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# The Special Theory of Relativity is an Excellent Theory

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## Abstract

Criteria for defining errors of a physical theory are formulated. It is shown that the Special Theory of Relativity (STR) has a solid mathematical basis. An enormous amount of experiments carried out in particle physics use beams of particles having a very high energy. The data of these experiments are consistent with STR and support our confidence that STR is an excellent theory. Several specific cases of this issue are discussed explicitly. Contrary to a common belief, it is proved that the contemporary mainstream of physicists adhere to some theoretical ideas that violate STR.

## 1. Introduction

The validity of physical theories should be tested time and again. Such a practice enables the increase of our confidence in good theories and the removal of erroneous ones. In order to carry out this task, one needs to define the structure of physical theories and their interrelations. Criteria for errors in physical theories can be created on this basis. This work presents the fundamental elements of the Special Theory of Relativity (STR) and explains why it should be regarded as a self-consistent and excellent theory. STR is used in classical physics and in quantum physics as well. The main part of the discussion carried out in this work is restricted to the validity domain of classical physics.

The second Section discusses the general structure of physical theories and defines criteria for a rejection

of a theory because of its erroneous properties. The third Section presents fundamental elements of STR pertaining to mechanics and to electrodynamics. The fourth Section examines some peculiar (and counterintuitive) predictions of STR and shows that these predictions are consistent with experimental data. Several examples proving that some widely accepted contemporary physical theories are inconsistent with STR, are discussed in the fifth Section. The last Section contains concluding remarks.

In this work, Greek indices run from 0 to 3 and Latin indices run from 1 to 3. Units where  $\hbar = c = 1$  are used. In this unit system, the celebrated relativistic formula  $E = mc^2$  reduces to  $E = m$ . For these reasons, the symbol  $c$  is removed in many cases and the symbol  $m$  denotes not the dynamic mass but the particle's mass in its instantaneous rest frame. The relativistic factor  $\gamma = (1 - v^2)^{-1/2}$ . The symbol  $_{,\mu}$  denotes the

partial differentiation with respect to  $x^\mu$ .

## 2. The Structure of Physical Theories

A physical theory resembles a mathematical theory. Both rely on a set of axioms and employ a deductive procedure for yielding theorems, corollaries, etc. The set of axioms and their results are regarded as elements of the structure of the theory. However, unlike a mathematical theory, a physical theory is required to explain existing experimental data and to predict results of new experiments.

This distinction between a mathematical theory and a physical theory has several aspects. First, experiments generally do not yield precise values but contain estimates of the associated errors. (Some quantum mechanical data, like spin, are the exception.) It follows that in many cases, a certain numerical difference between theoretical predictions and experimental data is quite acceptable.

Next, one does not expect that a physical theory should explain every phenomenon. For example, it is well known that physical theories yield very good predictions for the motion of planets around the sun. On the other hand, nobody expects that a physical theory be able to predict the specific motion of an eagle flying in the sky. This simple example proves that the validity of a physical theory should be evaluated only with respect to a limited set of experiments. The set of experiments which are relevant to a physical theory is called its domain of validity. (A good discussion of this issue can be found in [1], pp. 1-6.)

Relations between two physical theories can be deduced from an examination of their domain of validity. In particular, let  $D_A$  and  $D_B$  denote the domains of validity of theories  $A$  and  $B$ , respectively. Now, if  $D_A$  is a subset of  $D_B$  then one finds that the rank of theory  $B$  is higher than that of theory  $A$  (see [1], pp. 3-6). Hence, theory  $B$  is regarded as a theory having a more profound status. However, theory  $A$  is not “wrong”, because it yields good predictions for experiments belonging to its own (smaller) domain of validity. Generally, theory  $A$  takes a simpler mathematical form. Hence, wherever possible, it is used in actual calculations. Moreover, since theory  $A$  is good in its validity domain  $D_A$  and  $D_A$  is a part of  $D_B$  then one finds that *theory  $A$  imposes constraints on theory  $B$ , in spite of the fact that  $B$ ’s rank is higher than  $A$ ’s rank*. This self-evident relation between lower rank and higher rank theories is called here “restrictions imposed by a lower rank theory”. Thus, for example, although Newtonian mechanics is good only for cases where the velocity  $v$  satisfies  $v/c \rightarrow 0$ , relativistic mechanics should yield formulas which agree with corresponding formulas of Newtonian mechanics, provided  $v$  is small enough. As is very well known, STR satisfies this requirement.

Having these ideas in mind, a theoretical error is regarded here as a mathematical part of a theory that yields predictions which are clearly inconsistent with experimental results, where the latter are carried out within the theory’s validity domain. The direct meaning of this definition is obvious. It has, however, an indirect aspect too. Assume that a given theory has a certain part,  $P$ , which is regarded as well established. Thus, let  $Q$  denote another set of axioms and formulas which hold in (at least a part of)  $P$ ’s domain of validity. Now, assume that  $Q$  yields predictions that are inconsistent with those of  $P$  and the inconsistency holds in the common part of their domains of validity. In such a case,  $Q$  is regarded as a theoretical error. (Note that, as explained above,  $P$  may belong to a lower rank theory.) An error in the latter sense is analogous to an error in mathematics, where two elements of a theory are inconsistent with each other.

There are other aspects of a physical theory which have a certain value but are not well defined. These may be described as neatness, simplicity and physical acceptability of the theory. A general rule considers theory  $C$  as simpler (or neater) than theory  $D$  if theory  $C$  relies on a smaller number of axioms. These properties of a physical theory are relevant to a theory whose status is still undetermined because there is a lack of experimental data required for its acceptance or rejection.

The notions of neatness, simplicity and physical acceptability have a subjective nature and so it is unclear how disagreements based on them can be settled. In particular, one should note that ideas concerning physical acceptability changed dramatically during the 20th century. Thus, a 19th century physicist would have regarded many well established elements of contemporary physics as unphysical. An incomplete list of such elements contains the relativity of length and time intervals, the non-Euclidean structure of space-time, the corpuscular-wave nature of pointlike particles, parity violation and the nonlocal nature of quantum mechanics (which is manifested by the EPR effect).

For these reasons neatness, simplicity and physical acceptability of a theory have a secondary value. Thus, if there is no further evidence, then these aspects should not be used for taking a *final decision* concerning the acceptability of a physical theory.

Before concluding these introductory remarks, it should be stated that the erroneous nature of a physical theory  $E$  cannot be established merely by showing the existence of a different (or even a contradictory) theory  $F$ . This point is obvious. Indeed, if such a situation exists then one may conclude that either of the following relations holds: the two theories agree/disagree on predictions of experimental results belonging to a common domain of validity. If the theories agree on all predictions of experimental results then they are just two different mathematical formu-

lations of the *same* theory. (The Heisenberg and the Schroedinger pictures of quantum mechanics are an example of this case.) If the theories disagree then (at least) theory *E* or theory *F* is wrong. However, assuming that neither *E* nor *F* relies on a mathematical error, then one cannot decide on the issue without having an adequate amount of experimental data.

Another issue is the usage of models and phenomenological formulas. This approach is very common in cases where there is no established theory or where theoretical formulas are too complicated. A model is evaluated by its usefulness and not by its theoretical correctness. Hence, models apparently do not belong to the subject of this compilation of Articles.

### 3. The Mathematical Structure of the Special Theory of Relativity

Within the scope of this work, one certainly cannot write a comprehensive presentation of STR. As a matter of fact, there is no need for doing that, because there are many good textbooks on this subject. References [2,3] as well as many other textbooks may be used by readers who are still unacquainted with STR. Hence, fundamental elements of the mathematical structure of STR are presented here without a thorough pedagogical explanation. STR is based on 2 postulates:

1. The laws of mechanics and of electrodynamics take the same form in all inertial frames.
2. The speed of light in vacuum takes the same value  $c$  in all inertial frames (and it is independent of the velocity of the source).

The theory derived from these postulates can be formulated by using tensor calculus within Minkowski space of 4 dimensions. Three equivalent forms of this space can be found in the literature. In these forms the metric tensor (denoted by  $g_{\mu\nu}$ ) is diagonal and contains the numbers  $\pm 1$ . The signature of the three forms takes the values 4, 2 and  $-2$ , respectively. In the signature 4, the metric is the unit tensor and calculations use complex numbers. The metric used here is (1,-1,-1,-1). Apparently, this is the most popular metric used by modern textbooks.

The differential of the interval  $ds$  is obtained from  $ds^2 = dt^2 - dx^2$ . Lorentz transformations are second rank tensors  $L^\mu_\nu$  that conserve the length of the interval. They are used for transforming quantities from one inertial frame to another. Lorentz transformations form a group. A subgroup of this group is the group of rotations in the ordinary 3-dimensional space. The Poincare group is the group that contains the Lorentz group and the group of space-time translations.

There are some important physical quantities which

are invariant under Lorentz transformations (these invariants are also called Lorentz scalars). These invariants are the interval; the following relation of energy and momentum components of a closed system  $E^2 - P^2$ ;  $B^2 - E^2$  and  $\mathbf{E} \cdot \mathbf{B}$  of the electromagnetic fields. The electric charge is a Lorentz scalar too.

Some other physical quantities are entries of first rank tensors (also called 4-vectors). Thus, space-time coordinates are entries of a 4-vector denoted by  $x^\mu$ . For coordinates of the path of a moving massive particle, the square of the interval  $ds^2 = dt^2 - dx^2 > 0$ . Hence, the 4-velocity of a massive particle  $v^\mu \equiv dx^\mu/ds = \gamma(1, \mathbf{v})$  is a well defined 4-vector. Similarly, the 4-acceleration is defined as follows  $a^\mu \equiv dv^\mu/ds$ . Energy and momentum of a closed system are entries of the 4-vector  $P^\mu \equiv (E, \mathbf{P})$ . The scalar and vector potentials of electrodynamics are entries of the 4-vector  $A^\mu \equiv (\Phi, \mathbf{A})$ . The 4-current is another 4-vector. Here  $j^\mu \equiv (\rho, \rho\mathbf{v})$ , where  $\rho$  denotes charge density. This 4-current satisfies the continuity equation  $j^\mu_{,\mu} = 0$ , which proves charge conservation. The 4-current can be written in a different notation, where  $\rho$  denotes probability density and all entries of the 4-vector are multiplied by the electric charge  $e$ . An analogous 4-vector is the mass current where the rest mass  $m$  (which is a Lorentz scalar!) replaces the electric charge.

Electromagnetic fields are components of a second rank antisymmetric tensor which is the 4-curl of  $A_\mu$ . Thus  $F_{\mu\nu} \equiv A_{\nu,\mu} - A_{\mu,\nu}$ . Energy and momentum densities as well as energy and momentum currents are entries of a second rank symmetric tensor  $T^{\mu\nu}$ . This tensor is called the energy-momentum tensor (or the stress energy tensor). Thus,  $T^{00}$  is the energy density and  $T^{i0}$  are densities of momentum components.

The density of angular momentum components are entries of a third rank tensor  $S^{\lambda\mu\nu} \equiv x^\lambda T^{\mu\nu} - x^\mu T^{\lambda\nu}$ .

It is interesting to note that Maxwellian electrodynamics predicts the existence of transverse electromagnetic waves that satisfy the following equation

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0 \quad (1)$$

and a similar equation for the components of the magnetic field. In the vacuum, these waves travel in the speed of light. Moreover, since Maxwell's wave equation is independent of quantities of the inertial frame where the fields are measured (and of the velocity of the source of the fields as well), one concludes that Maxwellian fields travel in the speed of light  $c$  in all frames. This conclusion agrees completely with postulate 2 of STR.

The mathematical structure of Minkowski space is known to be self-consistent. Moreover, as stated above, STR agrees with Newtonian mechanics in cases where  $v/c \rightarrow 0$ . Thus, the mathematical aspect of STR is flawless and its validity should be examined by means of a comparison of its predictions with well established experimental data.

## 4. Experimental Data and Special Relativity

As explained in Section 2, the acceptability of STR should be examined within its validity domain. Thus experiments where effects of gravitational field or of noninertial frames can be ignored are examined. Hence, terrestrial experiments of strong, electromagnetic and weak interactions belong to the validity domain of STR. This section discusses several results of STR, some of which may look strange to everybody who follows his intuition (which has been developed on the basis of life experience in a macroscopic world and where  $v/c \ll 1$ ).

1. It is proved in STR that the speed of light is an upper bound for the velocity of massive particles  $v < c$ . This property is verified in many experiments. Take for example the CERN's LEP accelerator where beams of electrons and positrons are accelerated to a very high kinetic energy. The beams collide and their center of mass energy exceeds 200 GeV [4]. Thus, electrons and positrons of the beams have kinetic energy which is more than 200000 times  $mc^2$ . In spite of this gigantic kinetic energy, particles do not move faster than light.

Another kind of information are the neutrinos measured from the 1987A supernova. This supernova exploded about 164000 years ago (data taken from the Internet site of Wikipedia). Thus the number of seconds elapsed is about  $5 \cdot 10^{12}$ . On earth, the neutrino burst lasted about 13 seconds. A variation in the energy of these neutrinos is expected to hold, due to Doppler shift and other reasons. According to recent experimental measurements, neutrinos are massive particles (see [5], pp. 451-467). Therefore, one may conclude that the variation in speed of these very high energy particles is less than  $10^{-11}$  of their mean speed. This conclusion is consistent with STR. Indeed, in STR the speed of all very high energy massive particles is  $c(1 - \varepsilon)$ , where  $\varepsilon$  is a very small positive number.

2. The equivalence of mass and energy is another result of STR. This conclusion is seen in many experiments of particle physics. Thus, the positronium is a bound state of an electron and a positron. These particles annihilate each other and two or three photons are emitted. Photons are massless particles found in electromagnetic radiation. Hence, they are a form of energy (which can be converted into heat, etc.). Similarly, the particle  $\pi^0$  disintegrates into 2 photons. Another experimental example of the equivalence of mass and energy is the heat released from a fission of heavy nuclei like  $^{235}\text{U}$  and  $^{239}\text{Pu}$ . Here the sum of the masses of the nuclei produced by fission is smaller

than that of the original nucleus. The difference between the masses appears as a kinetic energy which is eventually converted into heat.

Processes taking the opposite direction are seen too. Thus, photons having energy greater than 1 MeV are absorbed by matter in a process called pair production, where an electron and a positron are created [6]. In higher energy processes, meson production [7] (namely a  $\bar{q}q$  bound state) is observed. In even higher energy, a pair of proton-antiproton are produced [8].

3. The Lorentz contraction of length is another result of STR. Thus, a rod of length  $l$  looks shorter, if it is measured in an inertial frame  $\Sigma$  where it moves in a direction which is not perpendicular to its length. Lorentz contraction is seen in an examination of  $\mu$  mesons having a very high energy. The half-life time of these particles is about  $2.2 \cdot 10^{-6}$  seconds. This time interval should be measured in the particle's rest frame  $\Sigma'$ . Hence, if Lorentz contraction does not hold, then after moving 4000 meters, their number should be about 1.5% of their original number. After passing 10000 meters, the number should be less than  $10^{-4}$  of the original number. Now, many  $\mu$  mesons are produced at the upper part of the atmosphere as a result of interactions initiated by a very energetic cosmic ray and a considerable part of these particles reach sea level. This effect is explained by measuring the time (and the half-life time) in the particle's rest frame  $\Sigma'$  and by the Lorentz contraction of the distance between the upper part of the atmosphere and sea level, which holds in  $\Sigma'$ .

This effect can also be seen in a  $\mu$  meson machine where processes are under control [9]. Here high energy  $\mu$  mesons move in a storage ring. Lorentz contraction of length in the  $\mu$  meson's instantaneous rest frame is seen as a time dilation in the laboratory frame. Thus, in this specific case, the time dilation factor is about 30. This outcome is a very convincing argument supporting the Lorentz contraction of length.

4. Landau and Lifshitz use STR and prove that an elementary classical particle must be pointlike (see [2], pp. 43-44). This result is supported by quantum mechanics and by quantum field theory. Indeed, in these theories the wave function/field function  $\psi(x^\mu)$  depends on a *single* set of space-time coordinates  $x^\mu$ . Hence, these functions describe pointlike particles. Experimental results of the elementary Dirac particles: electrons,  $\mu$  mesons and  $u$ ,  $d$  quarks are consistent with this property. This conclusion is inferred from the experimental support of the Bjorken scaling in very high energy scattering [10].

The foregoing examples show several kinds of experimental data, all of which are predicted by STR. In addition to these examples, it can also be stated that an enormous number of experiments in high energy physics have been carried out during the last 50 years. These experiments are designed, constructed and analyzed in accordance with the laws of STR. Therefore, beside yielding specific results, these experiments provide a solid basis for our confidence that STR is an excellent theory.

## 5. Violations of the Special Theory of Relativity by Contemporary Theoretical Ideas

This Section shows three examples where theoretical ideas adopted by the mainstream of contemporary physics are inconsistent with STR.

1. The data of high energy photons interacting with nucleons show that in this case, protons and neutrons are very much alike [7]. These data cannot be explained by an analysis of the photon interaction with the electric charge of nucleon constituents. Thus, an idea called Vector Meson Dominance (VMD) has been suggested for this purpose.

The main point of VMD is that the wave function of an energetic photon takes the form

$$|\gamma\rangle = c_0 |\gamma_0\rangle + c_h |h\rangle \quad (2)$$

where  $|\gamma\rangle$  denotes the wave function of a physical photon,  $|\gamma_0\rangle$  denotes the pure electromagnetic component of a physical photon and  $|h\rangle$  denotes its hypothetical hadronic component.  $c_0$  and  $c_h$  are appropriate numerical coefficients *whose values depend on the photon's energy* [7,11]. Thus, for soft photons  $c_h = 0$  whereas it begins to take a nonvanishing value for photons whose energy is not much less than the  $\rho$  meson's mass.

The fact that the Standard Model has no other explanation for the hard photon-nucleon interaction is probably the reason for the survival of VMD. An analysis published recently proves that VMD is inconsistent with many well established elements of physical theories [12]. In particular, VMD is inconsistent with Wigner's analysis of the Poincare group [13,14]. This outcome proves that VMD violates STR.

This conclusion can also be proved by the following specific example. Consider the experiment described in figure 1. In the laboratory frame  $\Sigma$  of

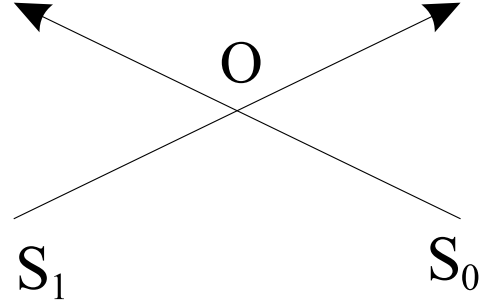


Fig. 1. Two rays of light are emitted from sources  $S_1$  and  $S_2$  which are located at  $x = \pm 1$ , respectively. The rays intersect at point  $O$  which is embedded in the  $(x, y)$  plane. (This figure is published in [12] and is used here with permission.)

fig. 1, the optical photons of the rays do not interact. Thus, neither energy nor momentum are exchanged between the rays. Therefore, after passing through  $O$ , the photons travel in their original direction. Let us examine the situation in a frame  $\Sigma'$ . In  $\Sigma$ , frame  $\Sigma'$  is seen moving very fast in the negative direction of the  $Y$  axis. Thus, in  $\Sigma'$ , photons of the two rays are very energetic. Hence, if VMD holds then photons of both rays contain hadrons and should exchange energy and momentum at point  $O$ . This is a contradiction because if the rays do not exchange energy and momentum in frame  $\Sigma$  then they obviously do not do that in any other frame of reference. Thus, this simple example proves that VMD violates STR.

2. The Yukawa interaction is derived from the interaction term of a Dirac spinor  $\psi(x^\mu)$  with a Klein-Gordon (KG) particle  $\phi(x^\mu)$  (see [15], p.79 and [16], p. 135)

$$L_{Yukawa} = L_{Dirac} + L_{KG} - g\bar{\psi}\psi\phi. \quad (3)$$

Here the KG particle plays a role which is analogous to that of the photon in electrodynamics. The following argument proves that a Lorentz scalar (like the KG particle) cannot be used as a basis for a field of force.

Consider the following Lorentz scalar  $v^\mu v_\mu$ . As a scalar, it takes a fixed value in all inertial frames. (In the units used here its value is unity.) Differentiating this expression with respect to the interval, one finds

$$\frac{d(v^\mu v_\mu)}{ds} = 2v^\mu a_\mu = 0. \quad (4)$$

This relation means that in STR the 4-velocity is orthogonal to the 4-acceleration.

Let an elementary classical particle  $W$  move in a field of force. The field quantities are independent of the 4-velocity of  $W$  but the associated 4-force

must be orthogonal to it. In electrodynamics this goal is attained by means of the Lorentz force. In this case, one finds

$$a^\mu v_\mu = \frac{e}{m} F^{\mu\nu} v_\nu v_\mu = 0, \quad (5)$$

where the null result is obtained from the antisymmetry of  $F^{\mu\nu}$  and the symmetry of the product  $v_\mu v_\nu$ . In electrodynamics, the antisymmetric field tensor  $F^{\mu\nu}$  is constructed as the 4-curl of the 4-potential  $A_\mu$ . Such a field of force cannot be obtained from the *scalar* KG field. Now, the notion of force holds in classical physics. Hence, the classical limit of the Yukawa interaction is inconsistent with STR.

3. Following historical ideas,  $\pi$  mesons are regarded as KG particles (see [15], pp. 79, 122). This is certainly wrong because it has recently been proved that the KG equation is inconsistent with well established physical theories [17,18]. This conclusion is in accordance with Dirac's negative opinion on the KG equation [19,20].

This matter has also an indirect aspect pertaining to STR. Indeed, as shown in point 4 of Section 4, STR proves that a truly elementary classical particle should be pointlike. This result is also obtained from the quantum mechanical wave function  $\Psi(x^\mu)$  which depends on a *single* set of space-time coordinates. Now, the KG equation, is supposed to be a quantum mechanical equation. As such, it must describe pointlike particles. On the other hand, it is now recognized that  $\pi$  mesons are not pointlike and that their size is not much smaller than the size of the proton (see [5], pp. 499, 854.). Therefore the usage of  $\pi$  mesons as KG particles violates STR indirectly.

solid basis for our confidence that STR is an excellent theory.

The discussion carried out above concentrates on phenomena belonging to classical physics. It should be noted that the Dirac equation is a relativistic quantum mechanical equation. It predicts correctly the spin of the electron and the existence of antiparticles. It yields very good predictions for the energy levels of the hydrogen atom and for the electron's g-factor. Corrections to these values are obtained from quantum field theory, which is a higher relativistic theory.

It is also proved that, contrary to a common belief, some theoretical ideas, adopted by the mainstream of contemporary physicists, violate STR. These ideas are VMD, the Yukawa theory of a field of force carried by a scalar meson and the idea that  $\pi$  mesons are Klein-Gordon particles.

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## 6. Concluding Remarks

The notion of a theoretical error is defined. It is explained that STR has a solid mathematical basis. The fact that its formulas agree with Newtonian mechanics in cases where  $v/c \rightarrow 0$  proves that it satisfies restrictions imposed by a lower rank theory. Next, it is shown that some peculiar predictions of STR are confirmed by experiments. The predictions discussed here are the relation  $v < c$  where  $v$  denotes the velocity of a massive particle; the equivalence of mass and energy; the Lorentz contraction; and the pointlike nature of elementary particles. The enormous number of experiments carried out in particle physics use particles whose velocity is in the relativistic domain where  $0 < 1 - v/c \ll 1$ . The design, construction and analysis of these experiments abide by the laws of STR. The data obtained are compatible with STR and provide a

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