

Number Systems And Codes

Positional Number System

- General form of a number :

$$d_{p-1}d_{p-2}\dots d_1d_0.d_{-1}d_{-2}\dots d_{-n}$$

- The value of the number :

$$D = \sum_{i=-n}^{p-1} d_i \cdot r^i$$

- . : radix point
- r : base or radix
- $dp-1$: the most significant digit
- $d-n$: the least significant digit

Binary System

- The form of a binary number

$$b_{p-1}b_{p-2}\dots b_1b_0.b_{-1}b_{-2}\dots b_{-n}$$

- The decimal value of the number is :

$$B = \sum_{i=-n}^{p-1} b_i \cdot r^i$$

- $r = 2$ (binary radix)
- . : binary point
- bi (binary digit = bit) : 0 , 1
- $bp-1$: the most significant bit (MSB)
- $b-n$: the least significant bit (LSB)

Binary,Decimal,Octal and Hexadecimal Numbers

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

General Positional Number Conversion

- *radix-r* to decimal :

$$D = \sum_{i=-n}^{p-1} d_i \cdot r^i$$

- decimal to *radix-r* :
 - Successive division of D by *r*
 - The remainder of the long division will give the digits starting from the *least significant digit*

Binary Addition

- Binary addition table :

carry(in)	x	y	x+y	carry(out)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Binary Subtraction

● Binary Subtraction table :

borrow(in) x	y	x-y	borrow(out)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Two's Complement Representation

- The MSB represents the sign bit (0 = +ve , 1 = -ve)
- To calculate the negative number :
1- Complement all bits of the positive number (one's complement)
2- Add 1

- For n-bit number the decimal value =

$$B = \left(\sum_{i=0}^{n-2} b_i \cdot 2^i \right) - b_{n-1} \cdot 2^{n-1}$$

- The range for n-bit is :

$$\text{from } -(2^{n-1}) \text{ to } (2^{n-1} - 1)$$

- Advantages :
1- Only one zero
2- Addition and subtraction can be performed directly
- Disadvantage : One extra negative number (not symmetric)

Comparison (n=4)

Decimal	Signed Magnitude	One's Complement	Two's Complement
-8	-	-	1000
-7	1111	1000	1001
-6	1110	1001	1010
-5	1101	1010	1011
-4	1100	1011	1100
-3	1011	1100	1101
-2	1010	1101	1110
-1	1001	1110	1111
0	0000 or 1000	0000 or 1111	0000
1	0001	0001	0001
2	0010	0010	0010
3	0011	0011	0011
4	0100	0100	0100
5	0101	0101	0101
6	0110	0110	0110
7	0111	0111	0111

Decimal Codes (Table)

Decimal	BCD(8421)	2421	Excess-3	Biquinary	1-out-of-10
0	0000	0000	0011	0100001	100000000
1	0001	0001	0100	0100010	010000000
2	0010	0010	0101	0100100	001000000
3	0011	0011	0110	0101000	000100000
4	0100	0100	0111	0110000	000010000
5	0101	1011	1000	1000001	000001000
6	0110	1100	1001	1000010	000000100
7	0111	1101	1010	1000100	000000010
8	1000	1110	1011	1001000	000000001
9	1001	1111	1100	1010000	000000000

Gray Code

- One bit changes between two successive code words
- Binary Code and Gray Code (n = 3) :

Decimal	Binary Code	Gray Code
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

ASCII Table

		b6b5b4							
		000	001	010	011	100	101	110	111
b3b2b1b0									
0000				SP	0	@		'	
0001				!	1	A		a	
0010				"	2	B		b	
0011				#	3	C		c	
1111									

- The Code for a is 1100001