The Importance of the Agenda in Bargaining

**Chaim Fershtman***

*Department of Economics, Tel Aviv University, Tel Aviv, Israel, and Department of Managerial Economics and Decision Sciences, J. L. Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois 60208

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This paper discusses a multi-issue bargaining game in which the players set up an agenda and negotiate on the issues sequentially according to this agenda. We demonstrate that the agenda might play an important role in the bargaining and discuss the relationship between the agenda and the final outcome of the bargaining game. Assuming that players have conflicting evaluations regarding the importance of the issues under negotiation, we identify the type of agenda each player prefers. We also show that in such cases the outcome of the bargaining game need not to be efficient. By demonstrating the strategic use of the agenda the paper explains the phenomenon of "bargaining on the agenda." *Journal of Economic Literature* Classification Number: 026.

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1. **Introduction**

In many bargaining situations the parties involved negotiate on more than one issue. In labor negotiation, for example, workers and management negotiate on wages, retirement programs, as well as on other issues related to the workers' working conditions. Negotiations between countries usually involve settlement of different issues on which the countries dispute. For example, in the peace talks between Israel and Egypt the two countries discussed the end of the state of war, the withdrawal from occupied territories, trade and tourist relationship, an arrangement regarding the oil fields in the Sinai desert, as well as other issues. In such

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multi-issue negotiations the players have the possibility to discuss all the issues simultaneously or, alternatively, they can set up an agenda and negotiate on the issues sequentially according to this agenda. The main objective of this paper is to discuss the relationship between the agenda and the outcome of the bargaining game. We show that the agenda really matters, so if players change the order of issues in the agenda, it will affect the outcome of the bargaining game.

In a multi-issue negotiation we sometimes face a situation in which players have different evaluations regarding the importance of the issues under negotiation. Some issues are more important to one player while the other player regards the other issues as the important ones. We investigate such a situation and discuss the type of agenda each player prefers and how it relates to the players’ evaluations regarding the importance of the different issues.

Using the axiomatic approach to bargaining, Kalai (1977) discussed the cooperative outcome of the bargaining when the negotiation is done by stages. The players consider first only a subset of the feasible alternatives, reach an agreement on this subset, and then consider the remaining alternatives. This structure can be formally described as an agenda. Restricting the set of solutions to ones which are invariant under decomposition of the bargaining process into stages, Kalai proved that only the proportional solution satisfies this restriction and that this solution involves interpersonal comparison of utility.

The framework that we adopt in this paper is Rubinstein’s (1982) strategic approach. Our point of deviation from Rubinstein’s paper is by assuming that instead of one pie the players have to agree on the partition of several pies and that the bargaining on these pies is done sequentially. The pies are of different sizes but we will differentiate between two situations: (i) the players are identical in their evaluations regarding the size of these pies; (ii) players have different (possibly conflicting) evaluation regarding the size of the pies.

Since the bargaining on the pies is done sequentially, an important part of such a model is the assumption regarding the time the players are allowed to eat these pies. There are three major possibilities regarding this assumption:

(a) After the players agree on a partition of a particular pie each one of them is allowed to eat his share.

(b) The players can eat their shares from all the pies only when the negotiations on all the pies are over.

1 See also Binmore (1982) and Shaked and Sutton (1984).
(c) The third possibility is a situation in which a pie is actually a flow of services. Once a partition of a certain pie is decided, players start to enjoy the benefits from their share of the pie. However, if the players fail to reach an agreement on the partition of all the pies and the negotiation terminates, all the partitions that were agreed upon are cancelled and each player has to give up the pieces of pies he already started to enjoy.

In the first case mentioned above, if the utility from the partition of one pie does not affect the utility generated from the partition of another pie then the bargaining on each pie can be discussed separately. Whatever is agreed upon in the bargaining on one pie does not affect the bargaining game on the other pies. In the second case, when players are allowed to eat their shares only at the end, a partition of one pie affects both the threat point and the impatience rate in the bargaining game on the second pie. It affects the threat point since if the bargaining on the second pie is broken, the players do not get to eat their shares from the first pie. It affects the impatience rate since if there is a delay in the agreement on the second pie, it implies a delay in eating their share from the first pie. In the third case, when the pies represent flows of services, the partition of one pie affects only the threat point in the bargaining game on the second pie. It does not affect the impatience rate since players start immediately to enjoy the services provided by their shares of the pie.

In this paper we discuss the more interesting case in which players are allowed to eat their shares only when the bargaining on all the pies is finished. A similar assumption was assumed by Horn and Wolinsky (1988) in which a bargaining game between an employer and two groups of workers was discussed. In this work the bargaining with the two groups is done simultaneously, but once an agreement with one group is reached the bargaining with the other group continues while the implementation of the first agreement is postponed until an agreement is reached also with the second group. In such a framework the bargaining with the two groups is interdependent as an agreement with one group of workers determines the surplus to be divided in the bargaining with the other group.

The paper is organized as follows. In Section 2 we discuss the case in which the pies have different sizes and players have identical evaluations regarding the sizes of the pies. Section 3 discusses the bargaining game when players have conflicting evaluations regarding the sizes of the pies. We show that in such a multi-issue bargaining game, the players' expected utilities depend on the agenda of the bargaining. Each player prefers an agenda such that the first pie to bargain on is the one which is the least important to him but the most important to his opponent. We also show that once players have conflicting evaluations and they bargain on the pies sequentially the equilibrium allocation might be inefficient.
2. Identical Preferences: The Order of Pies

2.1. The Model

Consider a two-person bargaining problem in which players bargain on the partition of two pies. One pie is of size 1 while the other is of size \( a > 1 \). Both players have identical evaluations regarding the size of these pies. The bargaining is done sequentially one pie at a time. Thus an agenda is an ordering of the pies. We assume that the pies are eaten only after the entire negotiation on the two pies has been completed.

We adopt the Rubinstein (1982) alternating offers bargaining model. Specifically the game we consider proceeds as follows: in the first period the first mover is selected randomly. After being identified the first mover offers a partition of the first pie to his opponent. The opponent can either accept or reject this offer. If he rejects the offer the game proceed to the next period in which the second player offers and the first players responds. If players agree on a partition of the first pie at period \( t \) then at period \( t + 1 \) the bargaining on the second pie starts. A lottery determines the first mover and the game proceeds in the standard alternating offers procedure. Note that in this formulation the first mover in the bargaining on the second pie is not known to the players while they bargain on the first pie.

When the equilibrium partition is such that the first player gets \( y \) from the first pie and \( x \) from the second pie and the bargaining game ends after \( t \) periods, the players' utilities are assumed to be

\[
\begin{align*}
u_1(y, x) &= (y + x)\delta^t, \\
u_2(y, x) &= (a - y + 1 - x)\delta^t,
\end{align*}
\]

where \( \delta \) is a constant and identical rate of time preferences.\(^2\) We further assume that \( a > 1/\delta > 1 \).

The total size of the two pies is \( 1 + a \). When players bargain over the two pies simultaneously the unique subgame perfect equilibrium is that the first mover gets \( (1 + a)/(1 + \delta) \) while his opponent gets \( \delta(1 + a)/(1 + \delta) \). We let \( E_i \) be the equilibrium payoffs combination in the simultaneous bargaining game when player \( i \) is the first mover.

Consider now the issue by issue bargaining. One can view the bargaining on the second pie as a bargaining on a partition of a pie of size \( 1 + a \) when the set of possible partitions is constrained by the agreement on the first pie. This situation is illustrated in Fig. 1.

\(^2\) In our formulation the agenda is important only when players are impatient. When \( \delta \to 1 \) the agenda ceases to be important. We thus assume that \( \delta < 1 \). For more on the rationale of such an assumption in the union–management context see Hart (1989).
When the first pie is of size 1 and the second is of size $a$ then once a partition $A$ of the first pie is agreed upon, the only feasible partitions of the combined pies are the ones that lie on the line $BC$.

In the following two subsections we examined the outcome of the bargaining game under two types of agendas. In the first one the players bargain on the small pie first while in the second case we reverse the order of pies.

2.2. Bargaining on the Small Pie First

**Proposition 1.** If the two players have identical evaluations regarding the size of the pies and they bargain on the small pie first, then the final equilibrium share of the two pies is either $E_1$ or $E_2$, depending on the identity of the first mover in the bargaining on the second pie.

**Proof.** Let $y$ be a partition of the first pie. The bargaining on the second pie can now be viewed as a bargaining on the partition of a pie of size $1 + a$ under the constraint that the first player gets at least $y$ while the second player gets at least $1 - y$. Now note that our assumption that $a > 1/8$ guarantees that for every $y \leq 1$, $\max\{y, 1 - y\} \leq 8(1 + a)/(1 + 8)$. Thus, as it is demonstrated in Fig. 1, the partitions $E_1$ and $E_2$ are both feasible in the second stage of the bargaining game, and therefore one can use standard arguments to claim that, depending on the identity of the first mover, one of them is the equilibrium partition. ■
Given the above equilibrium the players’ final share of the two pies is 
\[ A_1 = \frac{1 + a}{1 + \delta} \] if a player is the first mover in the bargaining on both pies. 
\[ A_2 = \frac{\delta(1 + a)}{1 + \delta} \] if he is first mover in the bargaining on the first pie and second in the bargaining on the second pie. Similarly, 
\[ A_3 = A_1 \] is his payoff if he is the second mover and then the first mover and 
\[ A_4 = A_2 \] is his payoff if he is the first mover and then the second mover.

2.3. Bargaining on the Large Pie First

Let us now change the order of pies and assume that players bargain first on the large pie. In this case the equilibrium combinations \( E_1 \) and \( E_2 \) are not necessarily in the feasible set in the bargaining on the second pie. The constraint imposed by the agreement on the first pie, denoted by \( y \), on the feasible set in the bargaining on the second pie can be one of six types, as is illustrated in Fig. 2.

We now analyze the bargaining on the second pie, discussing each of these six cases separately. We let \( x_i^*(y) \) denote the equilibrium partition of the second pie when player \( i \) is the first mover. As before \( x_i^*(y) \) is the share of the first player.

Case a. The equilibrium combinations \( E_1 \) and \( E_2 \) are feasible in the bargaining on the second pie. Thus the problem is identical to the one described in the previous section and the equilibrium partition is either \( E_1 \) or \( E_2 \) depending on the identity of the first mover. This case occurs when

\[ \text{Max}\{y, a - y\} < \frac{\delta(1 + a)}{1 + \delta} \] or, after simplifying, when

\[ \frac{(a - \delta)}{(1 + \delta)} < y < \frac{\delta(1 + a)}{(1 + \delta)}. \quad (2) \]

Lemma 1. When players bargain on the large pie first and \( a > 2\delta/(1 - \delta) \), the final equilibrium partition is not identical to the one of the game in which players bargain on the two pies simultaneously.

Proof. The condition \( a > 2\delta/(1 - \delta) \) implies that \( \frac{\delta(1 + a)}{(1 + \delta)} < \frac{(a - \delta)}{(1 + \delta)} \), which contradicts condition (2). Thus for any partition \( y \) of the first pie either \( E_1 \) or \( E_2 \) or both are not in the feasible set in the bargaining on the second pie.

Let us therefore continue our discussion under the assumption that \( a < 2\delta/(1 - \delta) \).

Case b. The equilibrium partition \( E_1 \) is not feasible but \( E_2 \) is feasible. This situation occurs when

\[ y \leq \frac{\delta(1 + a)}{(1 + \delta)} \] and

\[ \frac{(1 + a)}{(1 + \delta)} > a - y > \frac{\delta(1 + a)}{(1 + \delta)}, \quad (3) \]
FIGURE 2
which implies that

\[ y < \min\{\delta(1 + a)/(1 + \delta); (a - \delta)/(1 + \delta)\}. \quad (4) \]

**Lemma 2.** For a partition \( y \) of the first pie that satisfies condition (4) the equilibrium partition of the second pie is

\[ x_1^*(y) = 1; \quad x_2^*(y) = \max\{0; \delta - y(1 - \delta)\}. \]

**Proof.** First note that, as long as \( y \leq \delta/(1 - \delta) \), the partition \( x_2^*(y) = \delta - y(1 - \delta) \) is in the feasible set \([M_1, M_2]\). Using our assumption that \( a < 2\delta/(1 - \delta) \), we obtain

\[ y < (a - \delta)/(1 + \delta) < [2\delta/(1 - \delta) - \delta]/(1 + \delta) = \delta/(1 - \delta). \quad (5) \]

To establish our claim note that

\[ x_2^*(y) = \arg \max_{x_2 \in \mathcal{X}_2}\{0 \leq 1 - x_2 \leq 1 \mid y + x_2 \geq \delta(1 + y)\} \quad (6) \]

and

\[ a - y + 1 - x_1^*(y) \geq \delta[a - y + 1 - x_2^*(y)], \quad (7) \]

which implies that the first player is willing to accept \( x_2^*(y) \) rather than to wait another period and get all of the second pie, and the second player prefers to get \( 1 - x_1^*(y) \) from the second pie rather than to wait another period and get \( 1 - x_2^*(y) \) of this pie.

**Case c.** \( E_1 \) is feasible while \( E_2 \) is not feasible. This situation occurs when

\[ (1 + a)/(1 + \delta) > y > \delta(1 + a)/(1 + \delta) \text{ and } a - y < \delta(1 + a)/(1 + \delta). \]

This case is similar to Case b. We can thus use symmetry arguments to conclude that in this case

\[ x_1^*(y) = 1 - [\delta - (1 - \delta)(a - y)] = (1 - \delta)(1 + a - y); \quad x_2^*(y) = 0. \]

**Case d.** Both \( E_1 \) and \( E_2 \) are not feasible. Such a situation occurs when

\[ y > \delta(1 + a)/(1 + \delta) \text{ and } a - y > \delta(1 + a)/(1 + \delta), \]

which yields

\[ \delta(1 + a)/(1 + \delta) < y < (a - \delta)/(1 + \delta). \quad (8) \]

Our previous analysis (see proof of Lemma 1) indicates however that such a situation may occur only when \( a > 2\delta/(1 - \delta) \). Using this inequal-
ity and (8), one can easily show that \( y > \delta(y + 1) \) and \( a - y > \delta(a - y + 1) \), which implies that in Case d the equilibrium of the bargaining game on the second pie is such that each player demands and receives all of the second pie, i.e., \( x_1^*(y) = 1, \ x_2^*(y) = 0 \).

**Case e.** \( y < \delta(1 + a)/(1 + \delta) \) and \( a - y > (1 + a)/(1 + \delta) \). The analysis of this case is similar to that of Case b, thus

\[
x_1^*(y) = 1; \quad x_2^*(y) = \delta - y(1 - \delta).
\]

**Case f.** \( y > (1 + a)/(1 + \delta) \) and \( a - y < \delta(1 + a)/(1 + \delta) \). This case is completely symmetric to Case e and thus the equilibrium partitions are

\[
x_1^*(y) = (1 - \delta)(1 + a - y); \quad x_2^*(y) = 0.
\]

The above analysis indicates that when \((a - \delta)/(1 + \delta) \leq y \leq \delta(1 + a)/(1 + \delta)\) the final shares of the two pies together are not affected by the partition of the first pie and are identical to the equilibrium partition of the simultaneous bargaining game. However, when the second player gets a big piece from the first pie, i.e., \(0 \leq y < (a - \delta)/(1 + \delta)\) (Cases b and e), in the bargaining on the second pie he becomes impatient enough and prefers to get none of this pie rather than to wait another period and become the first mover. A similar situation occurs when \(\delta(1 + a)/(1 + \delta) < y \leq a\) (Cases c and f) in which the first player gets most of the first pie. Thus once \(a < 2\delta/(1 - \delta)\), as we assume, we have three possible ranges of \(y\) that we need to consider.

We now proceed and discuss the bargaining on the first pie and the final equilibrium allocation of both pies.

**Proposition 2.** When players bargain on the large pie first the equilibrium partitions, depending on the identities of the first movers, are as follows:

<table>
<thead>
<tr>
<th>First pie</th>
<th>Second mover</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mover</td>
<td>(B_1 = \frac{1}{1 + \delta^2}[(1 + a)](1 + \delta))</td>
</tr>
<tr>
<td>Second mover</td>
<td>(B_2 = \frac{1}{(1 + \delta^2)}[(1 + a)](1 + \delta))</td>
</tr>
</tbody>
</table>

**Proof.** Each player in this bargaining game is aware of the way the partition of the first pie determines the equilibrium of the bargaining game on the second pie. We assume that the first mover is selected randomly by
a \(\frac{1}{2}, \frac{1}{2}\) lottery. Letting \(R_i(y)\) be the \(i\)th player's expected share from both pies as a function of the partition of the first pie (i.e., \(R_1(y) = y + \frac{1}{2}(x_1^*(y) + x_2^*(y))\) and \(R_2(y) = a - y + \frac{1}{2}(1 - x_1^*(y) + 1 - x_2^*(y))\) yields

\[
R_1(y) = \begin{cases} 
(1 + \delta)(1 + y)/2, & 0 \leq y \leq (a - \delta)/(1 + \delta) \\
(1 + a)/2, & (a - \delta)/(1 + \delta) < y \leq \delta(1 + a)/(1 + \delta) \\
\frac{1}{2}[(1 + a)(1 - \delta) + (1 + \delta)y], & \delta(1 + a)/(1 + \delta) < y \leq a 
\end{cases}
\]

(9)

\[
R_2(y) = \begin{cases} 
[2a + (1 - \delta) - (1 + \delta)y]/2, & 0 \leq y \leq (a - \delta)/(1 + \delta) \\
(1 + a)/2, & (a - \delta)/(1 + \delta) < y \leq \delta(1 + a)/(1 + \delta) \\
\frac{1}{2}[(1 + a)(1 + \delta) - (1 + \delta)y], & \delta(1 + a)/(1 + \delta) < y \leq a. 
\end{cases}
\]

(10)

Note however that unlike in the simultaneous bargaining case, in this case players get their share in the second period so the expected equilibrium utilities are \(\delta R_i(y)\).

Having the payoff functions \(R_i(y)\) we can now resort to standard techniques to find the equilibrium of the bargaining game on the first pie.

\(R_i(y)\) is constant for \(y \in [(a - \delta)/(1 + \delta); \delta(1 + a)/(1 + \delta)]\). Thus any pair of strategies that suggest partitions in this region cannot be an equilibrium as they both yield for each player the expected share of \((1 + \delta)/2\), and if one player demands such a share the other player can use his impatience and offer his opponent a little less, which implies an offer not in the above region.

In a similar way we can eliminate the possibility of having an equilibrium in which the first player offers a partition not in \([\delta(1 + a)/(1 + \delta), a]\) or the second player offers a partition not in \([0, (a - \delta)/(1 + \delta)]\).\(^3\) Now let \(\alpha^* = (1 + \delta^2)(1 + a)/(1 + \delta)^3\) and \(\beta^* = (2\delta a - \delta^2 - 1)/(1 + \delta)^2\). Note that \(\delta(1 + a)/(1 + \delta) < \alpha^* < a\) and \(0 < \beta^* < (a - \delta)/(1 + \delta)\). Straightforward calculation indicates that \((\alpha^*, \beta^*)\) is the unique solution of the character-

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\(^3\) Although all these possibilities can be eliminated directly by showing that there are no such equilibria, we can also use symmetry arguments and claim that every pair of strategies in which offers are not in the middle region \([(a - \delta)/(1 + \delta), \delta(1 + a)/(1 + \delta)]\) or in the regions suggested above will break the symmetry between the two players.
istic equations

\[ R_1(\beta^*) = \delta R_1(\alpha^*) \]  \hspace{1cm} (11) \\
\[ R_2(\alpha^*) = \delta(R_2(\beta^*)). \]  \hspace{1cm} (12)

Thus the unique subgame perfect equilibrium of the bargaining on the first pie is for the first player to offer \( \alpha^* \) but to accept any offer that gives him at least \( \beta^* \), while the second player offers \( \beta^* \) but accepts any offer that gives him at least \( a - \alpha^* \). The final partition of both pies can now easily be calculated from the above equilibrium.

2.4. \textit{Comparison of the Two Agendas}

We discuss above two possible agendas. In the first one players bargain on the small pie first and in the second case they start by bargaining on the large pie.\(^4\) Comparing the two agendas implies the following:

(i) Since \( A_1 = A_3 \) and \( A_2 = A_4 \) when the players bargain on the small pie first, the lottery that determines the first mover in the bargaining on the first pie does not affect the final partition at all. When players bargain on the large pie first, it is the first lottery which is the more important one. Simple calculation indicates that \( B_1 > B_2 > B_3 > B_4 \); thus, if the first player is the second mover in the bargaining on the first pie he will be worse off regardless of the outcome of the second lottery.

(ii) Comparing the payoff distributions for the two possible agendas yields that since \( B_1 > A_1 \) and \( B_4 < A_4 \) the highest payoffs that the second agenda might yield are higher than the best that the first agenda might yield. At the same time if a player is the second mover in the bargaining on both pies, he will be better off with an agenda in which the smaller pie is discussed first.

(iii) Since \( \sum_{j=1}^{4} A_j = \sum_{j=1}^{4} B_j = \delta(1 + a)/2 \) the agenda does not affect the players' expected share.

3. \textbf{Opposite Evaluations and the Importance of the Agenda}

In many bargaining situations players do not assign the same importance to all the issues under negotiation. Some issues might be important to one group of players while other players might consider the other issues as the important ones. In such situations, the bargaining game

\(^4\) Note that we assume that \( a > 1/\delta \). Thus an attempt to find out the source of discontinuity by investigating the outcome of the bargaining game as \( a \to 1 \) is illegitimate in our formulation.
becomes more interesting since players have the option to give up on the issues which are less important to them and, in return, to get a favorable settlement on the issues which are important to them.

In this section we consider a two-player, two-pie bargaining game. We assume that each player evaluates the size of these pies differently. The first player regards the first pie as the important one while the second player regards the second pie as the important one. We assume that the different evaluations are common knowledge. Let \((y, x)\) be a partition of the two pies such that the first player receives \(y\) percentage of the first pie and \(x\) percentage of the second pie. Given \((y, x)\) we assume that the utility functions of the two players are

\[
\begin{align*}
    u_1(y, x) &= (ay + x)\delta^t \\
    u_2(y, x) &= (1 - y + a(1 - x))\delta^t,
\end{align*}
\]

where \(\delta\), \(t\), and \(a > 1/\delta\) are defined as in the previous section.

3.1. Multiissue Sequential Bargaining with Agenda

Let the players bargain on the two pies sequentially such that they first bargain on the pie which is more important to the first player. Any such agenda introduces an asymmetry between the two players. The main question is of course whether there is a player that benefits from such an asymmetry.

When players have opposite evaluation of the pies we cannot use our previous technique and analyze the bargaining on the second pie as a bargaining on the partition of both pies under the constraint that the final utility of each player will not be below his utility from his share of the first pie. Note that now each point on the efficient frontier of the bargaining set describes a unique allocation of the two pies. Moreover once players agree on a certain partition of the first pie there is only one allocation on the efficient frontier that they can reach by allocating the second pie. Every other partition implies that they end up in an inefficient point inside the bargaining set. For example, for every partition of the first pie \(y < 1\) the only partition of the second pie that leads to an efficient outcome is to give all of it to the second player. Indeed, as we prove later, the equilibrium of such a bargaining game is not necessarily efficient; i.e., the equilibrium utility combination is not on the Pareto frontier and players can benefit by trading among themselves their shares of the different pies.

**Proposition 3.** In a multiissue bargaining game the players expected equilibrium utilities depend on the agenda of the bargaining. In the two-pie bargaining game discussed here each player prefers an agenda such that the first pie to bargain on is the one which is the least important to him but the most important to his opponent.
Proof. Given a partition \( y \) of the first pie let

\[
x^*(y) = \begin{cases} 
\frac{[1 + a - (a^2 + 1)y]/a(1 + \delta)}{(1 - \delta)(1 + a)/a}, & y < \delta(1 + a)/(a^2 + \delta) \\
\frac{[\delta(1 + a) - (a^2 + \delta)y]/a(1 + \delta)}{(1 - \delta)(1 - y + a)/a}, & y \geq \delta(1 + a)/(a^2 + \delta)
\end{cases}
\]

(14a)

\[
z^*(y) = \begin{cases} 
\frac{[\delta(1 + a) - (a^2 + \delta)y]/a(1 + \delta)}{(1 - \delta)(1 + a)/a}, & y < \delta(1 + a)/(a^2 + \delta) \\
0, & y \geq \delta(1 + a)/(a^2 + \delta).
\end{cases}
\]

(14b)

Claim 1. The unique subgame perfect equilibrium of the bargaining on the second pie is for the first player to offer \( x^*(y) \) and to accept any offer that gives him at least \( z^*(y) \) and for the second player to offer \( z^*(y) \) and to accept any offer that gives him at least \( 1 - x^*(y) \) percentage of the second pie.

Proof. One can easily verify that the suggested strategies are the unique solution of the characteristic equations.

Claim 2. The unique equilibrium of the bargaining game on the first pie is that the first player's offer is that he will get all of the first pie but he accepts any offer that gives him at least \( q^* \) percentage of it, and the second player offers the partition \( q^* \) but he accepts any offer where

\[
q^* = \frac{[2a^2\delta - (1 - \delta) - a(1 - \delta)^2]/(2a^2 - 1 + \delta)}{[2a^2\delta - (1 - \delta) - a(1 - \delta)^2]/(2a^2 - 1 + \delta)}.
\]

(15)

Proof. Let \( R_i(y) \) be the \( i \)th player expected utility from both pies as a function of a partition \( y \) of the first pie. Using the equilibrium strategies identified in Claim 1 and keeping in mind that the first mover is selected randomly yields

\[
R_1(y) = \begin{cases} 
\frac{\delta(1 + a)y/2a + \delta(a^2 - 1)y/2a}{\delta(1 - \delta)(1 + a)/2a + \delta(2a^2 - 1 + \delta)/2a}, & 0 \leq y \leq \delta(1 + a)/(a^2 + \delta) \\
\frac{\delta(1 + \delta)(1 + a - y)/2}{\delta(1 + \delta)(a + 1 - y)/2}, & 1 \geq y \geq \delta(1 + a)/(a^2 + \delta)
\end{cases}
\]

(16a)

\[
R_2(y) = \begin{cases} 
\frac{\delta[a + 1 + (a^2 - 1)y]/2}{\delta(1 + \delta)(a + 1 - y)/2}, & 0 \leq y \leq \delta(1 + a)/(a^2 + \delta) \\
\frac{\delta(1 + \delta)(a + 1 - y)/2}{\delta(1 + \delta)(a + 1 - y)/2}, & 1 \geq y \geq \delta(1 + a)/(a^2 + \delta)
\end{cases}
\]

(16b)

Note that for \( 0 \leq y \leq \delta(1 + a)/(a^2 + \delta) \) the utilities of both players are increasing functions of \( y \). Thus, in discussing the bargaining game on the
first pie, giving an offer in this range cannot be a part of an equilibrium behavior, as a player that gives such an offer can clearly benefit from offering a higher $y$. Thus in the bargaining on the first pie both players offer partitions in $[\delta(1 + a)/(a^2 + \delta), 1]$. Now observe that $q^*$ is in this range and that long but simple calculations, which are omitted here, indicate that the suggested strategies are the unique subgame perfect equilibrium of the bargaining on the first pie.

Going back to Proposition 3 we can now use (16a, b) and Claim 2 to compute the players’ expected equilibrium utility:

$$E_{u_1} = \delta(2a + 1 - \delta)(1 + \delta)/4$$  \hspace{1cm} (17)

$$E_{u_2} = (\delta a(1 + \delta)/4)[(4a^2 - 1 + 2a - 2a\delta + \delta^2)/(2a^2 - 1 + \delta)].$$  \hspace{1cm} (18)

Comparing the two yields that $E_{u_2} > E_{u_1}$. ■

The above proposition indicates that in the two-issue conflicting evaluation bargaining game there is an advantage to the second player who evaluates the second pie more. By letting the first player have a large proportion from the first pie he can make the first player more impatient in the bargaining on the second pie. Impatience gives the second player an advantage in the bargaining on the second pie, the one which is important to him. As an example, we can see that, at the equilibrium, when the second player is the first mover in the bargaining on the second pie, he suggests that he will get all of it and the first player is too impatient to object. Also note that Eqs. (16a, b) indicate that for every $0 \leq y \leq \delta(1 + a)/(a^2 + \delta)$, the utility of the second player is an increasing function of the share of the first player from the first pie. When the players have identical evaluations regarding the size of the pies, the random selection of the first mover implies that players are symmetric and thus they have identical expected utility. But when players have conflicting evaluations, the random selection of the first mover does not imply symmetry. The source of such symmetry is the agenda which determines the sequence of pies over which the players bargain. Assuming that the agenda is selected randomly implies that players are symmetric and thus obtain identical expected utility from the bargaining game.

Comparing the bargaining with and without agenda yields that besides the asymmetry that the agenda introduces and the delayed realizations of the partition which affects the players’ equilibrium utilities, the existence of an agenda can introduce another source of inefficiency.

**Proposition 4.** When players have opposite evaluation regarding the size of the pies, the equilibrium allocation of the two pies is not necessarily on the efficient frontier and players can benefit from trading after the bargaining is over.
Proof. When the second player is the first mover in the bargaining on the first pie and the first player is the first mover in the bargaining on the second pie, Claims 1 and 2 indicate that at the equilibrium the second player has a share of the first pie while the first player has a share of the second pie, which implies that each player has a share of the pie that is important to his rival. Clearly the players can improve on such an allocation by trading. ■

References


