,9) are represented by 4 bit BCD code.
- - The logic circuit should have 4 input variables and one output.
- - There are 16 different input combinations but only 10 of them are used.
- - The logic function should produce 0 if the number is <5 , and 1 if it is >= 5

Example - The Truth table

- The Truth table for the function is specified as follows:

Row	W	X	Y	Z	F
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	d
11	1	0	1	1	d
12	1	1	0	0	d
13	1	1	0	1	d
14	1	1	1	0	d
15	1	1	1	1	d

Timing Hazards

- The Truth Table determines the Steady State behavior of a Combinational Logic Circuit

- Transient behavior :
  - Output could produce ***glitches*** when input variables change.
  - Glitches occur when the paths between inputs and output have different delays.
  - Timing ***Hazards*** refer to the possibility of having glitches during input transitions.

- Hazards :
  - Definitions.
  - Finding hazards.
  - Eliminating hazards.

Example

- F= YZ+XZ'
- Delay in each gate is T .
- Input changes from XYZ=111 to 110