Diffraction Free Beams

The idea that diffraction-free beams (also called propagation invariant beams) exist has been published in the literature [40]. The spatial part of such a beam is assumed to take the form (see [40], eq. (2))

$$\phi = e^{i\beta z} J_0(a\rho), \tag{1}$$

where ρ denotes the radius in cylindrical coordinates, J_0 is the zeroth order Bessel function of the first kind and a is a factor having the dimension $[L^{-1}]$. Article [40] has inspired a lot of activity and it has been cited more than 400 times. Following [40], a family of diffraction-free solutions of Maxwell equations has been published [41].

Taking the diffraction-free idea literally, one obviously realizes that it is an error, because it is inconsistent with the uncertainty principle. Indeed, the notion of a beam describes a set of physical objects moving in a specific direction and the linear dimension of the relevant cross section containing these objects is much smaller than the beam's length (see [40], p. 1499, near the bottom of the left column).

The ratio between the length and the diameter of the beam indicates that it may be evaluated at the wave zone. It is easy to realize that a Bessel beam like (1) cannot exist [42]. Indeed, let us examine a circle C at the wave zone having a diameter which equals that of the assumed beam (see fig. 1).

At the source, the beam's amplitude is a Bessel function, which means that it changes sign alternately. It follows that it interferes *destructively* at C. Hence, since energy is conserved in the process, one concludes that a part of the beam does not pass through C. This conclusion means that the beam is *not* diffraction free. Moreover, a Bessel beam spreads *faster* than a uniform beam because, at circle C, interference of the latter is constructive.

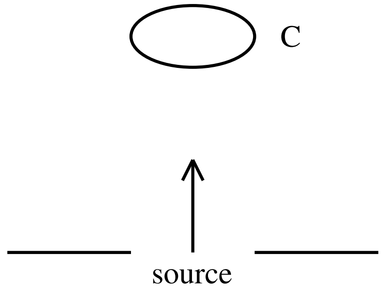

Figure 1: A beam of electromagnetic wave is emitted from a circular source S. The beam's intensity is calculated at a circle C whose radius is the same as that of the source. (This figure was published in [41]).

Using this result, one infers that the family of diffraction-free solutions of Maxwell equations [41] describe solutions of electromagnetic waves inside a perfect cylindrical wave guide.

Moreover, most (if not all) experiments that follow [40] use a φ-invariant setup and show a strong peak at the center. Now, the φ-invariant solutions of Maxwell equations [41] are derived from the following vector potential

$$\mathbf{A} = -iJ_1(ar)e^{i(bz-\omega t)}\mathbf{u}_\varphi. \tag{2}$$

The fields are

$$\mathbf{E} = -\partial\mathbf{A}/\partial t = \omega J_1(ar)e^{i(bz-\omega t)}\mathbf{u}_\varphi \tag{3}$$

and

$$\mathbf{B} = curl\mathbf{A} = -bJ_1(ar)e^{i(bz-\omega t)}\mathbf{u}_r - iaJ_0(ar)e^{i(bz-\omega t)}\mathbf{u}_z. \tag{4}$$

There is a dual solution where $\mathbf{E} \to \mathbf{B},\ \mathbf{B} \to -\mathbf{E}$. Now, the Bessel function $J_1(0) = 0$, which means that at the beam's center, the energy current $\mathbf{E} \times \mathbf{B}/4\pi$ of these solutions has a *minimum*. This prediction contradicts the data and provides another proof of the claim that the experiments should not be described as a superposition of diffraction free beams. A more detailed discussion of these topics can be found in

[42].

References:

[40] J. Durnin, J. J. Miceli, Jr. and J. H. Eberly, Phys. Rev. Lett. **58**, 1499 (1987).

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