The Dirac Monopole Theory

Monopoles are defined by the following duality transformation (called also duality rotation by $\pi/2$)

$$\mathbf{E} \to \mathbf{B}, \quad \mathbf{B} \to -\mathbf{E} \tag{1}$$

and

$$e \to g, \quad g \to -e, \tag{2}$$

where g denotes the magnetic charge of monopoles.

A theory of monopoles was published by Dirac in the first half of the previous century[2,3]. At present, there is no established experimental evidence of these monopoles[4]. This experimental status of monopoles led Dirac later in his life to state: "I am inclined now to believe that monopoles do not exist. So many years have gone by without any encouragement from the experimental side"[5].

Here the following question arises: Does the failure of the monopole quest stem from the fact that they do not exist in Nature or from erroneous elements in Dirac's monopole theory? It is shown here that the second possibility holds.

Let us examine the established part of electrodynamics. Here the system consists of electric charges carried by matter particles and electromagnetic fields. The equations of motion of the fields are Maxwell equations

$$F^{\mu\nu}_{(e)\ ,\nu} = -4\pi j^{\mu}_{(e)} \tag{3}$$

and

$$F^{*\mu\nu}_{(e)\ ,\nu} = 0, \tag{4}$$

and the 4-force exerted on charged matter is given by the Lorentz law

$$m a^{\mu}_{(e)} = e F^{\mu\nu}_{(e)} v_{\nu}. \tag{5}$$

Here $F^{\mu\nu}$ is the antisymmetric tensor of the electromagnetic fields, $F^{*\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, $\varepsilon^{\mu\nu\alpha\beta}$ is the completely antisymmetric unit tensor of the fourth rank and j^μ is the electric 4-current. Subscripts ${}_{(e)}$, ${}_{(m)}$ denote quantities related to charges and monopoles, respectively. The duality transformation of fields (1) can be written in a tensorial form $F^{\mu\nu} \rightarrow F^{*\mu\nu}$.

An important quantity is the electromagnetic 4-potential A_μ. This quantity is used in the Lagrangian density of the system. The fields' part of the Lagrangian density is (see [6], p. 71; [7], p. 596)

$$L_{fields} = -\frac{1}{16\pi}F^{\mu\nu}_{(e)}F_{(e)\mu\nu} - j^{\mu}_{(e)}A_{(e)\mu}. \tag{6}$$

Using the duality transformation (1), (2) and Maxwellian electrodynamics (3)-(6), one derives a dual Maxwellian theory for a system of monopoles and electromagnetic fields (namely, a system without charges)

$$F^{*\mu\nu}_{(m)\ ,\nu} = -4\pi j^{\mu}_{(m)}, \tag{7}$$

$$-F^{\mu\nu}_{(m)\ ,\nu} = 0, \tag{8}$$

$$ma^{\mu}_{(m)} = gF^{*\mu\nu}_{(m)}v_\nu, \tag{9}$$

and

$$L_{fields} = -\frac{1}{16\pi}F^{*\mu\nu}_{(m)}F^{*}_{(m)\mu\nu} - j^{\mu}_{(m)}A_{(m)\mu}. \tag{10}$$

At this point we have two theories: the ordinary Maxwellian electrodynamics whose domain of validity does not contain magnetic monopoles and a monopole related Maxwellian theory which does not contain electric charges. The problem is to determine the form of a covering theory of a system of charges, monopoles and their fields.

As explained in the introductory part of this site, the two subtheories mentioned above impose constraints on the required charge-monopole theory:

1. It should conform to Maxwellian electrodynamics (3)-(6) in the limit where monopoles do not exist.

2. It should conform to the dual Maxwellian electrodynamics (7)-(10) in the limit where charges do not exist.

It turns out that Dirac's monopole theory is inconsistent with requirement 2. Therefore, it is inconsistent with a restriction imposed by a lower rank theory.

As a matter of fact, Dirac also uses implicitly a new axiom which has no experimental support. Thus, his theory assumes that:

A. Electromagnetic fields of charges and electromagnetic fields of monopoles have identical dynamical properties.

This approach forces him to use just one kind of 4-potential A_μ and to confront a new kind of singularity. Indeed, if the 3-vector $\mathbf{A}$ is regular then

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0 \tag{11}$$

and monopoles do not exist. Dirac uses the term 'string' for this kind of singularity. The utilization of the new axiom A, and the introduction of a new kind of singularity into electrodynamics indicate a departure from simplicity.

Several additional errors of the Dirac monopole theory have been pointed out a long time ago. Thus, it was claimed that the Dirac monopole theory is inconsistent with the S-matrix theory (see [8,9]). A third article [10] claims that the inclusion of the Dirac monopole in electrodynamics is inconsistent with relativistic covariance. Another kind of error of the Dirac monopole theory was published recently [11]. It is shown there that a hypothetical quantum mechanical system that contains a charge and a Dirac monopole violates energy conservation (see [11] pp. 98-99).

Another problem is the definition of the interaction part of the angular momentum in a system containing an electric charge and a Dirac monopole. Here one finds that the interaction part of the fields' angular momentum *does not vanish for cases where the distance between the two particles tends to infinity* (see [7] p. 256; [11], pp. 97-98; [12] p. 1366). An interaction of this kind is unknown in classical electrodynamics and is regarded as unphysical.

The discussion carried out here shows several theoretical errors and a deviation from simplicity by using an additional axiom and unphysical properties of the Dirac monopole theory. These difficulties are completely consistent with the failure of the experimental efforts aiming to detect Dirac monopoles. It is interesting to note that a regular and self-consistent charge-monopole theory can be constructed without using axiom A [11,13,14]. This theory derives a different set of equations of motion. The failure of the attempts to detect the Dirac monopoles is predicted in [8] and it is derived from the equations of motion of the regular monopole theory [15] as well.

References:

[2] P. A. M. Dirac, Proc. Royal Soc. **A133**, 60 (1931).

[3] P. A. M. Dirac, Phys. Rev., **74**, 817 (1948).

[4] S. Eidelman et al. (Particle Data Group), Phys. Lett. **B592**, 1 (2004).

[5] P. A. M. Dirac, A letter to A. Salam, published in *Monopoles in Quantum Field Theory.* Ed. N. S. Craigie, P. Goddard and W. Nahm (World Scientific, Singapore, 1982).

[6] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1975).

[7] J. D. Jackson, *Classical Electrodynamics* (John Wiley, New York, 1975).

[8] D. Zwanziger, Phys. Rev., **B137**, 647 (1965).

[9] S. Weinberg, Phys. Rev., **B138**, 988 (1965).

[10]] C. R. Hagen, Phys. Rev., **B140**, 804 (1965).

[11] E. Comay, published in *Has the Last Word Been Said on Classical Electrodynamics?* Editors: A Chubykalo, V Onoochin, A Espinoza, and R Smirnov-Rueda (Rinton Press, Paramus, NJ, 2004). (The Article's title is "A Regular Theory of Magnetic Monopoles and Its Implications".)

[12] P. Goddard and D. I. Olive, Rep. Prog. Phys., **41**, 1357 (1978).

[13] E. Comay, Nuovo Cimento, **80B**, 159 (1984).

[14] E. Comay, Nuovo Cimento, **110B**, 1347 (1995).

[15] E. Comay, Lett. Nuovo Cimento **43**, 150 (1985).