The Yukawa Interaction

The Yukawa interaction is derived from a Lagrangian density containing an interaction term of a Dirac spinor with a KG particle (see [24], p.79 and [25], p. 135)

$$L_{Yukawa} = L_{Dirac} + L_{KG} - g\bar{\psi}\psi\phi. \tag{1}$$

Here the KG particle plays a role which is analogous to that of the photon in electrodynamics. The dependence of (1) on the KG Lagrangian density indicates that it suffers from all the difficulties of the KG theory which are pointed out in the appropriate part of this site. Furthermore, note that, due to the fact that all terms of the Lagrangian density are Lorentz scalars, the interaction term of (1) depends on the Dirac particle's *scalar density* $\bar{\psi}\psi$ which is *not* its actual density $\psi^\dagger\psi$. This situation is very strange because one expects that the intensity of the interaction of a Dirac particle should depend on its actual density $\psi^\dagger\psi$ which is a component of the Dirac 4-current and *not* on the scalar density $\bar{\psi}\psi$. Moreover, it is explained below that (1) is not free of covariance problems.

An analysis of the nonrelativistic limit of two Dirac particles interacting by means of a Yukawa field, yields the following expression for the interaction term (see [26], p. 211)

$$V(r) = -g^2\frac{e^{-\mu r}}{r}, \tag{2}$$

where μ denotes the mass of the KG particle. The Yukawa theory was suggested as a theoretical interpretation of the nucleon-nucleon interaction. This idea was proposed in the early days of nuclear theory when nucleons were regarded as elementary Dirac particles. Now it is known that nucleons are composite particles containing quarks and therefore this application of (1) is deprived of its theoretical basis. Furthermore, a recent discussion proves that the classical limit of the interaction (2) is inconsistent with special relativity (see [22], p. 13). This argument relies on the relativistic relation between the 4-velocity and the 4-acceleration

$$a^\mu v_\mu = 0. \tag{3}$$

Examining an elementary classical particle, one finds that relation (3) yields the following relation for the 4-force $f^\mu v_\mu = 0$. It is explained below why this relation is inconsistent with the Yukawa interaction (2).

Let an elementary classical particle W move in a field of force. The field quantities are independent of the 4-velocity v^μ of W but the associated 4-force must be orthogonal to v^μ. In electrodynamics this goal is attained by means of the Lorentz force

$$ma^\mu_{(e)} = eF^{\mu\nu}_{(e)}v_\nu. \tag{4}$$

In this case, one finds

$$a^\mu v_\mu = \frac{e}{m}F^{\mu\nu}v_\mu v_\nu = 0, \tag{5}$$

where the null result is obtained from the antisymmetry of $F^{\mu\nu}$ and the symmetry of the product $v_\mu v_\nu$. In electrodynamics, the antisymmetric field tensor $F^{\mu\nu}$ is constructed as the 4-curl of the 4-potential A_μ. Such a field of force cannot be obtained from the *scalar* KG field. Hence, the classical limit of the Yukawa interaction is inconsistent with special relativity.

Considering the experimental side, the application of the Yukawa theory to nuclear interactions cannot be regarded as a success. The nuclear force is characterized by a very hard (repulsive) core and a rapidly decreasing attractive force outside this core. Therefore, at a certain point of r, the nuclear potential *changes sign* (see [27], p. 97). The Yukawa formula (2) does not change sign. Hence, it is inconsistent with this property. The nuclear force has also a tensorial component as well as a spin-orbit dependence (see [27], pp. 68-78). Today people use phenomenological formulas for a description of the nucleon-nucleon interaction data (see [27], pp. 97-99).

References:

[22] E. Comay, Apeiron, **11**, No. 3, 1 (2004).

[24] M. E. Peshkin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Mass., 1995).

[25] G. Sterman *An Introduction to Quantum Field Theory* (University Press, Cambridge, 1993).

[26] J. D. Bjorken and S. D. Drell *Relativistic Quantum Mechanics* (Mc-Graw, New York, 1964).

[27] S. S. M. Wong, *Introductory Nuclear Physics* (Wiley, New York, 1998). 2nd edition.