Learning by Doing, Inventory and Optimal Pricing Policy*

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The notion of the "learning effect" suggests that when a firm introduces a new product, the costs of production will decline as the accumulated output increases. In this paper we analyze this effect on price paths and production rates along time. The distinction between production level and sales level affects the behavior of the firm that can utilize the option of holding inventory. In this case we show that the optimal price increases, even though production costs decrease over time due to "learning by doing."

1. Introduction

The notion of the "learning effect" suggests that when a firm introduces a new product (or technology), the costs of production will decline as the accumulated output increases. Empirical evidence of this phenomenon can be observed in many industries (see for example Alchain 1963 and Hedley 1976). In analyzing the effect of learning by doing (LBD) on prices and production rates, it is usually assumed that production and sales rates are identical (see Spence 1981 and Fershtman and Spiegel 1983).

Spence, for example, proved that in the case of nonstorable output, the nonmyopic firm produces an output above that of short-run profit maximization in order to gain experience and reduce costs of production. The firm equates the marginal revenue with the full long-run marginal cost, taking into account the long-run learning effect of an additional unit of output.

Discussion of the firm's optimal strategy raises two questions that will be the main focus of our paper. First, What is the resulting price path of such strategy? Does the optimal price necessarily decline over time or can it fluctuate or even increase monotonically? Second, if we allow the firm to hold inventory, will the firm utilize this option?

Investigation of the optimal pricing policy in the presence of a learning effect generates ambiguous results. Although intuition may suggest that prices will fall over time due to the learning effect, we prove in this paper that changes in the optimal

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price over time cannot be specified a priori. The optimal price may go up, go down, or fluctuate over time, depending on the cost function and on the specific shape of the learning effect.

The intuition behind this result is that there are two factors that affect the behavior of prices over time: on the one hand, falling marginal cost will encourage the firm to reduce prices over time, and on the other hand, the incentives that the firm has to produce beyond the short-run profit maximization output level are also changing over time. Because it is not clear which of these two effects is stronger, we cannot conclude whether sales and prices are going up or down over time.

The traditional argument for maintaining inventory is the existence of a random demand function (see Arrow et al. 1958 and Peterson 1979). Firms use inventory to absorb short-run shocks in demand and to speculate on future price changes. These models usually assume static cost functions that do not change over time, and, therefore, in case of a fully known deterministic demand function, which remains fixed over time, the theory does not indicate any reason for holding inventory. A recent attempt to fill this void was undertaken by Arvan and Moses (1982), who pointed out that when there are scale economies, firms can increase their profits by adopting a dynamic strategy that involves holding inventory. In the first time interval, the firm produces in excess of current sales and accumulated inventory. In the second time interval, the firm stops its production and during this period sales are made from inventory.

Introducing LBD can contribute to the study of inventory as a strategic variable in the case of a monopoly firm. Because the LBC effect implies that production costs decline with accumulated output, the monopoly (hereafter the "firm"), has incentives to increase its production beyond the short-run profit-maximization output. By so doing, the firm pushes the market price down and reduces its profit. But when the product is storable, the firm does not have to sell its extra output. The current production level may differ from the current sales level, so the firm can increase its output, store part of it, and sell that in future periods. By comparing the dynamic strategies with and without inventory, we will prove that when production is incorporated with LBD, holding inventory can be optimal.

Holding inventory will also affect the behavior of prices and sales over time. We will show that the existence of inventory guarantees that even though production costs decrease over time due to LBD, the optimal price increases in the period during which inventory is positive.

2. Optimal Pricing Policy for Nonstorable Goods

Let \( q(t) \) denotes the firm's output at time \( t \), and let

\[
Q(t) = \int_0^t q(t) \, dt
\]

be the accumulated output. According to the LBD assumption, the cost function can be written as \( C(q, Q) \), which is assumed to be twice continuously differentiable with the derivatives \( C_q(q, Q) > 0 \) and \( C_Q(q, Q) < 0 \). We assume that \( C_{qq} > 0 \), which can be interpreted as diminishing marginal learning. If \( R(q) \) denotes the revenue
function, the instantaneous profit function can be written as

$$\pi(q, Q) = R(q) - C(q, Q),$$

which is assumed to be concave with respect to both arguments.

The problem that the monopolist faces is to find a production time path that will maximize his discounted stream of profits, that is

$$\max \int_0^\infty e^{-rt} \pi(q(t), Q(t)) \, dt$$

subject to

$$\dot{Q}(t) = q; \ \dot{Q}(0) = 0,$$

where $r$ denotes the discount rate and $\dot{Q} = dQ/dt$.

This problem can be easily solved by applying standard maximization techniques to this optimal control problem. Define the Hamiltonian as follows

$$H(t, Q, \lambda, q) = e^{-rt} \pi(q, Q) + \lambda q.$$  

The concavity of the profit function $\pi(q, Q)$ guarantees the existence of an optimal path (see Baum 1976). The necessary conditions for optimality are given by

$$e^{-rt} \pi_q(q, Q) + \dot{\lambda}(t) = 0$$  

and

$$-e^{-rt} \pi_Q(q, Q) = \dot{\lambda}(t).$$

In addition to (5) and (6), the transversality condition for control problems with infinite horizons (proven by Michel 1982), is that the discounted Hamiltonian vanishes as $t$ approaches infinity.

Integrating (6) yields

$$\lambda(t) = \lambda(0) - \int_0^t e^{-r\tau} \pi_Q(q(\tau), Q(\tau)) \, d\tau.$$  

Using the transversality condition $\lambda(\infty) = 0$, which is sufficient for $H \to 0$ as $t \to \infty$, we obtain

$$\lambda(0) = \int_0^\infty e^{-r\tau} \pi_Q(q(\tau), Q(\tau)) \, d\tau.$$  

Substituting (8) into (7) yields

$$\lambda(t) = \int_t^\infty e^{-r\tau} \pi_Q(q(\tau), Q(\tau)) \, d\tau.$$  

Substituting (9) into (5) yields

$$\pi_q(q, Q) + \int_0^\infty e^{-r\tau} \pi_Q(q(\tau), Q(\tau)) \, d\tau = 0.$$  

The economic intuition of this condition is that the firm should produce beyond the
short-run profit-maximization output and take into consideration the present value of
the total profit increase of an additional unit of output for all future periods.

Using the above necessary conditions we can investigate the optimal output and
price policy. Following equation (5) we obtain

$$R_q(q) - C_q(q, Q) + \bar{\lambda}(t) = 0,$$

(11)

where $\bar{\lambda}(t)$ denotes the current value of the costate variable; i.e., $\bar{\lambda}(t) = e^{\eta t} \lambda(t)$. If we
denote $\eta$ as the demand elasticity and replace $R_q$ with $P(1 - 1/\eta)$, we can after
simple manipulation rewrite Equation (11) as

$$P(t) = \frac{\eta}{\eta - 1} (C_q(q, Q) - \bar{\lambda}).$$

(12)

Equation (12) resembles the monopolistic optimal pricing rule. The firm has to equate
the marginal revenue with the real marginal cost, which is $C_q(q, Q) - \bar{\lambda}$.

3. The Optimality of Holding Inventory under the LBD
Assumption

The optimal path $\hat{q}(t)$ that satisfies condition (10) is found under the assumption that
production and sales are identical. The possibility of holding inventory implies,
however, that the firm has another degree of freedom. But it still remains to be proved
that the firm can benefit from using an inventory policy.

We will demonstrate the optimality of holding inventory with a two-step
procedure. First we present three conditions such that if the optimal output path ($\hat{q}(t)$)
satisfies one of these conditions, then the firm can increase its profit by using
inventory policy. Second (see Section 5) we show that there are cost functions for
which the optimal production path satisfies one of these conditions. Thus, when the
firm faces such a cost function, it can benefit from using inventory policy.

Consider an optimal path $\hat{q}(t)$ such that there are $t_1$ and $t_2$; $t_2 > t_1$ for which

$$- C_q(\hat{q}(t_1)) + e^{-r(t_2-t_1)} C_q(\hat{q}(t_2))$$

$$- \int_{t_1}^{t_2} e^{-r(t_2-\tau)} C_q(\hat{q}(\tau), \dot{\hat{q}}(\tau)) \, d\tau > \int_{t_1}^{t_2} \delta e^{-r(t_2-\tau)} \, d\tau,$$

(13)

where $\delta > 0$ is the storage cost per unit of inventory.

Inequality (13) implies that the firm can benefit from changing its production plan
$\hat{q}(t)$ even without changing its sales path. For example, the firm can produce an
additional unit at $t = t_1$, hold this unit as inventory until $t_2$, and at $t = t_2$ the firm can
produce $\hat{q}(t_2) - 1$, but by using its inventory the firm can continue to sell $\hat{q}(t_2)$.
Because these changes do not imply any changes in the sales path, there is no loss of
revenues. According to (13) however, by making these changes the firm reduces its
production cost. The first expression in (13) describes the additional cost of
producing another unit at $t = t_1$. The second expression describes the discounted
saving from not producing the last unit at $t = t_2$. The third expression in (13)
describes the present value of all the reduction in cost between $t_1$ and $t_2$ due to
additional learning. The right-hand side of (13) describes the present value of the
storage cost of one unit from $t = t_1$ until $t = t_2$. Therefore, if the savings in
production cost from the suggested changes in the production plan are greater than the storage cost, as condition (13) suggests, the firm can increase its profit by altering its production path and holding inventory.

In the same way we can construct a situation in which both production and sales will be changed. If there are $t_1$, $t_2$; $t_2 > t_1$ such that the path $q(t)$ satisfies

\[ -C_q(q(t_1), \dot{q}(t_1)) + e^{-r(t_2-t_1)} R_q(q(t_2)) - \int_{t_1}^{t_2} e^{-r(t_2-t_1)} C_q(q(t), \dot{q}(t)) \, dt \geq \int_{t_1}^{t_2} \delta e^{-r(t_2-t_1)} \, dt, \]  

the firm can increase its profits by producing an extra unit at $t = t_1$, hold it as inventory until $t = t_2$, and then sell it. In this case both production and sale paths are changed.

In a similar way, if $\hat{q}(t)$ satisfies

\[ -R_q(q(t_1)) + e^{-r(t_2-t_1)} R_q(q(t_2)) > \int_{t_1}^{t_2} \delta e^{-r(t_2-t_1)} \, dt, \]  

the firm can increase its profits by using inventory to change its sales path. By selling one unit less at $t = t_1$, holding this unit as inventory until $t = t_2$, and then selling it, the firm can earn additional profit.

From the discussion above we conclude that if the optimal production path $q(t)$ satisfies any one of the conditions in Equations (13) through (15), the firm can benefit by using the option of holding inventory. Holding inventory means that it is optimal for the firm to suffer a decrease in its short-run profits in order to gain higher profits in the long run. Thus, an increase in the interest rate will decrease the likelihood that the firm will find it optimal to hold inventory. From the continuity of conditions (13) and (15), however, we can conclude that the above result holds for some small $r > 0$.

It is important to note that once the optimal output is changed, the production costs at every $t$ are changed, and thus the whole production and sales paths will change.

4. The Pattern of Price Adjustment when the Firm Holds Inventory

The existence of a positive level of inventory can provide us with some information about the changes of prices over time.

Consider the monopolistic maximization problem under the assumption that the firm may hold inventory. Let $I(t)$, $q(t)$, and $s(t)$ be the inventory, production, and sales rates, respectively, and let $\delta$ denote the storage cost. The monopolistic problem is to maximize the present value of the stream of profit, subject to the constraint that the inventory level must be nonnegative; that is

\[
\max_{q(0), r(t)} \int_0^{\infty} \left[ R(s) - C(q, Q) + \delta \cdot I \right] e^{-\gamma t} \, dt,
\]

subject to

\[
I(t) = q(t) - s(t),
\]

\[
Q(t) = q(t),
\]
\[ I \geq 0 \text{ for all } t; \ I(0) = q(0) = 0, \]

where \( q(t) \) and \( s(t) \) are piecewise continuous. This problem can be solved by setting up the Hamiltonian

\[ H(t) = e^{-rt}[R(s) - C(q, Q) + \delta(I)] + \lambda_1(q - s) + \lambda_2q + \mu I, \]

where \( \lambda_i; i = 1, 2 \) are the costate variables and \( \mu \) is the Lagrangian multiplier. The first order conditions for this problem are:

\[ -C_q(q, Q)e^{-rt} + \lambda_1 + \lambda_2 = 0 \quad (17) \]
\[ R'(s)e^{-rt} - \lambda_1 = 0 \quad (18) \]
\[ \dot{\delta}e^{-rt} - \mu = \dot{\lambda}_1 \quad (19) \]
\[ C_Q(q, Q)e^{-rt} = \dot{\lambda}_2 \quad (20) \]
\[ I \geq 0; \ \mu \geq 0; \ \mu \cdot I = 0. \quad (21) \]

These conditions yield an optimal path of the control variables \( q(t) \) and \( s(t) \).

**Proposition 1.** The optimal price increases whenever there is a positive level of inventory. However, if there is zero inventory the price may increase, remain the same, or even decrease.

**Proof.** Differentiating (18) and substitute \( \lambda_1 \) into (19) yields

\[ \dot{\delta}e^{-rt} - \mu = -rR'(s)e^{-rt} + R''(s)se^{-rt}. \quad (22) \]

From equation (21) we learn that when the inventory level is positive \( \mu = 0 \). Therefore, changes of sales over time will be according to:

\[ R''(s)s = \delta + rR'(s) > 0. \quad (23) \]

Because \( R'(s) \) is assumed to be a decreasing function of \( s \), we conclude from equation (23) that \( s < 0 \). Because for every \( t \) the pair \((s(t), p(t))\) lies on the demand function, \( s < 0 \) implies that \( p > 0 \). Thus, even though production costs decline due to LBD, the optimal price goes up over time as long as there is a positive level of inventory.

Q.E.D.

5. Fixed Cost, LBD, and the Optimal Price Policy

In the previous section we found that when firms hold a positive inventory prices must rise over time. In this section we analyze the specific case of LBD that affects only the fixed cost of production function and leaves the variable costs unchanged. We show in this specific case that the optimal price path over time is positively sloped even if inventory level is zero. We develop for this specific case the conditions under which holding inventory is the optimal policy.

Assume that \( C(q, Q) = F(Q) + mq \), which implies that learning reduces the fixed cost, although the variable cost remains unchanged. This situation occurs, for example, when the learning is at the management level and managers learn how to operate the firm more effectively, how to avoid certain expenses, and how to get better deals on equipment. At the same time, variable costs mainly reflect the cost of raw materials, which remain unchanged.
Because, under this assumption, \( C_q(q, Q) \) remains constant over time, equation (12) indicates that \( \hat{\lambda} \) will determine the changes of real marginal cost over time.

**Proposition 2.** If the learning affects only the fixed cost while the marginal cost remains constant, the optimal price increases monotonically over time.

**Proof.** Because \( \hat{\lambda}(t) = e^{rt} \), changes of \( \hat{\lambda} \) over time can be written as

\[
\frac{d\hat{\lambda}}{dt} = (\lambda + 1/r\lambda)re^{rt}. \tag{24}
\]

Substituting \( \hat{\lambda} \) and \( \lambda \) by equations (6) and (9), respectively, yields

\[
\lambda + 1/r\lambda = -\int_{t}^{\infty} e^{-r\tau}F'(Q(\tau)) \, d\tau + 1/rF'(Q(t))e^{-rt}. \tag{25}
\]

Because we assume diminishing marginal learning, we obtain \( |F'(Q(t))| > |F'(Q(\tau))| \) for every \( \tau > t \). Moreover, \( 1/rF'(Q(t))e^{-rt} \) can be written as

\[
\int_{t}^{\infty} e^{-r\tau}F'(Q(t)) \, d\tau \quad \text{and thus}
\]

\[
\lambda + 1/r\lambda = \int_{t}^{\infty} e^{-r\tau}[F'(Q(t)) - F'(Q(\tau))] \, d\tau < 0. \tag{26}
\]

Because \( \hat{\lambda} \) decreases over time, we can conclude that although production costs decrease due to LBD, the optimal price increases over time. Q.E.D.

The intuitive appeal of this result can be better understood by examining the two factors that affect the optimal pricing policy. In the general case, the decline of the marginal cost encourages the firm to reduce its price. In the special case that we discussed in this section, marginal cost remains constant. Thus, changes of the optimal price over time merely reflect the changes in the incentive for the firm to produce beyond the short-run profit-maximization output. Because we proved that this incentive (denoted by \( \hat{\lambda} \) declines over time, the firm will find it less and less attractive to produce beyond the short-run profit-maximization output. Therefore, the optimal output declines over time, which results in a price increase.

**6. Concluding Comments**

The standard analysis of the monopolist's optimal pricing policy in the presence of experience is based on the assumption of nonstorable output. On the other hand, the traditional justifications for holding inventory are the existence of a random demand function that inventories help and the desire to speculate on future price changes. In this paper we show that the learning effect can provide us with another justification for holding inventory. We proved that in order to gain more experience, the firm may increase its output and store the surplus in order to sell it in future periods. Therefore, the existence of learning effect can turn inventory policy into a strategic variable. (Because the possibility of holding inventory can not be ignored, we claim in this work that the analysis of the optimal pricing policy should be discussed in a broad framework in which it is possible for the firm to hold inventory). We also showed that when the possibility of holding inventory is ignored, the changes of the optimal price over time cannot be specified a priori, whereas when a positive level of inventory is held (even though production costs decline due to LBD), the optimal price increases, as long as that positive inventory level is maintained.
References


