We present a model in which purely monetary inflation systematically affects efficiency, welfare, and relative prices. The model focuses on the microeconomics of trade in search markets under inflation. Inflation, by increasing the cost of holding money, undermines the market’s ability to sustain long-term customer relationships. Because those relationships generate the most efficient transactions patterns, overall welfare unambiguously declines.

1. INTRODUCTION

It is widely believed that inflation is costly because it causes resources to be employed inefficiently. According to Fischer (1981), “Inflation is associated with relative price variability that is unrelated to relative scarcities and hence leads to misallocations of resources.” It has also been observed that inflation dulls the competitive edge of more efficient firms. For example, Bresciani-Turroni (1937) in his classic study of the German hyperinflation observes, “But it cannot be said that these savings became available to the most productive firms and to those entrepreneurs who were most able to employ rationally the capital at their disposal. On the contrary, inflation dispensed its favors blindly and the least meritorious enjoyed them. Firms socially less productive could continue to support themselves thanks to the profits derived from the inflation although in normal conditions they would have been eliminated from the market” (p. 219). Similarly, the unusually high incidence of bankruptcies that has been observed to follow inflation stabilities (Garber, 1982; Bruno and Pitterman, 1988, p. 35) suggests that inefficient firms, which would otherwise not be viable, are able to prosper under inflationary conditions. Various accounts of the German inflation also report sharp declines in labor productivity (e.g., Bresciani-Turroni, 1937; Feldman, 1977).

Motivated by such observations, this article develops a model in which inflation exacerbates inefficiencies from informational frictions in decentralized search markets. Like the earlier informal discussions of the costs of inflation by Carlton (1982) and Okun (1975), our model emphasizes the disruptive effects of inflation on the informational role of prices and long-term trading relationships: “The welfare costs usually attributed to inflation . . . should be viewed in a broader context...
as disturbances to a set of institutions which economize on information, prediction and transaction costs through continuing buyer-seller relationships” (Okun, 1975, p. 359). It is also consistent with historical accounts of high inflationary episodes: “Previously stable trading connections were severed, Transactions patterns were altered and normally well-functioning markets collapsed” (Casella and Feinstein, 1990, p. 2).

In our model, overlapping generations of consumers derive utility from consuming two goods. The first is a numeraire good, produced and sold competitively. The second good, which is produced by firms with heterogeneous costs, is a “search good” in the sense that consumers know only its price distribution across sellers but not that which firm charges what price. Money depreciates at the rate of inflation. The basic decision faced by a first-time buyer, faced with a price observation, is whether to: (i) buy only one unit of the search good, spending the rest of her current money balances on the numeraire good; or, (ii) to keep enough depreciating cash on hand to return to the same firm next period, and buy a second unit at its new, inflated price.

In equilibrium, low-cost, efficient producers sustain long-term relationships with their clients by charging prices which are low enough that young consumers are willing to retain the required money balances to buy again from the same seller next time. Inefficient, high-cost producers cannot afford to do so and hence sell only to first-time buyers. However, the higher the rate of inflation, the deeper the discount that consumers require to retain money and hence the more costly it is for firms to maintain long-term relationships. As those relationships are severed, there is a shift in production from efficient sellers to inefficient ones and a substitution of consumption from the search good toward the numeraire good. Because long-term relationships generate the most efficient transactions patterns, overall welfare declines.

The above described features of the model are shown to generate interesting predictions about the effects of inflation on price dispersion for the same good (the search good) and the relative prices of the two consumption goods, subjects which have been the focus of intensive empirical research (e.g., Vining and Elwertowski, 1976; Parks, 1978; Taylor, 1981; Pagan et al., 1983; Van Hoomissen, 1988; Lach and Tsiddon, 1991; Tommasi, 1993).

Our model forms part of a growing body of recent research that seeks to increase our understanding of the welfare costs of inflation by studying its disruptive effects on market organization. As in this article, and in the related papers of Casella and Feinstein (1990) and Tommasi (1999), those effects may originate at the demand side, as a consequence of the fact that inflation induces individuals holding depreciating nominal money to speed up purchases. Or they may originate at the supply side, by affecting the ways in which sellers set real prices when nominal price changes involve real costs (Sheshinski and Weiss, 1973; Benabou, 1988, 1992; Fishman, 1992; Diamond, 1993; Ball and Romer, 1993).

2. THE MODEL

Consider an overlapping generations economy with a continuum of households and firms. A new cohort of identical households enters the economy at every
period. Each household lives for two periods, working at its first period and consuming at both periods. We shall henceforth refer to a household at its first period as a young consumer and at its second period as an old consumer.

There are two different consumption goods. One, the “numeraire” good, is consumed in infinitely divisible units and produced by a perfectly competitive industry. The other good is termed the “search” good because consumers are imperfectly informed about its price; although consumers know the distribution of its price, they do not know the price that is charged by each individual firm. The search good is consumed in indivisible units and a household demands at most one such unit at each period. A natural interpretation is that the search good is a service which is consumed periodically.

Households. Each household inelastically supplies two units of labor and there is no disutility from labor. A household derives constant marginal utility from the numeraire good and values a unit of the search good as the equivalent of one unit of the numeraire good. More specifically, we assume the household’s lifetime utility function to be:

\[ u = c_1 + c_2 + k_1 + k_2 \]

where \( c_i \) is the amount of the numeraire good consumed at period \( i \) and \( k_i = 1 \) if a unit of the search good is consumed at period \( i \), and 0 otherwise. All transactions are negotiated with fiat money, which is the only store of value. All firms are owned by households. At its first period, a household receives income, denoted \( v \), consisting of wages, an equal share of firms’ profits and a monetary transfer. More specifically, at each period, the firms pay wages and distribute profits to young consumers and the government issues new money in the amount of \( \pi M_{-1} \) to young consumers, where \( M_{-1} \) is the money supply at the end of the preceding period and \( \pi > 0 \). Thus the money supply, \( M \), increases at the rate \( \pi \).

A consumer can observe only one price of the search good at each period (for example, because of prohibitively high search costs). A young consumer observes one randomly selected price of the search good, and either accepts this price or does without the search good at that period. An old consumer can either return to the firm whose price she observed last period or choose a new firm at random.

Households make the following interrelated decisions. After observing one price of the search good, a young consumer decides whether to buy a unit of the search good and how much of the numeraire good to buy. This compound decision determines the amount of money that remains available for the second period. Given the amount of money retained from last period, an old consumer first decides whether to follow a repeat purchasing strategy and return to last period’s firm, or to choose a new firm. In addition, given the price she observes, she decides whether to buy a unit of the search good at that period or spend any remaining money on the numeraire good. A consumers’ decision strategy is thus

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2 As will become clear below, this ensures that each household can afford to buy the search good at each period.
fully described by $R^c = \{pr_1, m_2(p_1), s(m_2, p_1), pr_2(m_2)\}$, where:

$pr_1$ is the highest price at which a young consumer is willing to buy a unit of the search good.

$m_2(p_1)$ is the amount of money a young consumer retains to the following period, as a function of the price she observed at her first period, $p_1$.

$s(m_2, p_1)$ is an old consumer’s search strategy, determining whether she returns to last period’s firm or chooses a new firm.

$pr_2(m_2)$ is the highest price at which an old consumer holding $m_2$ units of money is willing to buy a unit of the search good.

The assumption that income can only be transferred between periods as money suggests that periods are short. Such an interpretation, according to which inflation is significant even during short periods, is consistent with a high inflationary environment. A consumer’s “lifetime” may then be thought of as the time between successive receipts of salary.\(^3\)

**Production.** Both the numeraire good and the search good are produced using only labor as a productive input. One unit of labor produces one unit of the numeraire good. The search good is produced by a continuum of firms with heterogeneous production costs. Specifically, each producer of the search good employs $\lambda$ units of labor to produce one unit of output, where $\lambda$ is uniformly distributed over $[0, 1]$. We assume that firms cannot price discriminate between young and old consumers. This assumption, although convenient, is inessential for our main results.\(^4\)

There is a large measure of households per search good firm. For convenience we normalize to 1 the measure of young households per firm at each period.\(^5\)

\(^3\) Of course, as in all models of overlapping generations, the assumption that households live for two periods is a modeling abstraction. If consumers lived for “many” periods, consumers who observed a high price at their first period might leave money in the hope of finding lower prices in successive periods. Thus the profitability of low prices, relative to that of high prices, would be higher than in the two-period world. Nevertheless, it would still be profitable for sufficiently high-cost firms to charge high prices and forego repeat sales. Thus the essentials of Proposition 1 should continue to hold.

\(^4\) Even if firms could discriminate and offer lower prices to old repeat buyers, only low-cost firms would profitably do so. Firms with sufficiently high costs (near 1) could still not afford to offer repeat buyers a large enough discount for them to make repeat purchases. Hence Proposition 1 would be essentially unchanged.

\(^5\) More precisely, following Burdett and Judd (1983), consider a market with $n$ firms and $mn^2$ consumers and demand per consumer of $1/n$. As the number of consumers per firm, $mn$, goes to infinity, the variance of sales per firm goes to zero and the number of sales per firm becomes deterministic. Here we have normalized $m$ to 1.

Alternatively, the number of sales per firm may be considered deterministic by appealing to the Law of Large Numbers (Feller, 1968), for any arbitrarily small $\varepsilon > 0$ and any arbitrarily small $\delta > 0$, there is $N$ such that if $n > N$, then with probability $1 - \delta$ the number of customers arriving at any specific firm—and hence the number of units sold by that firm—lies between $\theta n - \varepsilon$ to $\theta n + \varepsilon$. Thus, for $n$ large enough, the probability that the number of sales per firm deviates from $\theta n$ by more than a preassigned $\varepsilon$ tends to zero.
Equilibrium. There are four markets: the labor market, the market for the numeraire good, the market for the search good, and the money market. Both the labor and numeraire good markets are perfectly competitive. We seek a stationary, symmetric, rational expectations equilibrium in which all nominal prices inflate at the same rate as the money supply, $\pi$. That is, an equilibrium in which all relative prices—denominated in units of the numeraire—are time invariant.

An equilibrium is given by $\{R^c, w, p(\lambda)\}$, where $R^c$ is the consumers’ decision strategy, defined above, $w$ is the relative wage, and $p(\lambda)$ is the (relative) price charged by a search good firm, as a function of its production cost, such that:

(E.1) Given $p(\lambda)$, $R^c$ is the optimal strategy for consumers.
(E.2) Given that all other firms charge $p(\lambda)$ and given $R^c$, $p(\lambda)$ is an individually profit maximizing pricing strategy.
(E.3) All markets clear.

3. EQUILIBRIUM ANALYSIS

Since the labor and numeraire markets are competitive, the constant returns to scale technology implies that $w = 1$.

PROPOSITION 1. (i) If $\pi < 1$, the unique equilibrium is characterized by two prices for the search good, $(1 + \pi)^{-1}$ and 1. A young consumer buys one unit of the search good at either period and at their first period return to buy a second unit of the search good from the same firm at their second period. Consumers who pay the high price, 1, at their first period only buy the search good at their first period and do not retain any money for their second period. Firms whose cost $\lambda > \lambda_\pi \equiv (1 - \pi)(1 + \pi)^{-1}$ charge the high price, 1, and firms for which $\lambda \leq \lambda_\pi$ charge the low price, $(1 + \pi)^{-1}$.

(ii) If $\pi \geq 1$, in the unique equilibrium the price of the search good is 1 at each firm, and all consumers buy only one unit.

PROOF. To show that the above is an equilibrium, it must be shown that conditions (E.1)–(E.3) are met. We prove only part (i) of the proposition. The proof of part (ii) follows directly from the analysis of part (i).

It is convenient, and involves no loss of generality, to consider an arbitrary period, call it period zero, in which the nominal price of the numeraire good is normalized to 1. That is, at period zero nominal and real (in terms of the numeraire) values are equivalent. In the rest of the proof, therefore, all prices and money balances are denoted in nominal terms unless explicitly stated otherwise.

(E.1). By the utility function (1), each unit of the numeraire increases utility by 1 and the consumption of one unit of the search good at either period increases utility by 1. Therefore, since the nominal price of the numeraire is 1, and consumers observe only one price of the search good per period, it follows that $pr_1 = 1$. 


Suppose that prices of the search good are indeed as specified in the proposition. Consider the optimal amount of money that a young consumer at period zero with income \( v \) retains for the following period.

Suppose she pays \( p_1 \leq (1 + \pi)^{-1} \) for the search good at her first period.\(^6\) Since she expects to be able to buy the search good from the same firm for the nominal price of \( p_1(1 + \pi) \) at the following period, \( m_2 > p_1(1 + \pi) \) cannot be optimal. And since, given the equilibrium price distribution in the market, she expects next period’s low and high nominal prices at all other firms to be 1 and \( 1 + \pi \), respectively, \( 0 < m_2 < p_1(1 + \pi) \leq 1 \) (which is insufficient to buy the search good from any seller at the following period) can also not be optimal. Thus for such a consumer, either \( m_2 = p_1(1 + \pi) \) or \( m_2 = 0 \). In the first instance, she buys a unit from the same firm each period, and \( v - p_1 - p_1(1 + \pi) \) units of numeraire at the first period, attaining a lifetime utility of \( 2 + v - p_1(2 + \pi) \). If \( m_2 = 0 \), she buys only one unit of the search good and attains a lifetime utility of \( 1 + v - p_1 \leq 2 + v - p_1(2 + \pi) \) for \( p_1 \leq (1 + \pi)^{-1} \). Thus, if \( p_1 \leq (1 + \pi)^{-1} \), \( m_2(p_1) \) is optimally \( p_1(1 + \pi) \).

Now consider a young consumer who pays \( p_1 > (1 + \pi)^{-1} \) at her first period. To buy from the same firm next period, she must retain \( m_2 = p_1(1 + \pi) > 1 \), which cannot be optimal since each unit of numeraire consumed at the first period and a second unit of the search good both increase utility by 1. Thus such a consumer either sets \( m_2 = 0 \) or plans to choose a different search good firm at her second period. In the first instance, her lifetime utility is \( v - p_1 + 1 \). In the second instance she sets \( m_2 = 1 \) (the lowest expected price next period), selects a new firm at random, and buys a second unit of the search good if and only if its price is not greater than 1. This strategy yields a utility of \( v - p_1 + 1 \) if the new firm’s price is 1, an event which occurs with probability \( \lambda_{\pi} \). If the new firm’s price is \( 1 + \pi \), which occurs with probability \( 1 - \lambda_{\pi} \), the retained money is spent on \((1 + \pi)^{-1}\) additional units of the numeraire, giving utility of \( v - p_1 + (1 + \pi)^{-1} < v - p_1 + 1 \). Thus if \( p_1 > (1 + \pi)^{-1} \), \( m_2(p_1) \) is optimally 0.

Now consider an old consumer at period zero (who entered the economy at period −1). Given that firms adhere to the equilibrium prices, the low and high nominal prices at the preceding period were \((1 + \pi)^{-2} \) and \((1 + \pi)^{-1} \), respectively. Therefore, by exactly the same reasoning as above, an old consumer who paid the low price at the preceding period optimally holds a money balance of \((1 + \pi)^{-1}\), which is therefore his reservation price, while old consumers who paid the high price at the preceding period are not in the market for the search good.

(E.2). Since both generations of consumers accept \((1 + \pi)^{-1}, (1 + \pi)^{-1}\) are more profitable than any lower price for any firm. Since \( pr_1 = 1 \) for all young consumers while \( pr_2 = (1 + \pi)^{-1} \) for all old consumers who paid the low price at

\(^6\) In deriving the equilibrium strategies, we formulate the consumers’ optimal response to any, not only equilibrium, prices, to verify that the proposed equilibrium prices are indeed more profitable for firms than any other prices. Our formulation implicitly assumes that a consumer who observes an out of equilibrium, nominal price \( p \) at some firm expects it to charge the nominal price \( p(1 + \pi) \) at the following period (i.e., the firm’s real price is expected to be unchanged at the following period, whether its observed price is an equilibrium price or not). Thus, since a young consumer who observes \( p_1 \leq (1 + \pi)^{-1} \) keeps \( p_1(1 + \pi) \) units of money for the next period, the equilibrium low price \((1 + \pi)^{-1}\) is more profitable for firms than any \( p_1 < (1 + \pi)^{-1} \).
their first period and zero otherwise, any price \( p, (1 + \pi)^{-1} < p < 1 \), is less profitable than 1. Therefore, the most profitable price for any firm is either \((1 + \pi)^{-1}\) or 1. Since a firm whose price is 1 sells only to young consumers, its profit is \(1 - \lambda\). Since a firm whose price is \((1 + \pi)^{-1}\) sells to both generations of consumers, its profit is \(2((1 + \pi)^{-1} - \lambda)\). Thus \((1 + \pi)^{-1}\) is more profitable than 1 iff \(2((1 + \pi)^{-1} - \lambda) > 1 - \lambda\), i.e., iff: \(\lambda < (1 - \pi)(1 + \pi)^{-1} = \pi\), while 1 is more profitable otherwise. 0 < \(\lambda\pi < (1 - \pi)(1 + \pi)^{-1}\) exists iff \((1 - \pi)(1 + \pi)^{-1} > 0\), i.e., if and only if \(\pi < 1\).

(E.3). It remains to show that under the proposed equilibrium, all markets clear. We proceed to show that all markets—the labor market, the product markets, and the money market—clear if and only if \(M = 2 + (1 + \pi)^{-2}\) and \(v = 2 + (1 + \pi)^{-1}\).

\[\text{Labor Market.} \quad \text{Given the proposed equilibrium, the average labor demand per unit of production of high-price and low-price search good firms is } (1 + \lambda\pi)/2 \text{ and } (\lambda\pi/2), \text{ respectively. Hence, and since there are } 1 - \lambda\pi \text{ firms of the first type, each producing 1 unit, and } \lambda\pi \text{ of the second type, each producing 2 units, the aggregate labor demand of the search good sector is } (1 - \lambda\pi^2)/2 + \lambda^2\pi. \]

Since each unit of labor produces 1 unit of the numeraire, the aggregate demand for workers in the numeraire sector equals the aggregate demand for the numeraire good. The \(1 - \lambda\pi\) consumers who pay the high price for the search good purchase only 1 unit of the search good. Hence their aggregate demand for the numeraire is \((1 - \lambda\pi)(v - 1)\). The \(\lambda\pi\) consumers who pay the low price each spend \(1 + (1 + \pi)^{-1}\) on the search good and hence demand \(v - 1 - (1 + \pi)^{-1}\) units of the numeraire. Hence, their total demand for the numeraire is \(\lambda\pi(v - 1 - (1 + \pi)^{-1})\). Thus, the aggregate demand for workers in the numeraire sector is \((1 - \lambda\pi)(v - 1) + \lambda\pi(v - 1 - (1 + \pi)^{-1})\).

To summarize, the demand for workers in both sectors is \((1 - \lambda\pi^2)/2 + \lambda^2\pi + (1 - \lambda\pi)(v - 1) + \lambda\pi(v - 1 - (1 + \pi)^{-1})\), which, after substituting \(v = 2 + (1 + \pi)^{-1}\) and \(\lambda\pi = (1 - \pi)(1 + \pi)^{-1}\), equals 2. Since the labor supply is inelastically equal to 2, the labor market clears.

\[\text{Product Market.} \quad \text{Every young consumer and } \lambda\pi \text{ old consumers demand 1 unit of the search good. Therefore, the aggregate demand for the search good is } 1 + \lambda\pi. \]

Every low-price firm produces 2 units and every high-price firm produces 1 unit. Hence, the aggregate supply of the search good is \(2\lambda\pi + (1 - \lambda\pi) = 1 + \lambda\pi\). Thus the search market clears.

Our preceding analysis of the labor market shows that the numeraire market clears. Hence both product markets clear.

\[\text{Money Market.} \quad \lambda\pi \text{ young consumers who pay the low price for the search good save 1 for their second period and spend the remainder of their income, } v - 1, \text{ at their first period. } 1 - \lambda\pi \text{ young consumers who pay the high price spend their entire income, } v, \text{ at the first period. Thus the money demand of all young consumers per period is } \lambda\pi \text{ and } \lambda\pi \text{ old consumers, who paid the low price at the preceding period, demand } (1 + \pi)^{-1} \text{ to buy a second unit of the search good while the } 1 - \lambda\pi \text{ old consumers who paid the high price at their} \]
first period do not purchase at the second period. Thus the money demand of all old consumers is \( \lambda_\pi (1 + \pi)^{-1} \). The total amount of money demanded by both generations per period is thus \( v + \lambda_\pi (1 + \pi)^{-1} \). Substituting \( v = 2 + (1 + \pi)^{-1} \) in the preceding expression gives aggregate money demand as \( 2 + 2(1 + \pi)^{-2} \). Thus the money market is in equilibrium if \( M = 2 + 2(1 + \pi)^{-2} \). This proves that all markets clear if and only if \( M = 2 + 2(1 + \pi)^{-2} \) and \( v = 2 + (1 + \pi)^{-1} \).

The preceding has shown that if the price of the numeraire good (and, equivalently the wage) is normalized to 1, the money market clears if \( M = 2 + 2(1 + \pi)^{-2} \). Conversely, normalize monetary units so that \( M = 2 + 2(1 + \pi)^{-2} \). Then it follows that the money market is in equilibrium if the price of the numeraire is 1 and the prices of the search good are as stated in the proposition.

This proves existence of the two-price equilibrium given by part (i) of the proposition.\(^7\) The proof of uniqueness is left to Appendix 1.

If \( \pi < 1 \), the equilibrium is characterized by two prices. Low-cost firms (with \( \lambda < \lambda_\pi \)) sell two units of the search good to each customer. Those firms generate repeat sales by charging prices that are low enough to compensate consumers for the cost of retaining money to the following period. High-cost firms (with \( \lambda > \lambda_\pi \)) sell to young consumers only. Those firms extract the highest possible price from young consumers, but by so doing forego repeat sales.

The higher the rate of inflation, the deeper the “discount” that consumers require to retain money and hence the more costly it is for firms to generate repeat sales. Therefore, as evidenced by the fact that \( \lambda_\pi \) is decreasing in \( \pi \), the higher the rate of inflation, the smaller the number of low-priced firms that produce two units and the fewer consumers which succeed in buying from such firms. This feature captures the frequently expressed notion that inflation affects real prices by reducing the extent of consumer search activity. If \( \pi > 1 \), the level at which the low price must be set to generate repeat sales is so low that all firms charge the high price and sell only to young consumers.

It is worthwhile to pause for a moment to appreciate the role of incomplete information in the search markets for these results. Suppose consumers were perfectly informed about prices in the search market. Then only the most efficient firms, i.e., those with the lowest \( \lambda \), say \( \lambda' \), could be operative and would sell two units to each consumer at the price \( \lambda' \), as long as \( (1 + \pi)\lambda' \leq 1 \), i.e., \( \lambda' \leq (1 + \pi)^{-1} \). Thus inflation would have no effects on production or prices for \( \pi \leq (1 - \lambda')/\lambda' \).

Only if the inflation rate rose above this level would there be a discontinuous shift, reducing each consumer’s consumption of the search good to only one unit and inducing a shift of labor away from the search good sector toward the numeraire sector.

\(^7\) It is useful to verify that a household’s income from firms’ profits and wages adds up to \( v \). The average profit of the low-priced firms is \( 2[(1 + \pi)^{-1} - \lambda_\pi/2] \) and therefore their aggregate profits are \( 2\lambda_\pi[(1 + \pi)^{-1} - \lambda_\pi/2] \). The average profit of the high-priced firms is \( 1 - 1 + \lambda_\pi/2 \), and therefore their aggregate profits are \((1 - \lambda_\pi)(1 - 1 + \lambda_\pi/2) \). Hence, the aggregate profits in the search sector (and in the economy) are \( 2\lambda_\pi[(1 + \pi)^{-1} - \lambda_\pi/2] + (1 - \lambda_\pi)(1 - 1 + \lambda_\pi/2) = \frac{1}{2}(1 + \lambda_\pi) \).

Since \( w = 1 \) and every household has 2 units of labor, it follows that the aggregate (and average) income is \( 2 + \frac{1}{2}(1 + \lambda_\pi) = 2 + \frac{1}{2} \frac{1 + \pi + 1 - \pi}{1 + \pi} = 2 + (1 + \pi)^{-1} = v \).
sector (without any change in the distribution of production between firms). Thus incomplete information is crucial for our analysis.

4. INFLATION, EFFICIENCY, AND WELFARE

The preceding section shows that inflation has nontrivial effects on total production, real prices, and the distribution of production between firms in the search good sector. The following proposition describes the impact of these effects on welfare. Since in our model all firms are owned by households, and there is no disutility of labor, aggregate welfare is simply the expected utility of a representative household.

Proposition 2. If \( \pi < 1 \), higher inflation reduces expected welfare.

Proof. In equilibrium, \( \lambda_\pi \) households consume two units of the search good and 1 unit of numeraire, obtaining utility of \( 3 - \lambda_\pi \) households consume one unit of the search good and \( 1 + (1 + \pi)^{-1} \) units of the numeraire good, obtaining utility of \( 2 + (1 + \pi)^{-1} \). Thus a household’s expected utility (before entering the market) is \( W_\pi = 3 - \lambda_\pi + (1 - \lambda_\pi)(2 + (1 + \pi)^{-1}) \). Substituting for \( \lambda_\pi \) and differentiating with respect to \( \pi \) reveals that \( W_\pi \) is decreasing in \( \pi \).

Since agents have no disutility of labor and each unit of the numeraire gives the same utility as each unit of the search good, welfare can be measured by aggregate output. In the numeraire sector, a unit of labor produces one unit of output. In the search good sector, a unit of labor employed by a search good firm with cost \( \lambda < 1 \) produces \( 1/\lambda > 1 \) units. Hence, given that a unit of the search good provides the same utility as a unit of the numeraire, per capita utility can be increased by diverting labor from the numeraire sector to the search good sector as long as total production of the search good is less than 2 (2 units for each household). In equilibrium, however, \( 1 - \lambda_\pi > 0 \) firms produce only one unit of the search good at any positive rate of inflation. Moreover, the higher the rate of inflation, the fewer the number of search good firms which produce two units. Thus, one channel through which inflation reduces welfare in our model is the inefficient diversion of labor from the search good sector to the numeraire sector, which reduces total production of the search good.

A second, more subtle channel arises from the effect of inflation on the distribution of output between firms of varying efficiency in the search good sector. Specifically, when output of the search good is reduced, it is precisely the marginally efficient firms—who would otherwise produce two units—which cut back production, while the output of the least efficient firms—which in either case produce only one unit—remains unchanged. Thus the inefficient firms’ relative share of total production increases at the expense of the more efficient firms.

This effect is reflected by the impact of inflation on firms’ profits. Surprisingly, in our model, inflation reduces the relative profitability only of the more efficient firms. While higher inflation decreases the price of the most efficient firms, which produce two units, the price—and hence the profit—of the least efficient firms,
which produce only one unit, remains unchanged. Conversely, a lower rate of inflation enhances the profitability of efficient firms and lowers the relative profitability of the inefficient ones. Under our linear utility function, (1), this effect on firms’ relative profitability never leads to the exit or entry of inefficient firms (firms with $\lambda < 1$ are operative at any inflation rate, while firms with $\lambda > 1$ are inoperative at any inflation rate). More generally, however, in our model, an increase in the rate of inflation can induce the entry of firms that are otherwise too inefficient to survive, an observation supported empirically by the surprisingly large incidence of bankruptcies following inflation stabilizations (Garber, 1982; Bruno and Piterman, 1988, p. 35).

Consider, for example, the utility function: $U = k_1 + k_2 + \min\{c_1, c_2\}$. It is shown in Appendix 3 that the equilibrium corresponding to this utility function has essentially the same features to those described by Proposition 1. Specifically, if $\sqrt{2} - 1 < \pi < 1$, there are two prices, $p_H = 2 + \pi$ and $p_L = (2 + \pi)/(1 + \pi)$, such that consumers who pay $p_H$ at their first period only buy the search good at their first period, consumers who pay $p_L$ at their first period buy the search good at both periods, firms with $\lambda > 2/(1 + \pi) - \pi$ charge $p_H$ and sell one unit each period, and firms with $\lambda < 2/(1 + \pi) - \pi$ charge $p_L$ and sell two units each period. And as above, an increase in the rate of inflation reduces the number of firms which charge the low price, total production of the search good and, therefore, welfare. In contrast to the linear case, however, here higher inflation increases $p_H$, thereby increasing the real profits of high-priced firms. Therefore, a decrease in the rate of inflation, say from $\pi’$ to $\pi’’ < \pi’$, forces the exit of the most inefficient firms (with $2 + \pi’ > \lambda > 2 + \pi’’$). Conversely, a higher inflation rate induces the entry of firms which are inoperative at lower rates.

5. INFLATION AND PRICE VARIABILITY

The effect of inflation on relative prices and price dispersion has been the focus of intensive research. Empirically, inflation has been found to affect relative prices both across sectors (e.g., Pagan et al., 1983; Parks, 1978; Taylor, 1981; Vining and Elwertoowski, 1976) and across different firms within the same sector (Lach and Tsiddon, 1991; Tommasi, 1993; Van Hoomissen, 1988). Accordingly, this section examines the effect of inflation on equilibrium relative prices in our model.

Proposition 1 established that an increase in inflation reduces the low price without changing the high price. Thus inflation determines real price dispersion in the search good market. Changes to the distribution of prices within the search good sector also affect relative prices between sectors. More specifically:

**Proposition 3.**

(i) An increase in the rate of inflation increases the coefficient of variation of the search good’s price if $\pi < 0.58$ and decreases it if $1 > \pi > 0.58$.\(^8\)

\(^8\) Lach and Tsiddon report that the empirical evidence is consistent with the hypothesis that inflation increases price dispersion but is not strong enough to be decisive. Benabou (1988, 1992) also finds
(ii) An increase in the rate of inflation decreases (increases) the relative average price of the search good with respect to the numeraire good if $\pi < 1/3$ ($\pi > 1/3$).

PROOF. See Appendix 2.

Thus the effects of inflation on relative quantities, described in the previous section, are reflected by corresponding changes in equilibrium relative prices. Our model thus formalizes the perception, as expressed by Fischer (1981), that inflation subverts the allocative efficiency of the price system.

6. DISCUSSION OF THE ASSUMPTIONS

Since the utility function (1) is highly stylized, it is important to clarify which of its features are crucial for our results. In particular, (1) assumes that the search good is consumed in indivisible units, but the numeraire is consumed in divisible quantities. We argue informally that our main results should continue to apply if both goods are consumed in divisible quantities. As has been seen, in our model, inflation, by increasing the firms’ cost of generating repeat business, erodes the efficient firms’ relative advantage and profitability, reducing overall production of the search good. If the search good, like the numeraire, were consumed in continuous quantities, even consumers who pay high prices would retain some money for the second period but still less than those who pay low prices. Therefore it would still be true that an increase in the rate of inflation, by increasing the consumers’ propensity to shift consumption toward the present, would decrease the incentive to charge low prices, blurring the differences between more and less efficient firms and reducing total production of the search good. Thus the main features of our equilibrium should continue to obtain when both goods are continuously divisible.

(1) also imposes a second asymmetry between the goods—the search good is perfectly intertemporally insubstitutable (the additional utility from more than one unit at the first period is zero) whereas the numeraire good exhibits perfect intertemporal substitutability (lifetime utility is unaffected by the allocation of its consumption between periods). However, our above discussion, in Section 3, of the utility function $U = k_1 + k_2 + \min\{c_1, c_2\}$ (which exhibits imperfect intertemporal substitution of the numeraire good) shows that this feature may also be relaxed.9

The feature that appears most crucial for our results is that the search good be unstorable, which, after all, is the reason that repeat sales are important in our model. If the search good could be stored costlessly, consumers who find the low price at the first period would buy enough for both periods at their first period and inflation would have no real effects.

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9 Similarly, the main features of the model apply when the utility function is $U = k_1 + k_2 + \ln c_1 + \ln c_2$. That inflation increases price dispersion. In Benabou (1988) this unambiguously increases welfare. In Benabou (1992), the effect on welfare is ambiguous.
7. CONCLUDING REMARKS

There is a gap between the perception that inflation is a major economic problem, and its representation in conventional economic theory. Recent research seeks to narrow this gap by focusing on the microeconomics of trade and the details of market organization. This body of research may be divided into two strands. In the first, which includes this article and the related papers of Casella and Feinstein (1990) and Tommasi (1999), rapid inflation disrupts market organization by inducing individuals holding depreciating nominal money to speed up purchases. Casella studies the welfare consequences of this effect, via its effect on real prices, in search markets with homogenous firms. Tomassi (1999), and this article, focus on the consequences of this effect for production efficiency. In all these models, the inability to shield liquidity from the inflation tax constitutes a rigidity on the demand side of the market. Relative quantities and prices would be unaffected by inflation if liquid assets could be costlessly indexed to prices.

The other strand of the literature emphasizes the effects of inflation on real prices when nominal price changes involve real costs on the part of sellers (Sheshinski and Weiss, 1973; Benabou, 1988, 1992; Fishman, 1992; Diamond, 1993; Ball and Romer, 1993). In those models, inflation imposes a rigidity on the supply side. In those models inflation would be neutral if the rigidity were removed, for example, if prices could be quoted in real terms (i.e., by being indexed to the CPI).

Undoubtedly, in reality both demand side and supply side rigidities are important. Our view is, therefore, that the two modeling approaches should be seen as complementary. Broadly, both point to the role of specific institutional features of markets in gaining a better understanding of inflation’s effects.

APPENDIX 1

PROOF OF UNIQUENESS. We prove that the equilibrium of Proposition 1 is unique in a series of steps.

Step 1. If \( \pi < 1 \), in any equilibrium, there are at least two prices for the search good. There is a positive measure of firms whose price is 1 and a positive measure of firms whose price is less than or equal to \( (1 + \pi)^{-1} \).

PROOF. By our assumptions, \( v \geq 2 \). Given the utility function (1), it is never optimal to retain \( m_2 > 1 \) for the second period. Therefore, buying a unit at any price \( p \leq 1 \) does not restrict a young consumer’s ability to buy a unit of the search good at her next period. Since only one price of the search good can be observed at each period, it follows immediately from the utility function (1) that \( pr_1 = 1 \). Thus the highest price in the market cannot be greater than 1.

Suppose that the highest price of any firm is \( p < 1 \). Then firms with production cost \( \lambda' < p < \lambda' < 1 \) are inoperative. But such firms could earn positive profit by charging a price \( p' < \lambda' < p' < 1 \). Thus the highest price in the market cannot be less than 1.

Since the utility function (1) implies that no consumer will retain more than 1 to buy the search good at her second period, firms whose price is \( p, (1 + \pi)^{-1} < p < 1 \), sell only to young consumers. Since young consumers accept any price less than
1, such firms could increase profits by raising prices. Thus no firms charge \( p, (1 + \pi)^{-1} < p < 1 \). This proves that there must be a positive measure of firms whose price is 1.

Suppose that the price of each firm is 1. Then, each firm sells only one unit and earns a profit of \( 1 - \lambda \). Consider \( p' < (1 + \pi)^{-1} \). By (1), if \( p' \) is the lowest price in the market, young consumers who observe a firm with price \( p' \) optimally set \( m_2(p') = p'(1 + \pi) \) and return to buy a second unit from that firm at their second period. Thus, given that the price of all other firms is greater than \( p' \), the profit from \( p' \) is \( 2(p' - \lambda) > 1 - \lambda \) for \( \lambda < 2p' - 1 \). Since \( 0 < \lambda < 2p' - 1 \) exists if \( \pi < 1 \), there are firms for whom \( p' \) is more profitable than 1. Thus in any equilibrium there must be firms whose price is less than or equal to \( (1 + \pi)^{-1} \). This completes the proof.

Step 2. In any equilibrium, there are at most three different prices.

Proof. We show that a four-price equilibrium cannot exist. The proof for more than four prices is analogous. Suppose, a contrario, that there are four prices, \( 1 > p_2 > p_3 > p_4 \), where, by step 1, \( p_2 \leq (1 + \pi)^{-1} \). Let us refer to a consumer who retains money to her second period and chooses a new search good firm at her second period, as a “searching” consumer. Since all searching consumers face the same second period price distribution, regardless of which price they observed at their first period, \( m_2 \) is identical for all searching consumers. Since \( p_4 \) is the lowest price in the market, searching consumers must retain at least \( p_4(1 + \pi) \).

If no consumers search or if searching consumers retain more than \( p_4(1 + \pi) \), firms whose price is \( p_4 \) could increase prices slightly without losing any sales and increase profit. Thus searching consumers retain exactly \( p_4(1 + \pi) \). If young consumers who observe \( p_2 \) search, or if \( m_2(p_2) = 0 \), firms whose price is \( p_2 \) sell to young consumers only and could increase profit by charging \( 1 \). If young consumers who pay \( p_2 \) buy a second unit without search, firms that charge \( p_2 \) and \( p_3 \) sell the same number of units, implying that \( p_2 \) is more profitable than \( p_3 \) for each firm. This completes the proof.

Step 3. A three-price equilibrium does not exist.

Suppose, a contrario, that there are three prices, \( 1 > p_2 > p_3 \), and let \( \sigma , \theta - \sigma \) and \( 1 - \theta \) be the proportions of firms that charge \( p_3, p_2 \), and 1 respectively; i.e., firms whose cost is less than \( \sigma \) charge \( p_3 \), firms whose cost is between \( \sigma \) and \( \theta \) charge \( p_2 \), and firms whose cost is greater than \( \theta \) charge 1.

Suppose that \( m_2(p_2) < p_2(1 + \pi) \). Then firms that charge \( p_2 \) sell only to young, first-time buyers, in which case they could increase profit by charging 1 (since young consumers accept any price not exceeding 1). Thus \( m_2(p_2) = p_2(1 + \pi) \). Suppose that \( m_2(1) < p_3(1 + \pi) \). Then the only old consumers who patronize \( p_3 \) firms are repeat buyers (who were their clients at their first period). But, since \( p_3 \) is the lowest price in the market, those customers would also return for a repeat purchase at some slightly higher price \( p' > p_3 \), implying that \( p_3 \) cannot be an optimal price for any firm. Hence, \( m_2(1) \geq p_3(1 + \pi) \). If \( m_2(1) = p_2(1 + \pi) \), \( p_2 \) and \( p_3 \) firms sell the same number of units, implying that \( p_2 \) is more profitable.
than $p_3$ for every firm. Thus $m_2(1) = p_3(1 + \pi)$, i.e., consumers who observe the price 1 at their first period retain exactly $p_3(1 + \pi)$ and search for a $p_3$ firm at their second period. Hence, in a three price equilibrium, firms who charge 1 sell one unit each period, firms who charge $p_2$ sell 2 units each period, and firms who charge $p_3$ sell $3 - \theta$ units each period (one unit to first-time customers, one unit to repeat buyers, and $1 - \theta$ units to searching consumers who paid 1 at their first period). Firms with cost $\sigma$ must be indifferent between $p_3$ and $p_2$ and firms with cost $\theta$ must be indifferent between $p_2$ and 1. That is

\begin{equation}
(3 - \theta)(p_3 - \sigma) = 2(p_2 - \sigma)
\end{equation}

\begin{equation}
2(p_2 - \theta) = 1 - \theta
\end{equation}

Solving (A.2) for $\theta$ and substituting into (A.1) gives

\begin{equation}
p_3 = \frac{\sigma + (1 - \sigma)p_2}{2 - p_2}
\end{equation}

The fact that $m_2(1) = (1 + \pi)p_3$ implies that $1 + v - 1 - (1 + \pi)p_3 + \sigma + (1 - \sigma)p_3 \geq 1 + v - 1$, where the LHS of the preceding inequality is the expected utility of a consumer who pays 1 at the first period and searches at the second period and the RHS is her utility from spending all her income at the first period. Rearranging yields

\begin{equation}
\sigma/(\pi + \sigma) - p_3 \geq 0
\end{equation}

The fact that $m_2(p_2) = p_2(1 + \pi)$ implies that $(1 + \pi)^{-1} \geq p_2$. Suppose $(1 + \pi)^{-1} > p_2$. Then it must be the case that a consumer who observes $p_2$ at her first period obtains the same expected utility from returning to make a repeat purchase at $p_2$ as from searching for a $p_3$ firm. Otherwise, if she strictly preferred the former option, $p_2$ firms could slightly raise prices without losing customers. Thus, if $p_2 < (1 + \pi)^{-1},$

\[ 2 + v - (2 + \pi)p_2 = 1 + v - p_2 - (1 + \pi)p_3 + \sigma + (1 - \sigma)p_3 \]

where the RHS of the preceding equation is her expected utility from searching for a $p_3$ firm and the LHS is the expected utility from making a repeat purchase at $p_2$.\footnote{A third possibility is that those consumers, being indifferent between the two options, randomize between searching for a $p_3$ firm and returning to buy from the $p_2$ firm. However, this cannot be the case in equilibrium. Since consumers are indifferent between search and buying a second unit at $p_2$, a $p_2$ firm could profit by slightly lowering its price below $p_2$. This would make buying a second unit at that price strictly dominate searching for $p_3$ firm, and guarantee the repeated patronage of all its customers. Thus if consumers are indifferent between the two options, it must be the case that in equilibrium consumers follow a pure strategy and return to buy a second unit at $p_2$ with probability 1.} Simplifying the preceding equation gives

\begin{equation}
1 - (1 + \pi)p_2 = \sigma - (\pi + \sigma)p_3
\end{equation}
To summarize, either \( p_2 < (1 + \pi)^{-1} \) and (A.5) holds, or \( p_2 = (1 + \pi)^{-1} \). We consider each of these possibilities in turn and show that each leads to a contradiction.

**Possibility (i).** \( p_2 = (1 + \pi)^{-1} \).

Substituting \( p_2 = (1 + \pi)^{-1} \) into Equation (A.3) and rearranging yields

\[
\frac{1 + \pi \sigma}{1 + 2 \pi} = p_3 \quad \text{(A.6)}
\]

However, substituting the preceding expression in the LHS of (A.4) gives

\[
\frac{\sigma}{\pi + \sigma} - \frac{1 + \pi \sigma}{1 + 2 \pi} = \frac{-\pi [\sigma^2 - (2 - \pi)\sigma + 1]}{(1 + 2 \pi)(\pi + \sigma)} < 0
\]

for any \( 0 < \pi < 1 \), violating condition (A.4).

**Possibility (ii).** \( p_2 < (1 + \pi)^{-1} \).

Substituting \( p_3 \) from (A.3) into (A.5) and rearranging gives the condition:

\[
\sigma^2 - (2 - \pi)\sigma - \pi p_2 + 2 - p_2 = 0
\]

which has no real roots for any \( p_2 < (1 + \pi)^{-1} \). Thus a three-price equilibrium does not exist.

The preceding has established that if \( \pi < 1 \), there are two prices in equilibrium, the high price is 1 and that the low price is \( p_L \leq (1 + \pi)^{-1} \). It remains to show that \( p_L = (1 + \pi)^{-1} \) in any two price equilibrium.

Let \( \sigma \) denote the proportion of low-priced firms and suppose a contrario that \( p_L < (1 + \pi)^{-1} \). Suppose that \( m_2(1) = 0 \), i.e., consumers who encounter the price 1 at their first period only buy the search good at their first period (and do not search for \( p_L \) at their second period). Then \( p_L \) firms sell only to young and repeat buyers. But since \( p_L \) is the lowest price in the market, those firms would not lose any customers by charging slightly more than \( p_L \). Thus \( m_2(1) = p_L(1 + \pi) \) and firms whose price is 1 sell one unit per period while low-priced firms sell \((3 - \sigma)\) units—one unit to young consumers, one to old repeat buyers, and \(1 - \sigma\) units to old searching consumers who paid 1 when young. In equilibrium, firms with cost \( \sigma \) must earn identical profit from the two prices, i.e.,

\[
(3 - \sigma)(p_L - \sigma) = 1 - \sigma
\]

or

\[
p_L = \sigma + \frac{1 - \sigma}{3 - \sigma}
\]

Consider a young consumer who encounters a firm with the price \( p \), \( p_L < p < (1 + \pi)^{-1} \). This consumer prefers to retain \( p(1 + \pi) \) and make a repeat purchase from that firm at her second period, rather than search for \( p_L \), if

\[
2 + v - p - (1 + \pi)p \geq 1 + v - p - (1 + \pi)p_L + \sigma + (1 - \sigma)p_L
\]

(the LHS of the preceding expression is the utility from making a repeat purchase at \( p \), the RHS the expected utility from searching for a \( p_L \) firm). Let \( p_2 \) solve the
preceding inequality with equality, i.e.,

(A.8) \[ p_2 = \frac{1 - \sigma + (\pi + \sigma)p_L}{1 + \pi} \]

Thus, by charging \( p_2 \), a firm with cost \( \sigma \) sells two units and earns a profit of \( 2(p_2 - \sigma) \). Let \( \Delta \) denote the difference between its profit from \( p_2 \) and its profit from \( p_L \). Then,

\[ \Delta = 2(p_2 - \sigma) - (3 - \sigma)(p_L - \sigma) \]

Substituting from (A.7) and (A.8) yields,

\[ \Delta = \frac{2[1 - \sigma + (\pi + \sigma)p_L]}{1 + \pi} - \sigma - 1 \]

Given \( p_L \), \( \Delta \) attains its minimum at \( \pi = 1 \). Therefore, to show that \( \Delta \) is positive for every \( \pi \), it is enough to show it for \( \pi = 1 \). Substituting \( \pi = 1 \), \( p_L \) from (A.7), and rearranging yields

\[ \Delta|_{\pi=1} = \frac{1 - \sigma^2}{3 - \sigma} + \sigma^2 - \sigma \geq 0 \quad \forall 0 < \sigma \leq 1 \quad \text{with strict inequality for } \sigma < 1. \]

Thus a deviation to \( p_2 \) is profitable for firms with cost \( \sigma \), and by continuity, for firms with costs sufficiently near \( \sigma \). Thus a two price equilibrium with \( p_L < (1 + \pi)^{-1} \) does not exist.

This completes the proof of Proposition 1. \[ \blacksquare \]

**APPENDIX 2**

**PROOF OF PROPOSITION 3.**

(i) Since the proportion \((1 - \pi)(1 + \pi)^{-1}\) of firms charges \((1 + \pi)^{-1}\) and the remaining firms charge 1, the average price of the search good is

(A.9) \[ A(\pi) = (1 - \pi)(1 + \pi)^{-2} + 1 - (1 - \pi)(1 + \pi)^{-1} \]

The variance of the price distribution is

(A.10) \[ \text{var}(\pi) = (1 - \pi)(1 + \pi)^{-1}((1 + \pi)^{-1} - A(\pi))^2 + (1 - (1 - \pi)(1 + \pi)^{-1})(1 - A(\pi))^2 \]

Routine calculation of the coefficient of variation shows it to be increasing (decreasing) in \( \pi \) for \( \pi < 0.58 \) (1 > \( \pi > 0.58 \)).

(ii) Analysis of Equation (A.9) reveals that \( A(\pi) \) is decreasing (increasing) for \( \pi < 1/3 \) (\( \pi > 1/3 \)). \[ \blacksquare \]
INFLATION AND EFFICIENCY IN A SEARCH ECONOMY

APPENDIX 3

First note that if $x$ is the amount of money allocated for the search good, $v - x$ is optimally used to buy equal amounts of the numeraire in the two periods, at prices 1 and $1 + \pi$ respectively. Thus, since to purchase a unit of the numeraire at both periods costs $2 + \pi$, $\min\{c_1, c_2\} = c_1 = c_2 = (v - x)/(2 + \pi)$.

Suppose that the prices of the search good are as specified in the claim and consider a consumer who encounters $p_H$ when young. Buying a unit of the search good costs $2 + \pi$ units of money and yields 1 unit of utility. Alternatively, $p_H$ could be used to purchase an additional unit of numeraire at each period, which also yields 1 unit of utility. Thus, it is optimal for a young consumer to buy a unit of the search good if the price is $p_H$ and not to buy a unit if the price is greater than $p_H$.

Next, we show that it is optimal for a consumer who encounters $p_H$ at her first period not to buy a unit of the search good at her second period. Her utility from buying a second unit cannot be greater than if she were certain of finding a low-priced firm at her second period. In that case her utility from purchasing the search good at both periods is $2 + (v - p_H - (1 + \pi) p_L)/(2 + \pi) = v/(2 + \pi)$. But her utility from buying the search good only at her first period is also $1 + (v - p_H)/(2 + \pi) = v/(2 + \pi)$. Thus it is optimal for her not to buy a second unit.

Consider a young consumer who encounters $p_L$. Since $p_L < p_H$, it is optimal for her to buy a unit of the search good at her first period. Buying a unit of the search good at each period yields the utility

$$2 + (v - (2 + \pi) p_L)/(2 + \pi) = 1 + v/(2 + \pi) - (1 + \pi)^{-1}$$

Buying the search good only at the first period also yields

$$1 + (v - p_L)/(2 + \pi) = 1 + v/(2 + \pi) - (1 + \pi)^{-1}$$

Thus the behavior of consumers who observe $p_L$ is optimal. This establishes the optimality of consumer behavior.

Given the consumers’ behavior, it is straightforward to check that firms with cost greater than $\theta$ earn greater profit by charging $p_H$ and selling one unit than by charging $p_L$, and firms with cost less than $\theta$ earn greater profit by charging $p_L$ and selling two units.

Finally, since the number of firms that sell two units decreases with inflation, the argument that inflation reduces welfare is exactly as in Proposition 1.

REFERENCES


