Survival of Small Firms: Guerrilla Warfare

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Duopolistic interaction between a small firm and a large established firm is considered and compared to guerrilla warfare. The paper investigates a "hit and run" equilibrium in which the small firm enters the market, stays there for several periods, exits, stays out for several periods, and then reenters. Occasionally there may be a price war (or retaliation), but the small firm may also exit voluntarily, thereby avoiding possible confrontation. The amount of time that the small firm stays in the market and the timing of the price wars do not follow any predictable pattern, which is part of the mixed strategies that both firms play in equilibrium.

1. Introduction

When a small firm coexists in a duopolistic market with a large firm, the strategies used by the small firm can resemble guerrilla warfare. According to Huntington (1962), "Guerrilla warfare is a form of warfare by which the strategically weaker side assumes the tactical offensive in selected forms, time and places." This paper applies game-theory tools to understanding the military strategy issue of surprise and guerrilla warfare. Specifically, the paper considers a duopolistic market in which a large firm is capable of forcing a small firm out of the market. The small firm, however, would like to enter the market and remain there as many periods as possible, but at the same time, it tries to avoid any confrontation with the large firm. The coexistence of these different types of firms in the market creates a special type of duopoly that can give rise to an interesting nonstationary pattern of equilibrium prices.

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The element of *surprise* is an important part of guerrilla warfare.¹ Sun Tzu, the famous military philosopher, wrote 2400 years ago that speed, surprise, and deception were the primary essentials of an attack. Surprise may also be an important element in the survival of small firms. An entrant, in order to avoid (or delay) an aggressive response by an established firm, would like to achieve surprise with its decisions to enter and to stay in the market, while an established firm would like to surprise the entrant with its retaliation plans. A surprise can take the form of timing, location, characteristics of the good, or the action itself.

In this paper I adopt a very limited interpretation of surprise. In equilibrium, each firm can use mixed strategies without having to adopt a particular pattern of behavior that is predictable.² Guerrilla warfare-style competition intrinsically gives rise to randomized strategies.

I distinguish between the large incumbent and the small firm in the following manner. First, the established firm can retaliate and force the small firm out of the market whenever there is entry. Second, the established firm faces a delay in detecting entry or in responding to it. The assumption that the established firm can force out the small firm represents some competitive advantage of the large firm such as lower unit costs, superior market information, better technological know-how, or greater brand recognition. These factors often have been associated with large firms. The assumption that the incumbent is slow to detect or respond to entry represents bureaucratic inertia or organizational inefficiencies that are often characteristic of large companies. I do not discuss the historical factors leading to an asymmetric duopoly market structure. The model is a short-run analysis of market interaction between two types of firms rather than a long-run analysis of their different market positions.

The paper investigates a “hit-and-run equilibrium” in which the small firm enters the market, stays there for a few periods, exits, stays out for a few periods, and then reenters. Occasionally, there can be a price war (or some other type of retaliation by the established firm), but the small firm can also exit voluntarily, avoiding confrontation with the established firm. The number of periods that the small firm

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1. “Uproar in the east, strike in the west” (Sun-Tzu, 1963); “Hit and run, wait, lie in ambush, again hit and run and thus repeatedly, without giving any rest to the enemy” (Guevara, 1961, p. 20); “Avoid the solid, attack the hollow” (Mao, 1961, p. 46).
2. For an early analysis of mixed strategy equilibrium in oligopoly see Shilony (1977).
stays in the market does not follow any predictable pattern, thus creating an element of surprise.

The effect of market interaction on price formation is a central topic of research in industrial-organization economics. Guerrilla interaction, characterized by random hit-and-run strategies, gives rise to an interesting price pattern. Although the model is characterized by complete information and a stationary environment, it predicts a non-stationary price pattern in equilibrium. The industry goes through phases of monopoly pricing, duopoly pricing, and occasional price wars. The length of each phase is determined randomly. The standard explanation for the occurrence of price wars is Green and Porter’s (1984) analysis of collusive oligopoly with imperfect monitoring. They demonstrate that price wars do not necessarily indicate a failure of collusive agreements but may be a necessary part of those agreements. In contrast, the structure of the hit-and-run equilibrium, investigated in this paper, gives rise to periodic price wars as part of the large firm’s strategy of preventing the small firm from establishing itself in the market.

Examples of asymmetric structure can be found in many industries. One may think about a clothing manufacturer that produces a full line of apparel and competes with a smaller clothing designer that offers only one kind of jacket or suit in any given year. Similarly, a large publishing company offers a full line of books and faces a challenge from a smaller publishing house that markets books in only one category in any given year.

The guerrilla-warfare model differs from standard entry models in the industrial organization (IO) literature. Entry models consider the possible asymmetries between an incumbent and a potential entrant and the implication of these asymmetries for the entry decision. Entry, by definition, is a one-shot action. Once a firm enters, it becomes an insider and thus participates in the market game like all the other incumbent players. The discussion in the IO literature focuses on two issues: (1) the strategic implications of having a first-mover, i.e., the incumbent, who is able to commit to a particular strategy, and (2) the possible asymmetries that the incumbent may create, such as cost or capital asymmetries. The main focus of the guerrilla-warfare model is not on the entry act itself but on the survival of a small firm that cannot afford a direct confrontation with a big firm. The present

3. Abreu et al. (1985) generalize this analysis by considering a general strategy space and investigating the optimal cartel agreement.
4. See Carlton and Perloff (1990), Gilbert (1989), and Tirole (1989) for a detailed discussion.
model takes as given the asymmetry between the two types of firms and concentrates on the possible patterns of interaction. The model does not view entry as a one-shot event. Repeated entry and exit is the only equilibrium pattern of behavior. Moreover this pattern of interaction may persist for many periods.

The concept of hit-and-run entry has been used already in the contestable-markets literature. In a setting with no sunk cost, a price vector is sustainable if no firm can use a hit-and-run strategy and gain by entering the market for several periods and cutting the incumbent’s price. The possibility of such an action is a constraint, in the contestable-market literature, on the possible prices a firm can charge. In the guerrilla-warfare literature, the hit-and-run outcome is the result of the strategic interaction between the two types of firms.

2. The Model

Consider an industry with \( n \) market segments divided by geography or by differentiated products. There are two firms in the industry, a big established firm that operates in all the market segments and a small firm that has the ability (or capacity) to serve only one market segment at a time. The big firm’s monopolistic profit in each of the submarkets equals \( \pi_b^n \). Following entry into one of the market segments, the big firm has two options: either to accommodate the entrant or to retaliate by an aggressive response. I assume that the big firm enjoys a marketing advantage due to its size, so that by retaliating it can force the small firm out of the market. For example, the big firm can exert pressure on distribution channels by pressuring retailers not to carry the entrant’s product. In addition, I assume that the retaliation policy causes the small firm to incur losses during the retaliation period.

Let \( \pi_b^a > 0 \) and \( \pi_s^a > 0 \) be the per-period profits of the big and the small firm respectively, when the big firm accommodates the small firm, and let \( \pi_b^r \) and \( \pi_s^r < 0 \) be their respective profits when the big firm retaliates. The small firm’s profit from staying out of the market is normalized to zero. Further, assume that \( \pi_b^r > \pi_b^a > \pi_b^r > 0 \).

The big incumbent has one disadvantage. Due to its size, it cannot easily, or immediately, detect and respond to entry. For example, it may be too costly to monitor all market segments simultaneously. The delay in responding to an entry can be due to the need to announce the prices to retailers ahead of time, or the time required to prepare an advertising campaign or a sale. The model represents this
disadvantage by assuming that the big firm can retaliate only after a delay of $d$ periods.$^5$

On the other hand, I assume that the small firm has also a constraint on its entry and exit flexibility. Specifically, I assume that the small firm needs a preparation of $k$ periods before entering a new market segment. This can be due to limited resources such as a small number of employees or to other physical or logistical constraints. Consequently, after exiting from one market, the small firm cannot immediately enter another market segment. It needs to stay out for $k$ periods, and then it may decide to reenter.$^6$

Given the above assumptions, the game proceeds as follows: at period $t = 0$ the small firm decides whether or not to enter one of the market segments. If it does not enter, it will again face the same decision problem at $t = 1$. If it enters, then there is no retaliation for $d$ periods, and the firms' payoffs are $(\pi^b, \pi^s)$ per period. At the end of any period $t$ in which the entrant is in the market, it needs to decide whether to stay another period (i.e., stay for period $t + 1$) or exit the market. If it exits, it must wait out for $k$ periods, after which the game starts from the beginning, i.e., the small firm needs to make an entry choice as at $t = 0$. If the small firm stays in the market for more than $d$ periods, it runs the risk of facing a retaliatory response. If at period $t$ the small firm has been in the market for at least $d$ periods, it is now the turn of the big firm to decide if it is going to retaliate or to accommodate in the following period. This decision must be made at the end of period $t$ before observing the period $t + 1$ exit or stay decision of the small firm. I thus assume that retaliation needs preparation: preparing an advertising campaign; announcing price reductions, rebates, and coupons; etc. Since all these preparations are irreversible in the short run, I assume, for the sake of simplicity, that once the big firm decides to retaliate at period $t + 1$, its profit at that period is $\pi^b$ even if the small firm exits the market.$^7$ If, however, the small firm stays in the market for period $t + 1$, then the big firm's retaliation will force the small firm out of the market. After the small firm stays out for $k$ periods, the game starts all over again.

I allow firms to randomize whenever it is their turn to make a

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5. Our simplifying assumption here is that the delay period, $d$, is exogenously given. It is possible, however, that $d$ can be changed by investing in monitoring or by preparing a rapid retaliation ahead of time.

6. An alternative formulation would be to assume a direct fixed entry cost.

7. Once the big firm announces a future price cut, we assume that this announcement is irreversible. Thus even if the small firm exits the market after period $t$, the big firm does not enjoy the monopolistic profits at period $t + 1$. 
decision, and I assume that both firms are risk neutral. I assume that both firms maximize their expected discounted profits letting $0 < \delta < 1$ be the common discount factor for the two firms. I denote by $\pi^s(j, \delta)$ the discounted value of $j$ periods' profits of $\pi^s$. Letting $\pi^w(j, \delta)$ and $\pi^w(j, \delta)$ be defined similarly.

Let $h_t$ be the history of the game up to period $t$, and $H_t$ be the set of all possible such histories. A strategy of the small firm is a pair of functions $E : H^o_t \to [0, 1], S : H^i_t \to [0, 1]$ such that $H^o_t \subset H_t$ is the set of all histories of length $t$ where the small firm is out of the market at periods $t - k, \ldots, t$. The function $E(h_t)$ specifies the probability of entry, $H^i_t$ is the set of all histories of length $t$ where the small firm is in the market at period $t$, and $S(h_t)$ specifies the probability of staying in the market an additional period. Note that $S(h_t) = 0$ for every history $h_t$ in which the big firm retaliates at period $t$. A strategy for the big firm is a function $R : H^i_t \to [0, 1]$ which specifies the probability of retaliation given a history $h_t \in H^i_t$ where the small firm is in the market for at least $d$ periods.

I distinguish between three types of equilibria: (1) a monopoly equilibrium in which the small firm stays out of the market; (2) a hit-and-run equilibrium in which the small firm keeps entering and exiting the market, operating in one market segment for a couple of periods and then exiting, staying out for a couple of periods and then again reentering another segment; and (3) an accommodation equilibrium characterized by the permanent existence of the small firm in the market without any type of retaliation by the big firm.

When $k = 0$, the only market equilibrium is an accommodation equilibrium. In such a case the small firm is completely flexible to enter a new market segment immediately following its exit from another market. Consequently, as long as $d > 0$, the small firm can guarantee for itself the profits of $\pi^s(\infty, \delta)$ by entering a different market segment in each period. Although this is a hit-and-run strategy, the small firm has the ability to exit the market before the big firm is able to retaliate. The big firm's best response in such a case is never to retaliate, as preparing a retaliation is also costly. Even when the small firm stays in the same market for more than $d$ periods, retaliation by the big firm will reduce its profits in the short run without any long-run benefits, since the small firm will return to one of the markets in the following periods. Thus, in this case, the big firm realizes that it cannot gain by retaliating and that its best strategy is to let the small firm operate in one of the markets. I therefore assume hereinafter that $k > 0$.

Similarly, as long as $d > 0$, $\pi^w > 0$, and there is no entry cost [or the entry cost is lower than $\pi^w(d, \delta)$], there is no monopoly equilib-
rium. The small firm can enter one market, stay there for \( z \) periods, and guarantee positive profits for itself.

Define a simple stationary strategy (SSS) for the small firm as a number \( n_s \) that specifies the number of periods it stays in the market given that it does not face retaliation. An SSS for the big firm is a number \( n_b \) that specifies the number of periods it waits before retaliating. The first observation is that, in the above game, there is no equilibrium with SSSs. Given a strategy \( n_b \) for the big firm, the small firm's best response would be to stay in the market for exactly \( n_b \) periods, i.e., there is no reason to exit the market as long as the big firm does not plan to retaliate, while staying for the \( n_b + 1 \)th period yields only the losses of \( \pi_f^r \). On the other hand, given that the small firm plans to exit after period \( n_s \), the big firm's best response is either to retaliate at an earlier period, i.e., at some \( n_b < n_s \), or to let the small firm leave the market at period \( n_s + 1 \) without retaliating, since accommodating at period \( n_s \) yields the profits \( \pi_f^r > \pi_f^r \) without having any effect on the continuation of the game. Clearly there are no \((n_s, n_b)\) that satisfy these two conditions.

The game under consideration has a very simple structure. All subgames induced by a history \( h_t \) constructed by some history \( h_{t-d} \in H_{t-d} \) and \( d \) periods in which the small firm is in the market without retaliation by the big firm are strategically equivalent. That is, each player's action set, at any subsequent period, and his preferences over sequence of actions beginning at period \( t \) do not depend on the history \( h_t \). Similarly, all subgames induced by a history \( h_t \) constructed by some \( h_{t-k} \in H_{t-k} \) and \( k \) periods in which the small firm is out of the market are strategically equivalent. In the remainder of the paper, I examine the Markov perfect equilibrium of the above game, which is a subgame perfect equilibrium in which players play the same strategy at any strategically equivalent subgame (see Maskin and Tirole, 1992).

Pure Markov strategies in this game have a simple structure. Since all subgames in which the big firm may retaliate are strategically equivalent, a pure Markov strategy implies that the big firm either always retaliates or never retaliates. Similarly, for the small firm, a pure Markov strategy implies that for every \( h_t \in H^O_t \), \( E(h_t) \) is either zero or one, i.e., the small firm either enters immediately after waiting \( k \) periods outside the market, or never enters. Once the small firm is in the market for at least \( d \) periods, the pure Markov strategy is either to stay until forced out, or exit the market immediately. Since such strategies are special cases of SSS, it is clear that there is no perfect Markov equilibrium with pure strategies.

A Markov mixed strategy for the small firm is a pair \((p_s, p_e)\) such that \( p_s \) is the probability of staying another period given that the small
firm is in the market at least $d$ periods, and $p_e$ is the probability of entering a market once the small firm is out for at least $k$ periods. A Markov mixed strategy for the big firm is a probability $p_b$ of retaliating at period $t + 1$ once the big firm observes that the small firm is in the market at period $t$ and that it has been in the market for at least $d$ periods.

The fact that there is no pure Markov equilibrium or equilibrium with pure SSS strategies coincides with the intuition that the element of surprise is crucial in guerrilla warfare. Had the players been using pure strategies with a specific rule when to attack and when to withdraw, they would lose any element of surprise and could thus be easily outmaneuvered. I thus interpret surprise as randomization. Since the structure of the model is that of a multiperiod game, randomization should not be interpreted as a "throw of the dice." One can alternatively think of a player who decides on the number of periods he stays in the market (or on the timing of the retaliatory action) according to his intuition without forming any recognizable pattern of decisions. Such a strategy, however, can be viewed by the opponent as randomization. Clearly, in order to establish such a purification interpretation of mixed strategies it is important to assume that the above intuition is based on variables that are not payoff relevant.

3. **Hit-and-Run Equilibrium**

As was pointed out before, the hit-and-run strategy characterizes guerrilla behavior. On the one hand, the small firm wishes to stay in the market for as many periods as possible, while on the other hand, it tries to avoid any confrontation with the established firm. Clearly, any pair of mixed strategies will imply hit-and-run behavior. In what follows, I establish the existence of a hit-and-run equilibrium and identify the equilibrium mixed strategies and the value of the game for each player.

First note that as long as $\pi^2_e > 0$ at every Markov perfect equilibrium, it must be that at equilibrium $p_e = 1$. That is, if entry is profitable, there is no advantage in delaying it, as the retaliation strategy of the big firm is not affected by the entry date. In such a case, the small firm always enters the market (with probability 1) whenever it is able to do so. Similarly, in any Markov perfect equilibrium, the small firm stays in the market at least for the initial $d$ periods. Thus, for convenience, we refer in the rest of the analysis to the strategies $(p_e, p_b)$ only.

9. Such a characterization clearly need not necessarily hold in a general perfect equilibrium.
In the following two subsections, I examine the behavior of each firm, given that its opponent is playing a random strategy.

3.1 The Decision Problem of the Big Firm

After the small firm has been in the market for \(d\) (or more) periods, it is time for the incumbent firm to decide whether or not to retaliate. Retaliation is costly, as it reduces profits in the short run, and it may also be unnecessary, because the small firm might leave the market in the next period even without retaliation.

For convenience, I define three value functions. Let \(V_b(p_s, p_b)\) be the value of the game for the big firm when the small firm has been out of the market for at least \(k\) periods (and is about to enter) and the players play the mixed strategies \((p_s, p_b)\). The function \(V_b(p_s, p_b)\) is similarly defined as the value of the game for player \(b\) after a period in which the small firm exits the market, and \(V_s(p_s, p_b)\) is the value of the game for firm \(b\) when firm \(s\) has been in the market for at least \(d\) periods.

Consider the case where the small firm enters the market at period \(t\). Once it enters, it will stay there for \(d\) periods, during which the big firm obtains the discounted profits of \(\pi_b(d, \delta)\). At the end of period \(t + d\) the big firm must decide if it is going to retaliate at period \(t + d + 1\). Given that the big firm is using a randomized strategy, then at equilibrium the big firm must be indifferent to either retaliating or accommodating at period \(t + d + 1\). Assume that the big firm chooses to retaliate. In such a case its discounted profit for that period is \(\delta d \pi_f\). After retaliation, at period \(t + d + 1\), the small firm is out of the market for \(k\) periods. During these periods the big firm earns the monopolistic profits, \(\pi_m\), per period. At the end of \(k\) periods, the small firm is ready to reenter, yielding the discounted value of the continuation of the game, \(\delta^{d+k+1}V_b(p_s, p_b)\). Collecting the above terms yields that at equilibrium

\[
V_b(p_s, p_b) = \pi_b(d, \delta) + \delta d \pi_f + \delta^{d+1} \pi_m(k, \delta) + \delta^{d+k+1}V_b(p_s, p_b),
\]

which, after rearranging, yields the following value function:

\[
V_b(p_s, p_b) = \frac{\pi_b(d, \delta) + \delta d \pi_f + \delta^{d+1} \pi_m(k, \delta)}{1 - \delta^{d+k+1}}.
\]
\[ V_b^e(p_s, p_b) = \pi_b^e(k, \delta) + \delta^k V_b^e(p_s, p_b). \]  

(3)

Substituting for \( V_b^e(p_s, p_b) \) from eq. (2) yields that
\[ V_b^e(p_s, p_b) = \frac{\pi_b^w(k, \delta) + \delta^k \pi_b^w(d, \delta) + \delta^{d+k} \pi_b^f}{1 - \delta^{d+k+1}}. \]  

(4)

Once the small firm has been in the market for more than \( d \) periods, retaliation by the big firm yields it \( \pi_b^f + \delta V_b^e(p_s, p_b) \), while accommodation implies that with probability \( p_s \) the small firm stays in the market for another period, which yields profits of \( \pi_b^e + \delta V_b^e(p_s, p_b) \), and with probability \( 1 - p_s \) the small firm exits the market, yielding the value \( V_b^e(p_s, p_b) \). At equilibrium, the incumbent firm must be indifferent between the two alternatives, and therefore,
\[ \pi_b^f + \delta V_b^e(p_s, p_b) = p_s[\pi_b^e + \delta V_b^e(p_s, p_b)] + (1 - p_s) V_b^e(p_s, p_b). \]  

(5)

Note also that by definition, both sides of eq. (5) have the value \( V_b^e(p_s, p_b) \). Substituting for \( V_b^e(p_s, p_b) \) in eq. (5) and solving for the equilibrium \( p_s \) yields that
\[ p_s = \frac{(1 - \delta)V_b^e(p_s, p_b) - \pi_b^f}{(1 - \delta^2)V_b^e(p_s, p_b) - \delta \pi_b^e - \pi_b^f}. \]  

(6)

Note that on using eq. (4) to substitute for \( V_b^e(p_s, p_b) \), \( p_s \) is determined by the parameters of the model. First note that \( p_s > 0 \), since \( V_b^e(p_s, p_b) > \pi_b^f/(1 - \delta) \), and also that \( p_s < 1 \) iff
\[ \delta V_b^e(p_s, p_b) + \pi_b^f - \frac{\pi_b^e}{1 - \delta} > 0. \]  

(7)

The condition (7) simply implies that the big firm is better off retaliating and forcing the small firm out of the market rather than facing the small firm in the market forever.

### 3.2 The Decision Problem of the Small Firm

Once the small firm is in the market, it needs to decide whether or not to stay there. Clearly the small firm will stay in the market for the first \( d \) periods, during which the big firm is unable to retaliate. The dilemma begins when the small firm is in the market for more than \( d \) periods. Exiting the market will force the small firm to wait \( k \) periods before being able to reenter. Staying in the market exposes the firm to the risk of retaliation by the big firm.

Let \( V_s^e(p_s, p_b) \) be the value of the game for firm \( s \) at the beginning of a period in which it can enter the market and when the firms play
the Markov mixed strategy \((p_s, p_b)\). Entry will therefore occur only if \(V_s^e(p_s, p_b) > 0\). Similarly, let \(V_s^o(p_s, p_b)\) be the value of the game for the small firm after exit has occurred, and let \(V_s^d(p_s, p_b)\) denote the value of the game for the small firm after being in the market for at least \(d\) periods and when the firms play the mixed strategies \((p_s, p_b)\). Note that the value \(V_s^e(p_s, p_b)\) does not depend on whether the exit was voluntary or followed a retaliatory action.

Leaving the market requires the small firm to wait \(k\) periods before being able to reenter again. Therefore,

\[
V_s^o(p_s, p_b) = \delta^k V_s^e(p_s, p_b).
\]

If after being in the market for more than \(d\) periods the small firm decides to stay in the market for another period, then, given the big firm's strategy, with probability \(p_b\) the big firm retaliates and the entrant suffers the loss \(\pi_s^e < 0\) and is forced out of the market, and with probability \(1 - p_b\) the big firm accommodates and the entrant enjoys the profits of \(\pi_s^o\) for one period, but it will face the same decision problem at the next period. Consequently, the small firm's expected profit from staying in the market is

\[
V_s^d(p_s, p_b) = p_b[\pi_s^e + \delta V_s^o(p_s, p_b)] + (1 - p_b)[\pi_s^o + \delta V_s^d(p_s, p_b)].
\]

Since the small firm randomizes, it must be that at equilibrium the firm is indifferent between staying in and exiting the market, which implies that \(V_s^d(p_s, p_b) = V_s^o(p_s, p_b)\). Using eq. (8) to replace \(V_s^o(p_s, p_b)\) and \(V_s^d(p_s, p_b)\) with \(\delta^k V_s^e(p_s, p_b)\) and collecting terms, we get

\[
\delta^k(1 - \delta)V_s^e(p_s, p_b) = p_b\pi_s^e + (1 - p_b)\pi_s^o.
\]

Since after an entry the small firm stays in the market for \(d\) periods, \(V_s^e(p_s, p_b)\) represents the discounted profits of staying in the market for \(d\) periods unchallenged by the incumbent firm, plus the value of continuing the game, which is the value \(V_s^o(p_s, p_b)\) of exiting the market (or the value of staying for another period). Consequently,

\[
V_s^o(p_s, p_b) = \pi_s^o(d, \delta) + \delta^d V_s^o(p_s, p_b),
\]

However, \(V_s^o(p_s, p_b)\) is the value of waiting \(d\) periods and then reentering the market [see eq. (8)]. Thus, after simplification we obtain

\[
V_s^e(p_s, p_b) = \pi_s^e(d, \delta)(1 - \delta^{d+k})^{-1}.
\]
Using eq. (12) to substitute $V_s'(p, p_b)$ in eq. (10) and letting $\pi_s^*(d, \delta) = \pi_s^*(1 - \delta^d)(1 - \delta)^{-1}$ yields the equilibrium mixed strategy

$$p_b = \frac{\pi_s^a}{\pi_s^a - \pi_s^l} \left(1 - \frac{(1 - \delta^d)\delta^k}{1 - \delta^{d+k}}\right).$$

(13)

Note that since $\delta < 1$ and $\pi_s^l < 0$, it is guaranteed that $1 < p_b < 1$. The following result summarizes the analysis of the game.

**Proposition 1**: The unique Markov perfect equilibrium of the guerrilla-warfare game is such that the big firm retaliates (whenever possible) with probability

$$p_b = \frac{\pi_s^a}{\pi_s^a - \pi_s^l} \left(1 - \frac{(1 - \delta^d)\delta^k}{1 - \delta^{d+k}}\right),$$

(14)

and the small firm plays the mixed strategy

$$p_s = \frac{(1 - \delta)V_b'(p, p_b) - \pi_s^l}{(1 - \delta^2)V_b'(p, p_b) - \delta\pi_s^l - \pi_s^*}.$$  

(15)

That is, after staying in the market for $d$ periods, the small firm stays for an additional period with probability $p_s$.

Proposition 1 indicates that the guerrilla-warfare game has a unique Markov perfect equilibrium. The equilibrium is of the hit-and-run type in which both firms use nondegenerate mixed strategies. Consequently, the industry is characterized by the following pattern of behavior. The small firm enters one of the market segments. After a couple of periods the small firm either exits the market voluntarily or is forced out by the established firm. Retaliation occurs randomly according to eq. (14).

In formulating the above duopolistic game, we defined two variables, $d$ and $k$, which reflect the relative disadvantage of each type of firm: $d$ reflects the big firm's inflexibility toward quick response, while $k$ reflects the limited resources of the small firm. Having solved above for the equilibrium value of the game for both players, we now check if the above restrictions are indeed disadvantages for the firms. Differentiating $V_b'(p, p_b)$ [eq. (2)] yields that $\partial V_b'/\partial k > 0$ and $\partial V_b'/\partial d < 0$, which implies that the value of the game for the big firm declines with increase in its own delay $d$, and increases with $k$, the delay of the small firm. Similarly, differentiating $V_s'(p, p_b)$ [eq. (12)] yields that $\partial V_s'/\partial k < 0$ and $\partial V_s'/\partial d > 0$, which implies that the value of the game for the small firm declines with increase in its own delay and increases with the delay of the big firm.
4. Price Wars

I now consider equilibrium price patterns that occur when both firms are in the market and the big firm either accommodates or retaliates against the entrant. Competition occurs in a stationary environment, without demand or cost shocks. I show that a nonstationary price pattern results from the nature of the market interaction alone.

Assume now that the two firms compete through prices. Define three price levels: the monopolistic price $p^m$, the duopolistic (accommodation) price $p^d$, and the retaliation price $p^f$, such that $p^m > p^d > p^f$. Note that the two firms may choose different prices. In such a case we view the above prices as the weighted average market price.\(^{10}\)

In an accommodation equilibrium, the small firm enters and stays in the market forever, while the big firm does not retaliate. In such a case, the market price is $p^d$ for each period. In a monopoly equilibrium, firm s is out and the big firm charges the monopolistic price in each period. Hit-and-run equilibrium, however, gives rise to a nonstationary price pattern. Following the small firm’s entry into one of the markets, there are $d$ periods in which the market price is $p^d$. In period $d + 1$ there is the following distribution of prices: the duopolistic price $p^d$ [with probability $(1 - p_b)p_s]$ if the small firm stays in the market and the big firm does not retaliate, the retaliation price $p^f$ [with probability $p_b$], or the monopolistic price $p^m$ [with probability $(1 - p_b)(1 - p_s)$] if the small firm voluntarily exits the market and the incumbent firm does not plan to retaliate.\(^{11}\) The continuation of the price pattern depends on the event that has taken place at period $d + 1$. The monopolistic price will be followed by $k - 1$ periods of monopolistic price; the retaliation price will be followed by $k$ periods of monopolistic price; while after the duopolistic price $p^d$, we will have the same price distribution as in period $d + 1$.

Given the equilibrium strategies $(p_s, p_b)$, the probability that a specific market entry of the small firm will eventually lead to a price war is

$$P_f = \sum_{j=0}^{\infty} (1 - p_s)^j (1 - p_b)^j p_s p_b = \frac{p_b p_s}{p_b + p_s - p_b p_s}. \quad (16)$$

10. We have chosen in this model not to specify the price competition between the firms. Thus we view the above three price levels as equilibrium prices. Alternatively, one can think about a model in which there are only three levels of prices and at any period, once the firms realize the type of market interaction, they automatically select the appropriate price.

11. The price distribution may be slightly different when the average retaliation price depends on whether firm s stays in the market or exits voluntarily.
The probability $P_f$ is an increasing function of both $p_s$ and $p_b$, since a higher probability of the small firm staying in the market or a higher probability of retaliation raises the likelihood of a price war.

5. Predation and the Severity of Retaliation

Retaliation may take the form of any action or policy that is intended to inflict losses on the entrant and force it out of the market. Assume now that the big firm has a menu of different possible retaliatory actions, each implying a different pair of payoffs $(\pi_f^L, \pi_f^S)$. Clearly, if different forms of retaliation imply the same $\pi_f^L$, the big firm will choose the one that yields the highest possible $\pi_f^S$. But the question is whether there is any advantage in choosing a severe retaliatory action (lower $\pi_f^S$) rather than one which is just enough to force the small firm out of the market. The severity of the retaliation may affect the small firm's exit-or-stay decision, as it enhances the downside of such a decision. At the same time, changing the severity of the retaliation may also affect the equilibrium probability of using such a policy. In particular note that the equilibrium value $V^e_s(p_s, p_b)$ [as defined by eq. (12)] does not depend on $\pi_f^S$ and that adopting a more severe retaliation does not affect the entry decision in this model. On the other hand, differentiating eq. (3) yields

$$\frac{\partial V^e_s}{\partial \pi_f^L} = \frac{\delta^d}{1 - \delta^{d+k+1}} > 0. \quad (17)$$

That is, a retaliation policy with a lower $\pi_f^S$ will reduce the value of the game for the big firm. Thus the big firm's optimal retaliation policy is one that is sufficient to drive the small firm out of the market but yields the highest possible $\pi_f^L$.

6. Possibility of Cooperation

The hit-and-run equilibrium is clearly not Pareto optimal from the firms' point of view. Such an equilibrium involves periods of retaliation in which both firms have lower payoffs. Any Pareto efficient outcome of the above game involves a sequence of numbers $n_p = n_1$, $n_2$, . . . and $m_p = m_1$, $m_2$, . . . such that the first time the small firm enters, it stays in the market for $n_1$ periods and then exits and stays out for $m_1$ periods; the second time, it stays $n_2$ periods and then exits for $m_2$ periods; etc. The big firm's strategy is to never retaliate. A special case of such strategies comprises stationary strategies such that $n_i = n_i = d + 1$ and $m_i = m_i = k$. In such a case the small
firm's discounted payoffs will be
\[
\frac{\pi^s(d + 1)}{1 - \delta^{d+k+1}} > V^s(p_s, p_b),
\]
(18)
and the discounted payoffs of the big firm will be
\[
\frac{\pi^b(d + 1, \delta) + \delta^{d+1}\pi^b(k, \delta)}{1 - \delta^{d+k+1}} > V^b(p_s, p_b).
\]
(19)

Given the multiperiod nature of the game, standard analysis indicates that, for \(\delta\) sufficiently close to 1, one can support (as a noncooperative equilibrium outcome) outcomes that Pareto-dominate the Markov perfect equilibrium values \((V^s, V^b)\). The game under consideration, however, is not a repeated game. The game that starts after firm \(s\) enters the market is different from the game that starts after firm \(s\) exits. Note also that the "folk theorem" does not hold, in general, for dynamic games.\(^{12}\) But given the specific simple structure of our model, one can easily demonstrate that the firms can support Pareto optimal outcomes by using non-Markov strategies. For example, consider the strategies suggested by the example above, in which firm \(s\) stays in the market for exactly \(n = d + 1\) periods and then exits and stays out for \(k\) periods, while firm \(b\) never retaliates. Now consider the strategies in which both players continue the above cooperative behavior for as long as the history \(h_t\) is consistent with such behavior, and revert to the Markov perfect equilibrium strategies once the history is no longer consistent with the cooperative behavior. One can easily check that for \(\delta\) sufficiently close to 1, the above strategies are a subgame perfect equilibrium.

Note that the implication of our specific setting is that immediate punishment is not necessarily suffered for every deviation from the collusive path. In a regular repeated game setting, once a deviation from the collusive path is detected, it is possible for the players to revert, in the next period, to the punishment mode. In our setting, there might be a greater delay between crime and punishment. Assume for example that the cooperative program is such that firm \(s\), after being in the market for \(z \geq d\) periods, is supposed to exit, letting the big firm enjoy the monopolistic position for \(y \geq k\) periods. As we noted before, the Pareto optimal plan does not call for retaliation, and thus the big firm does not plan (or commit to) one. Assume now that

\(^{12}\) The "folk theorem" states that in a repeated game every feasible individual rational payoff can be obtained as a Nash equilibrium payoff providing that the discount rate is not too big. This common wisdom cannot be extended to games with dynamic structure.
the small firm deviates from the collusive program and stays in the market for period \( z + 1 \). There is no retaliation at that period, and thus firm \( s \)'s net gain is \( \pi^s \). In the following period, the small firm may exit and stays out for \( k \) periods, its profits during these periods being zero. Had the firm followed the original plan, its profits would have also been zero for at least \( k - 1 \) of those periods (perhaps even more, if the cooperative plan implies staying out for more than \( k \) periods). Thus, if the two firms switch to the punishment mode after the deviation, it will take at least \( k - 1 \) periods until firm \( s \) begins to be punished for its deviation. Thus, since there is a possible delay between crime and punishment, the condition on \( \delta \) that facilitates supporting a cooperative outcome might be very restrictive.

7. Concluding Remarks

Analysis of military conflict started many centuries before economists began to study rivalry between firms. Although until recently the two disciplines used different frameworks of analysis, there is a clear similarity between the two types of conflict. Economists have used military examples many times to illustrate some of the main ideas of strategic competition. In this paper, I have drawn attention to the similarity between the analysis of guerrilla warfare and competition in an asymmetric duopoly with one large established firm and a small entrant.

Although I discussed only the hit-and-run equilibrium, the guerrilla-warfare literature also discusses other forms of strategies that facilitate the survival of small vulnerable forces. In particular, the literature emphasizes not only the survival of small forces but their ability to gain an "ultimate victory." This aspect of the analysis is missing from our model. While the interpretation of the term "victory" is well understood in the military context, the analogy to market interaction is less clear. One can imagine, however, a dynamic version of the guerrilla-warfare model in which staying in the market for longer periods yields more resources for the small firm, which subsequently can be employed to enter new markets. Thus, given sufficient resources, the small firm can enter two markets simultaneously, or more frequently. In this case, early retaliation by the big firm may prevent the small firm from becoming established.

References


