1. Introduction

The influence of government bonds and social security on the perceived net wealth of households is a problem that has been extensively discussed in the literature. Robert Barro [1] reexamined the problem and presented a model in which under reasonable conditions government bonds are not perceived by households as an addition to their net wealth. It was due to Barro’s paper that the notion of “Ricardian equivalence” started to play such an important role in economic thought (for recent survey see Bernheim [2]). Barro’s conclusion is based on the existence of intergenerational transfers and on the assumption that the utility of members of the i’th generation depends not only on their own consumption but also on the utility of their immediate heirs: Under these conditions Barro argues that, though government bonds create wealth for the current generation, they also create liabilities for the future generation, yielding zero net wealth for the current generation.

The purpose of this paper is to provide a graphic presentation of Barro’s results. Using this we prove that any criticism of Barro’s results (see for example Feldstein [7], Buchanan [4]) must involve shifting the individuals’ opportunity curve. We illustrate this argument by showing that if individuals differ in the number of their heirs, Barro’s result does not hold. That is to say that when individuals differ in the number of their heirs, some of them respond to government bonds issue by changing their consumption. These changes do not necessarily cancel one another so we can conclude that under such assumptions, government bonds may alter aggregate consumption.

2. The Graphic Description of Barro’s Results

The framework of Barro’s analysis is an intertemporal model with overlapping generations. Each individual lives two periods (young and old). It is assumed that the utility of each individual is a function of his own consumption and also an increasing function of the utility of his immediate successor. A member of the i’th generation earns w while young and allocates it between current consumption $c_i^y$ and a transfer to the second period. The total assets that a member of the i’th generation has in his second period is $A_i^o + A_{i-1}^o$ where $A_i^y$ is the transfer from the first period and $A_{i-1}^o$ is the bequest received from the i — 1 generation. The problem facing a member of the i’th generation is to choose $C_i^y$ — consumption while young, $C_i^o$ — consumption while old and the bequest provision $A_i^o$ in order to maximize his utility and given the budget constraints

\[ (i) \quad w = C_i^y + (1 - r)A_i^y \]

\[ (ii) \quad A_i^y + A_{i-1}^o = C_i^o + (1 - r)A_i^o \]

where $r$ is the rate of returns on assets. Clearly, by using the above budget constraints, the problem that each individual faces can be reduced to the optimal choice of $C_i^y$ and $A_i^o$.

The point made by Barro is better illustrated by considering the problem facing each individual in his second period. Given the assets of $A_i^y + A_{i-1}^o$, he has to decide how to allocate these assets between consumption ($C_i^o$) and net bequest ($((1 - r)A_i^o$). The relevant opportunity set is described in Figure 1. Barro’s key restriction is that the bequest must be nonnegative which means that the old generation does not have the option to borrow from its successors. The optimal consumption-bequest point will be the tangency point between an indifference curve and the opportunity curve. Clearly we might have a corner solution in which there is no bequest and in which the indifference curve does
not have the slope of $-1$ at the zero bequest point. However Barro's assumption of positive bequest rules out this possibility.

Since consumption, as well as bequest are measured in dollars, the slope of the opportunity curve is negative one. Consequently the marginal rate of substitution between consumption and bequest, at the optimal point $E$, is also negative one. This is clearly not a surprise since the individual is free to transfer money from consumption to bequest and vice versa. The choice of positive bequest implies that at the optimal point $E$ the individual evaluates identically an additional dollar of consumption and an additional dollar of net bequest.

Adding government bonds to the model enables the old generation to leave debts to its successors and therefore there are changes in the opportunity sets. Assuming that each member of the $i$'th generation receives the quantity $B$ of bonds it is now possible for him to consume more than $A_i^0 + A_{i-1}^0$. Since, after the bond issue, the total assets the individual has (while old) is $A_i^0 + A_{i-1}^0 + B$, he can consume it all. In this case his net bequest will be negative, and his heirs will have to pay the principal $B$ and the interest $rB$ as a tax. Thus, if prior to the bonds issue the actual bequest that the $i+1$ generation receives as $A_i^0$ is $(1 + r)(A_i^0 + A_{i-1}^0)$ or when the net bequest is at its minimum values of $-B$ the $i+1$ generation will have to pay $(1 + r)B$.

Clearly a member of the $i$'th generation who had selected a positive bequest prior to the government bonds issue will remain at the same point after the bonds issue (viz., the tangency point $L$ at which he maximize his utility). Therefore, we need not assume that the members of the $i$'th generation have identical tastes, but that they all wish to leave positive bequest prior to the bonds issue. In that case, Barro's result holds and bonds are not perceived as net wealth. The intuition of this claim is clear from Figure 2. The government bonds issue adds irrelevant opportunities and thus does not affect the individual's consumption-bequest choice.

On the other hand, as Barro pointed out, if a member of the $i$'th generation was initially at a corner point, such as point $M$ in figure 2, in which $A_i^0 = 0$ then after the government bond issue this individual can increase his utility by moving from point $M$ to point $P$. From the quasiconcavity of the utility function we know that unless the indifference curve is tangent to the opportunity set at point $M$, there are points on $MN$ that this individual prefers to point $M$. Therefore as a result of the new options arising from the government bond issue this individual will increase his consumption and leave a negative bequest to his heirs.

Finally, the only role of government bonds in this model is to provide a means by which the
old generation can borrow from its successors. If the old generation already has the option to leave debt to its descendants by some other means, the government bonds issue will not affect the opportunity curve and therefore the consumption level will remain unchanged.

### 3. Shifts of the Opportunity Curve

Suppose now that the government bond issue causes a shift of the opportunity curve, for example, when the future tax liabilities are not fully capitalized (Buchanan, 1976), which implies that the opportunity curve shifts outside. Using the graphical analysis above, it is clear that if the bonds issue causes a shift of the entire individual’s opportunity curve, Barro’s results do not hold and the government bonds are perceived as net wealth.

We illustrate this possibility by considering the case in which there are three kinds of individuals in the i'th generation, a, b and c, and the number of their heirs are respectively higher, equal and less than the average number of heirs. In order to avoid discussion about economic growth we assume that the numbers of heirs are respectively 1 + n, 1 and 1 - n where 0 ≤ n ≤ 1. Assuming that the future tax liabilities will be financed by a poll tax, each member of the i + 1 generation will pay equal tax. Therefore it is evident that the taxes that the heirs of a, b and c will have to pay are (1 + r)(1 - n)B, (1 + r)B, (1 + r)(1 - n)B respectively which implies that the net present value of the taxes that the heirs of a, b and c will pay are respectively higher, equal or less than (1 + r)B.

In figure 3 the three opportunity sets are depicted. The introduction of government bonds does not cause a shift of the opportunity curve of individual b. If prior to the bond issue individual b was in point L, the government bonds issue does not alter his consumption-bequest choice. This result does not hold however for individuals a and c. The net liabilities of the heirs of type a (type c) derived from the bonds issue are higher (less) than B. Thus for every level of consumption C^\circ_i the possible bequest is less (higher) than prior to the bonds issue. This result can be described by shifting the opportunity curve. For example, the maximum net bequest that type a can give to his heirs after the bonds issue is A^\circ_i + A^\circ_{i-1} - nB while type c can give $A^\circ_i - (1 - n)B$.

**FIGURE 3**

The maximum consumption while old, C^\circ_i, that type a (type c) can consume remains the same, $A^\circ_i + A^\circ_{i-1} + B$, but if type a (type c) consumes that much the net bequest that he leaves to his heirs is $-(1 - n)B$ for type a and $-(1 - n)B$ for type c. The new opportunity curves are depicted in Figure 3. For type a the opportunity curve shifts inward (curve a), and for type c the opportunity curve shifts outward (curve c).

If type a chooses point L prior to the government bonds issue, then after the bonds issue this point is not feasible, as the relevant opportunity curve shifts downward from curve b to curve a. Thus type a has to choose a new point according to his new opportunity set. Assume that he chooses point L'; this means that he chooses to reduce his consumption to C^\circ_i. Similarly the relevant opportunity curve for type c is curve c so if he chooses point L'' after the bonds issue, it implies that there is an increase in his consumption.

Clearly when some individuals increase their consumption while others decrease it there is no way to determine the total change in aggregate consumption. Therefore the bonds issue may alter total consumption and accumulation of private capital. The direction of change depends on individuals’ preferences as well as on the distribution of types.

To illustrate this problem let us consider a simple example. Consider three different individuals. The assets available to each one of them is 10. We assume that $n = 0.5$, so we can think of the individuals as one couple with three children, another couple with two children and a
third couple with one child. The utility of each individual (or couple) is assumed to be $U_i(C_i, N_i) = (C_i^{3/2} + 2N_i^{1/3}$ where $N_i$ is the net bequest provision. When $B = 0$, i.e. there is no government bonds issue, all three individuals consume $c_i^o = 7.3$. But now let us assume that $B = 10$. Each individual is maximizing his utility function subject to a different budget constraint. Since the number of heirs of the first individual is above the average the share of his heirs in the future tax liability is above his own share of the bonds windfall. Simple optimization yields that for the first individual $C_1^o = 3.2$, for the second $C_2^o = 7.3$ and for the third $c_3^o = 11.6$. Thus it is evident that in the case under discussion individuals change their consumption as a result of government bonds issue. Also note that in this example there is a small change in the aggregate consumption.

6. Summary

Barro's result that government bonds are not perceived by households as an addition to their net wealth is described and proved graphically in this work. Using this graphic analysis we claim that any shift of the opportunity set, derived for example, from incomplete capitalization of future tax, will alter Barro's results. As an example we analyze the case of nonidentical numbers of heirs. We show that in this case the bonds issue may cause a shift of the opportunity curve. As a result of this shift individuals respond to government bonds issue by changing their consumption. Therefore in this case government bonds issue may alter aggregate consumption and accumulation of private capital.

Notes
1. For early discussions on the problem see: Buchanan [3], Modigliani [9], Diamond [5] and Kochin [8].
2. As initially presented by Samuelson [10].

References